Interference-Free Energy Efficient Scheduling in Wireless Ad Hoc Networks[1]

Liron Levin[a], Michael Segal[a], Hanan Shpungin[b]

[a] *Department of Communication Systems Engineering, Ben-Gurion University of the Negev, Beer-Sheva, 84105, Israel*
[b] *Deptartment of Electrical and Computer Engineering, University of Waterloo, Waterloo, ON, N2L 3G1, Canada*

Abstract

This paper studies the problem of interference-free broadcast in wireless ad hoc networks. In particular, we are interested in asymmetric power assignments so that the induced broadcast communication graph is both, energy efficient and has a short collision-free broadcast schedule. We consider both random and deterministic node layouts and develop four different broadcast schemes with provable performance guarantees on three optimization objectives simultaneously: total energy consumption, network lifetime and collision-free schedule length. We also show extensive numerical results which support our findings.

Keywords: Broadcast, energy efficiency, approximation algorithms, scheduling

1. Introduction

The technological and theoretical advances in the study of wireless communications lead to a rapid introduction of wireless ad hoc networks to a wide spectrum of applications, from scientific monitoring to military and rescue operations. As wireless nodes are often deployed in areas where battery replacement is infeasible, energy efficiency remains one of the most critical issues in any wireless network design [9]. The high levels of energy consumption are mainly due to the power used for wireless communication. It is customary to assume that the power required to transmit to distance r is proportional to r^α, where α is the *distance-power gradient*. In perfect conditions $\alpha = 2$, however in more realistic settings (in presence of obstructions) it can have a value between 2 and 4 (see [23]). Thus, the assignment of transmission powers and the relative node disposition constitute the communication backbone of the network. In this paper we assume $\alpha = 2$ for simplicity, although our results could be easily generalized for any $\alpha > 2$. Energy efficiency is usually measured through two parameters: *total energy consumption* [10] (also referred as total cost) and *network lifetime* [26]. The former refers to the total energy used for a specific network task (e.g. broadcast), while the latter estimates the expected endurance of network nodes until they run out of battery charge (typically, the network lifetime is defined as the time it takes the first node to run out of its battery charge).

Energy efficiency is not the only challenge faced by the network designer. As nodes communicate through radio signals, *wireless interference* becomes inevitable. Simultaneous transmissions are sensed at every node, which may lead to incorrect signal receptions. We consider omnidirectional antennas, where the transmission of a single node is propagated in all directions. The level of interference depends on the transmitting nodes proximity and the transmission ranges. High levels of interference decrease the number of transmissions that can happen simultaneously, which has a direct affect on the *schedule length* of the network, which is the required number of time slots for the message to propagate from the source to all the other nodes in

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the network. It should be noted that traditional works which aim to minimize the hop-diameter in order to minimize the schedule length fail to do so as they neglect the presence of interference.

In this paper we consider a fundamental topology control problem which is to induce an energy efficient *broadcast communication backbone* with a low schedule length. That is, given a special source node s (also referred to as the root node), we wish to induce a communication graph by adjusting transmission powers, so that there is a directed path from s to every other node in the network; the total energy consumption, network lifetime and feasible schedule length are used to measure the efficiency of the scheme

The broadcast problem, under various optimization criteria has been extensively studied in the research community. It was first introduced by Wieselthier et al. ([31, 32]), where the authors proposed several heuristics (*Minimum Spanning Tree* (MST), *Shortest Path Tree* (SPT) and *Broadcast Incremental Power* (BIP)), without provable bounds, for the minimum total energy broadcast problem. Wan et al. [30] presented the first analytical results for this problem; in particular, they showed that the approximation ratio of an MST is between 6 and 12, for BIP it is between $\frac{13}{3}$ and 12 and for SPT it is at least $\frac{n}{2}$, where n is the number of receiving nodes. Cagalj et al. [8] gave a proof of NP-hardness of the minimum-energy broadcast problem in a Euclidean space and Ambühl et al. [1] showed that the MST heuristics achieves an approximation ratio of 6, which matched the lower bound by [30]. All of the results above address only the issue of energy efficiency without considering the schedule length in the presence of interference and the network lifetime. One possible approach to minimize the schedule length is to bound the hop-diameter of the induced communication graph [4]. This alone, however, is not sufficient as possible collisions need to be taken into account. There has been some extensive work on collision-free scheduling as well. Parthasarathy et al. [24] show that minimum schedule length broadcasting is NP-hard for ad hoc wireless networks. They present a distributed collision-free broadcasting algorithm with an approximation ratio of $O(1)$ on the schedule length and the number of message propagations in the case that the transmission ranges are bounded. Onus et al. [22] develop efficient self-stabilizing collision free broadcasting and data gathering algorithms. See some additional results in [15, 16]. These works however do not address the energy efficiency of the proposed schemes. The works [6] and [33] address a minimum energy delay constrained broadcast scheduling problem without taking interference or network lifetime into consideration. See [20] and [35] for some results on interference and energy aware scheduling in networks with directional antennas.

Our main contribution in this paper is the development of interference and energy aware broadcast schedules with provable analytical bounds for three optimization criteria simultaneously: total energy consumption, network lifetime and schedule length. To the best of our knowledge, this is the first paper to consider all three objectives simultaneously. A summary of our results appears in Table 1. Note that the total cost and network lifetime are compared with the optimal cost and network lifetime of *any* broadcast, without schedule length restrictions, in all cases except for the FORK[2] scheme, where the network lifetime is compared with the optimal network lifetime of a broadcast schedule of length m. As it can be seen the first two schemes, *Grid based Broadcast Tree* (GBT) and *MST based broadcast tree* (MST) are developed for the case of random node distribution in a unit square, while the latter two, *Hamiltonian Circuit based broadcast Tree* (HCT) and FORK, consider deterministic node layouts. As can be seen from Table 1 which summarizes our results, our results are partitioned into 3 major groups of topologies: random, arbitrary and grid. Thus, we expect that practically oriented users will use them depending on the type of topology they are using, Moreover, with the respect of random topology, we suggest to use the GBT scheme since it has slightly better conflicting set size as we will be shown later. In addition, we have explained different settings in the simulation section where for each scheme we encounter its advantages and disadvantages.

The rest of this paper is organized as follows. Some definitions, the wireless model, and problem formulation are given in Section 2, while Section 3 holds some preliminary results. Sections 4 and 5 present our results for the random and deterministic cases, respectively. Section 6 discusses a possible distributed implementation of our schemes. In Section 7 we verify our results through simulations. Finally, in Section 8 we conclude our work and note some possible future research directions.

[2]FORK is not an acronym. The name follows from the visual resemblance between the topology and the well-known cutlery.

Scheme	Schedule length	Total cost approx.	Lifetime approx.	Node layout	Section
GBT	$O\left(\sqrt{n/\log n}\right)$	$O(1)$	$\Omega(1)$	Random	4.1
MST	$O\left(\log n \cdot h(MST_V)\right)$	$O(1)$	1 [optimal]	Random	4.2
HCT	$\lfloor (n-1)/k \rfloor$	$O(k)$	$\Omega(1/k^2)$	Arbitrary	5.1
FORK	m	$O(1)$	1 [optimal for m]	Grid	5.2

Table 1: Summary of our contribution. Note that two of the schemes are parameterized: in HCT, k is a schedule length-lifetime tradeoff parameter; in FORK, m is the desired schedule length (note that the network lifetime is optimal for a given m, while the total cost bound is universal, i.e. it is at most $O(1)$ times the cost of any broadcast.

2. System settings

Let $G_V = (V, E_V)$ be a complete directed graph of the wireless nodes V, $|V| = n$, positioned in the plane. We define the weight function, $w : E_V \to \mathbb{R}^+$, on the edge set E_V as $w(u, v) = d(u, v)^\alpha$, where $d(u, v)$ is the Euclidean distance between u and v. Note that the weight of an edge (u, v) matches the amount of energy which is required to transmit from u to v.

We proceed by first presenting some general graph theory related definitions. This is followed by the wireless ad-hoc network model used in the paper. In the end, we formally define the *Minimum Energy and Schedule Broadcast* problem.

2.1. General definitions

The following definitions are used for both, directed and undirected graphs. Let $G = (V, E)$ be a subgraph of G_V. Denote the total weight of G by $w(G) = \sum_{e \in E} w(e)$. For an edge $e = (u, v) \in E$, let $|e| = d(u, v)$. Also, let $e^*_u(G)$ be the longest outgoing edge from u in G, and by $e^*(G)$, $|e^*(G)| = \max_{u \in V} |e^*_u(G)|$, the longest edge in G. Let MST_V be a minimum weight spanning tree of the undirected version of G_V (which is obtained easily by omitting the edge directions). The *hop-distance* from u to v in G, $h_{u,v}(G)$, is defined as the minimum number of edges in any path from u to v. The height of u in G, $h_u(G)$, is the maximum hop-distance from u to any node, i.e. $h_u(G) = \max_{v \in V} h_{u,v}(G)$. The hop-diameter of G is defined as $h(G) = \max_{u \in V} h_u(G)$. A directed graph $G = (V, E)$ is a *broadcast arborescence* rooted at $s \in V$ if for any node $u \in V$ there is a path from s to u in G.

We assume the following scenario of a broadcast task. A message from a source $s \in V$ to all the other nodes in the network over a broadcast arborescence G is propagated over a directed subtree of G (in the tree there is exactly one path from s to every other node). Once a node receives a message from its parent, it forwards it to all its children.

2.2. Wireless ad-hoc network model

A power assignment is a function $p : V \to \mathbb{R}^+$, which assigns each node $v \in V$ a transmission range $r_v = \sqrt[\alpha]{p(v)}$. The transmission possibilities resulting from a power assignment induce a directed communication graph $H_p = (V, E_p)$, where $E_p = \{(u, v) : r_u \geq d_{u,v}\}$ is a set of directed edges. Recall that we assume that $\alpha = 2$.

The total energy consumption, also referred to as the cost, of the power assignment is given by $c(p) = \sum_{v \in V} p(v)$. Each node v has some initial battery charge $b(v)$, which is sufficient for a limited amount of time, proportional to the power assignment $p(v)$. It is common to take the lifetime of a wireless node v to be $l(v) = b(v)/p(v)$. The network lifetime is defined as the time it takes the first node to run out of its battery charge. For a power assignment p and initial battery charges b, the network lifetime is defined as $l(p) = \min_{v \in V} l(v)$. Again, for simplicity we assume $b(v) = 1$ for every $v \in V$.

Interference is a direct consequence of any power assignment p. A signal transmitted over one communication link may interfere with the correct reception of a transmission over some other link. We adopt the **protocol interference model** which defines for each node u a set of nodes, $I_p(u, T)$, referred to as the *conflict set* of u, which consists of nodes which cannot be scheduled to transmit simultaneously with u because of interference to either the recipients of u or v, in a broadcast subtree T of H_p which is used for the broadcast task. That is, node u cannot be scheduled simultaneously with v iff there exists a child of u in T which is interfered by v or vice versa (a child of v interfered by u). To simplify the notations we use $I_p(u)$ instead of $I_p(u,T)$ throughout the paper as the broadcast subtree is clear from the context. There are several variations for the definition of $I_p(\cdot)$ ([7, 18, 21]); for simplicity we use the following definition, however it can be easily generalized.

Definition 2.1. *For a power assignment p and a broadcast subtree $T=(V,E_T)\subseteq H_p$, $v\in I_p(u)$ iff there exists $(u,x)\in E_T$ such that $(v,x)\in E_p$ or there exists $(v,y)\in E_T$ such that $(u,y)\in E_p$.*

Note that $v\in I_p(u)$ iff $u\in I_p(v)$. Let I_p^* be the maximum size conflict set for a power assignment p, i.e. $|I_p^*|=\max\limits_{u\in V}|I_p(u)|$.

For the broadcast operation we assume a time slot based scheduling, where links are assigned to time slots. Let $S=\{U_1,U_2,\ldots,U_k\}$ be a schedule based on some communication graph H_p, where $U_i\subseteq V$, $1\le i\le k$, is a non-empty set of nodes scheduled to transmit at time slot i. The length of S is defined as $|S|=k$. A schedule S is feasible if all the transmitting nodes in every time slot are non-interfering, i.e. for any two nodes, $u,v\in U_i$, scheduled at the same time slot i, it holds $u\notin I_p(v)$ and $v\notin I_p(u)$. When schedule S is *executed*, all the nodes in U_i, $1\le i\le k$, are activated (simultaneously) before any of the nodes in U_j, $i<j\le k$, are allowed to transmit. Below we give a definition of a *feasible broadcast schedule*.

Definition 2.2. *S is a feasible broadcast schedule for a broadcast arborescence $G=(V,E)$ rooted at $s\in V$ if S is a feasible schedule and it is possible to broadcast a message from s to all the nodes in $V\setminus\{s\}$ when S is executed.*

Let $FBS(G,s)$ be a set of feasible broadcast schedules for a broadcast arborescence G rooted at s. The scheduling efficiency of a broadcast arborescence G rooted at $s\in V$ is the length of the shortest feasible broadcast schedule, $Len(G,s)=\min_{S\in FBS(G,s)}|S|$.

2.3. Problem definition

The problem we address in this paper is defined as follows.

Problem. *Given a set V of wireless nodes in the Euclidean plane, and a source node $s\in V$, find a power assignment p, such that H_p is a broadcast arborescence rooted at s. The objectives are: minimize $Len(H_p,s)$, minimize $c(p)$, and maximize $l(p)$.*

The cost and network lifetime of our power assignments is compared to the best possible for any broadcast arborescence, i.e. for any $s\in V$, let p_s^{COST} and p^{LIFE} be the minimum cost power and maximum network lifetime power assignments, respectively so that $H_{p_s^{COST}}$ and $H_{p_s^{LIFE}}$ are broadcast arborescences rooted at s. We define $c_s^*=c(p_s^{COST})$ and $l_s^*=l(p_s^{LIFE})$.

Note that the optimization objectives can be both contradicting and complimentary. Our objective is to simultaneously: maximize the network lifetime, minimize the total cost and minimize schedule length. For instance, a lower total cost would mean that many nodes are assigned low powers which consequently leads to the fact that the underlying communication graph has a large hop-diameter. As a result, the schedule length is increased. On the other hand, having a small total cost, leads to lower interference, which in turn reduces the schedule length. Another example of contradiction can be found between the total cost and lifetime measures. Both indicate energy efficiency and in that sense go hand in hand. However, if one node is using an extremely large power, it does not influence the total cost much, but has a devastating effect on the network lifetime. In our solutions we provide analytical guarantees for all three measures simultaneously.

3. Preliminaries

In what follows we present several theoretical results which are used throughout the paper. The cited theorems and lemmas are adapted to our model.

In [30] the authors derived a bound on the minimum cost of any power assignment that induces a broadcast arborescence.

Theorem 3.1 ([30]). *$c_s^* = \Omega(w(MST_V))$, for any $s \in V$.*

Interestingly, using the edges of the minimum spanning tree produces an optimal network lifetime for broadcast if all the initial battery charges are equal ($\forall u \in V, b(u) = 1$), as shown in the next theorem.

Theorem 3.2. *$l_s^* \leq 1/w(e^*(MST_V))$, for any $s \in V$.*

Proof. Let p be a maximum network lifetime power assignment such that H_p is a broadcast arborescence rooted at s and $l_s^* = l(p)$. Let T be a minimum bottleneck directed spanning tree of H_p, so that the weight of the maximum weight edge in T is minimized. Clearly $l(p) \leq 1/w(e^*(T))$. A well known property of MST_V is that its bottleneck edge is the minimum possible one of all spanning trees ([5] Lemma 3.3). Therefore, $1/w(e^*(T)) \leq 1/w(e^*(MST_V))$, which completes our proof. □

Two major factors have influence on the schedule efficiency of a broadcast arborescence; these are the hop-diameter of the arborescence and the conflict sets of communication links. In the next theorem we derive a simple upper bound on the schedule length as a function of the hop-diameter and the maximum size of a conflict set.

Theorem 3.3. *If a power assignment p induces a broadcast arborescence H_p rooted at node $s \in V$ then there exists a feasible broadcast schedule S so that $Len(S) \leq h_s(H_p) \cdot (|I_p^*| + 1)$.*

Proof. Let T be a directed spanning tree of H_p rooted at s which results from running the BFS algorithm ([11]) from s (all the edges are directed from the parent to the child nodes). Clearly, $h_s(T) = h(H_p)$. Let L_i, $0 \leq i \leq h_s(T) - 1$, be a set of nodes which are at distance $i - 1$ from s, i.e. for $u \in L_i$, $h_{s,u}(T) = i$. We show that it is possible to schedule every set of nodes L_i in $|I_p^*| + 1$ time slots.

For every L_i, $0 \leq i \leq h_s(T) - 1$, we divide the nodes into l time slots $U_1, U_2, \ldots, U_l$, which form a feasible schedule. The sets U_j, $1 \leq j \leq l$, are constructed in incremental order; first U_1, then U_2, etc. Each set U_j is constructed in a greedy fashion as follows.

- **Step 1** Initialize $U_j \leftarrow \emptyset$ and $L \leftarrow L_i \setminus \{U_1 \cup U_2 \ldots \cup U_{j-1}\}$.
- **Step 2** Pick an arbitrary node $u \in L$ and add it to U_j.
- **Step 3** Remove u and $I(u)$ from L.
- **Step 4** If $L \neq \emptyset$ goto step 2.

The above routine is executed until all the nodes are assigned to one of the time slots U_j, $1 \leq j \leq l$. It is important to note that this assignment of nodes to time slots forms a feasible schedule since all the sets are disjoint and in each set U_j there are no interfering nodes due to step 3. To upper bound l we first show that for any j, $1 \leq j \leq l$,

$$|U_1 \cup U_2 \cup \ldots \cup U_j| \geq \frac{|L_i| \cdot j}{|I_p^*| + 1}.$$

We show this fact by induction. In order to simplify the notation we denote $m = |L_i|$ and $k = |I_p^*|$. It is easy to verify that $|U_1| \geq m/(k+1)$ as for each node which is picked in step 2, at most $k+1$ nodes are removed from L, which is initialized to be L_i for the construction of U_1. Therefore, step 2 will be repeated at least $m/(k+1)$ times.

Suppose that the inequality holds for some j; we will prove it for $j+1$. Let L be a set of nodes which are yet to be assigned a time slot after the construction of $U_1, U_2, \ldots, U_j$ (as initialized in step 1). According to the induction assumption

$$|L| \leq m - \frac{m \cdot j}{k+1} = \frac{m \cdot (k - j + 1)}{k+1}.$$

We observe that every node $u \in L$ was not assigned to any of the time slots $1, \ldots, j$. This could happen only if it was removed in step 3 of the construction of each of the node sets $U_1, U_2, \ldots, U_j$. From the definition of $I_p(\cdot)$ it follows that $u \in I_p(v)$ iff $v \in I_p(u)$, and therefore for any U_q, $1 \leq q \leq j$, there exists a node $v \in U_q$

so that $v \in I_p(u)$. We conclude that for any node $u \in L$ it holds $|I_p(u) \cap L| \leq k - j$. Following the same reasoning as for U_1, $|U_{j+1}| \geq \frac{|L|}{(k-j+1)}$. As a result,

$$
\begin{aligned}
|L| - |U_{j+1}| \leq |L| - \frac{|L|}{k-j+1} &= \frac{|L|(k-j)}{k-j+1} \\
&\leq \frac{m \cdot (k-j)}{k+1}.
\end{aligned}
$$

Note that $L \setminus U_{j+1}$ are exactly the nodes which are left unassigned after U_{j+1} is constructed. Therefore,

$$
|U_1 \cup U_2 \cup \ldots \cup U_{j+1}| \geq m - (|L| - |U_{j+1}|) \geq \frac{m \cdot (j+1)}{k+1}.
$$

Clearly, after at most $k+1$ rounds all nodes will be assigned and therefore $l \leq k+1 = |I_p^*| + 1$.

To create a feasible broadcast schedule for H_p we simply schedule the node sets L_i, $0 \leq i \leq h_s(T) - 1$ consecutively so that each node set is scheduled as described above. It is easy to verify that the schedule is a feasible broadcast schedule and its length is at most $h_s(H_p) \cdot (|I_p^*| + 1)$. □

4. Random node layout

In this section we consider a random network, where n wireless nodes are uniformly and independently distributed in a unit square. Due to the probabilistic nature of the network, the results are *with high probability*, or in short *w.h.p.*, which means that the probability of the result converges to one as the number of network nodes, n, increases. First, we introduce some bounds which are used throughout the section. Then, we present two power assignment schemes (GBT and MST), with different performance guarantees. The theorem below derives an additional lower bound on the minimum cost of a power assignment which induces a broadcast arborescence.

Theorem 4.1. *$c_s^* = \Omega(1)$, for any $s \in V$.*

Proof. The proof relies on two previous results for strong connectivity. A graph $G = (V, E)$ is strongly connected iff for any two nodes $u, v \in V$ there is a path from u to v and from v to u in G. Kirousis et al. [19] showed it was possible to construct a power assignment p so that H_p is strongly connected and $c(p) \leq 2w(MST_V)$. Let p^* be a minimum cost power assignment so that H_{p^*} is strongly connected. Zhang and Hou in [34] derived a lower bound on the cost of p^* under the assumption that the nodes form a homogeneous Poisson point process with density λ. According to [14], a random, uniform and independent n-point process in a unit square is essentially a Poisson process with $\lambda = n$, for large values of n – which allows us to use the results of [34]. In particular they showed that $c(p^*) = \Omega(1)$. Therefore, $w(MST) \geq c(p)/2 \geq c(p^*)/2 = \Omega(1)$. Combining with Theorem 3.1, we derive $c_s^* = \Omega(1)$, for any $s \in V$. □

In [28] it was derived that $w(e^*(MST_V)) = \Theta\left(\frac{\log n}{n}\right)$. In conjunction with Theorem 3.2 it is possible to yield the following upper bound on the maximum network lifetime.

Corollary 4.2. *$l_s^* = O(n/\log n)$, for any $s \in V$.*

4.1. Grid based broadcast tree (GBT)

The first scheme (GBT) is based on dividing the unit square into grid cells and then using nodes from each cell to deliver the message to other nodes within the cell. The idea is to exploit the probabilistic properties of the random node distribution to guarantee the existence of at least one node in each cell. We make use of the following technical lemma which was developed in [29].

Lemma 4.3 ([29])**.** *Let D^* be a maximum radius disk, which can be placed inside a unit square, so that there are no nodes inside D^*. Let ε be the radius of D^*. Then, $\varepsilon < \sqrt{2 \log n / n}$.*

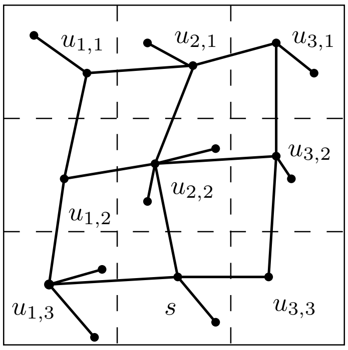


Figure 1: An example of GBT. There are 9 cells; the representative node in each cell, $u_{i,j}$, broadcasts to the nodes in the cell (the edges E_1) and to the representatives of adjacent cells (the edges E_2).

Next, we describe the construction of the power assignment p_1 for any source node $s \in V$. Let $\delta = \sqrt{\frac{8 \log n}{n}}$.

- **Step 1** We divide the unit square into cells of size[3] $\delta \times \delta$. Let us denote by $N(i, j)$ all the nodes which are in a grid cell (i, j), $1 \leq i, j \leq 1/\delta$. Note that due to Lemma 4.3, each grid cell contains at least one node.
- **Step 2** We arbitrarily pick one *representative* node $u_{i,j}$ from each grid cell $N(i, j)$, $1 \leq i, j \leq 1/\delta$, with one exception. If the cell contains s we pick s.
- **Step 3** Let $H = (V, E)$ be an undirected graph with two types of edges, $E = E_1 \cup E_2$. The first type of edges is between the representative nodes to other nodes in their cell.

$$E_1 = \{(u_{i,j}, v) : v \in N(i, v),\ 1 \leq i, j \leq 1/\delta\}.$$

The second type of edges connect between representatives of adjacent cells. Two cells $N(i, j)$ and $N(k, l)$ are adjacent if they have a common wall; this is denoted by $(i, j) \perp (k, l)$.

$$E_2 = \{(u_{i,j}, u_{k,l}) : (i, j) \perp (k, l),\ 1 \leq i, j, k, l \leq 1/\delta\}.$$

An example of H is given in Figure 1 for a grid with 9 cells.
- **Step 4** Let T be a **directed** spanning tree of H rooted at s which results from running the BFS algorithm from s.
- **Step 5** Finally, we can define the power assignment p_1. For every $u \in V$, $p_1(u) = w(e^*_u(T))$.

The performance analysis of p_1 is based on the following two lemmas.

Lemma 4.4. *$c(p_1) = O(1) \cdot c^*_s$ and $l(p_1) = \Omega(1) \cdot l^*_s$.*

Proof. From the construction of T it follows that $|e^*(T)| \leq \sqrt{5}\delta$ due to the fact that the distance between any two points in two adjacent grid cells is at most $\sqrt{5}\delta$. Therefore, for any $u \in V$, $p(u) \leq 5\delta^2$, which immediately results in $l(p_1) = \Omega(1) \cdot l^*_s$ due to Corollary 4.2.

To bound the cost of p_1 we look closely at the tree T. Note that nodes which are not cell representatives have no outgoing edges in T. This is due to the fact that the degree of any *not representative* node in H is 1. Since s is a representative node, every non representative must be a leaf in T. Thus, only cell representatives will have some power assigned. There are $1/\delta^2$ cells, each assigned a power of at most $5\delta^2$ and hence $c(p_1) = O(1)$. In conjunction with Theorem 4.1 we obtain $c(p_1) = O(1) \cdot c^*_s$. □

[3]We omit the use of floors and ceilings to simplify the notation.

Lemma 4.5. $Len(H_{p_1}, s) = O\left(\sqrt{\frac{n}{\log n}}\right)$.

Proof. Clearly $h(T) \leq 2/\delta = \sqrt{\frac{n}{2\log n}}$ as every pair of adjacent cells is connected. To bound the length of a feasible schedule we argue that $I^*_{p_1} = O(1)$. As it was stated in the proof of Lemma 4.4, representative nodes are the only ones to transmit and their transmission range is at most $\sqrt{5}\delta$. We can derive that each representative nodes *interferes with* and is *interfered by* at most a constant number of nodes and hence $|I^*_{p_1}| = O(1)$. By using Theorem 3.3 we obtain $Len(H_{p_1}, s) = O\left(\sqrt{\frac{n}{\log n}}\right)$. □

The properties of p_1 are summarized in the following theorem.

Theorem 4.6. *For n wireless nodes, randomly, uniformly, and independently distributed in a unit square and a source node s, the power assignment p_1 induces a broadcast arborescence H_{p_1} rooted at s such that $Len(H_{p_1}, s) = O\left(\sqrt{\frac{n}{\log n}}\right)$, $c(p_1) = O(1) \cdot c^*_s$, and $l(p_1) = \Omega(1) \cdot l^*_s$.*

4.2. MST based broadcast tree (MST)

The second scheme (MST) is based on a minimum spanning tree MST_V. We describe a power assignment p_2 which was first introduced by Kirousis [19]. For every $u \in V$ we define $p(u) = w(e^*_u(MST_V))$. In [19] the authors showed that $c(p_2) \leq 2w(MST_V)$. The energy efficiency of p_2 is easily derived in the following lemma based on Theorems 3.1 and 3.2.

Lemma 4.7. $c(p_2) = O(1) \cdot c^*_s$ *and* $l(p_2) = l^*_s$.

Next we analyze the length of the shortest schedule from **any** source node $s \in V$ we first derive the following technical lemma by applying the "balls and bins" analysis from [25].

Lemma 4.8. $|I^*_{p_2}| = O(\log n)$.

Proof. Note that from the definition of $I_{p_2}(\cdot)$ it follows that two nodes cannot be scheduled to transmit simultaneously if there exists a designated recipient that is within the transmission range of both nodes. Which means that two conflicting nodes can be at a distance of at most twice the maximum transmission range. Therefore, according to the construction of p_2, for any node u, the nodes in $I_{p_2}(u)$ are within distance $2|e^*(MST_V)|$ from u. Since $w(e^*(MST_V)) = \Theta\left(\frac{\log n}{n}\right)$, for any $u \in V$ and $v \in I_{p_2}(u)$, $d(u,v) \leq a \cdot \log n/n$, where $a > 1$ is some constant.

Next we apply the "balls and bins" analysis. Divide the unit square into $\frac{n}{\log n}$ grid cells, each of size $\sqrt{\frac{\log n}{n}} \times \sqrt{\frac{\log n}{n}}$. We can look at the distribution of nodes in the grid as a random process where we independently and uniformly throw n balls into $\frac{n}{\log n}$ bins. The authors in [25] analyzed the maximum number of balls in each bin (in our case, nodes in a grid cell). They showed that w.h.p. each grid cell contains at most $O(\log n)$ nodes. We can conclude that for each node u there are at most $O(\log n)$ nodes at distance $a \cdot \log n/n$ from it. Thus, $|I^*_{p_2}| = O(\log n)$. □

By using Theorem 3.3 and Lemmas 4.7, 4.8 we can easily obtain the properties of p_2.

Theorem 4.9. *For any source node $s \in V$, $Len(H_{p_2}, s) = O(\log n \cdot h(MST_V))$, $c(p_2) = O(1) \cdot c^*_s$, and $l(p_2) = l^*_s$.*

4.3. Single hop broadcast

Surprisingly, the naive power assignment, $p_s(s) = \max_{u \in V} w(u)$ and $p_s(u) = 0$ for any $u \in V \setminus \{s\}$, which assigns s with enough power to reach all the other nodes in a single hop while all the other nodes do not transmit, has good energy efficiency in terms of total cost. The following theorem shows the performance guarantees of p_s based on Theorem 4.1 and Corollary 4.2.

Theorem 4.10. $Len(H_{p_s}, s) = 1$, $c(p_s) = O(1) \cdot c^*_s$, *and* $l(p_s) = \Omega(\log n/n) \cdot l^*_s$.

5. Deterministic node layout

In this section we present two broadcast schemes for deterministic node layout, i.e. the nodes are not placed randomly. First we develop a Hamiltonian circuit[4] based scheme (HCT) for an arbitrary node layout, and then present the FORK scheme for a grid layout of nodes.

5.1. Hamilton circuit based broadcast tree (HCT)

Our third construction (HCT) is based on a Hamiltonian circuit. Sekanina [27] showed that the cube of any tree[5] $T = (V, E)$, with $|V| \geq 3$, is Hamiltonian. Andrea and Bandelt [2] give a linear time algorithm for the construction of the Hamiltonian circuit P_h in T^3, given T. They also show that $w(h) \leq w(T) \cdot (\frac{3}{2}\tau^2 + \frac{1}{2}\tau)$, where τ is the weak triangle inequality parameter. Note that $\tau = 2^{\alpha-1} = 2$ under our assumption that $\alpha = 2$ (recall that α is the *distance-power gradient*). Moreover, it can be shown that the weight of the longest edge in P_h is at most $O(1)$ times the weight of the longest edge in T. The following theorem applies the above on $T = MST_V$.

Theorem 5.1 ([2])**.** *Let $P_h = \langle u_0 = s, u_1, \ldots, u_n = u_0 \rangle$, where $u_i \in V$ for $0 \leq i \leq n$, be a Hamiltonian circuit as a result of applying the construction in [2] on MST_V. Let H be a path graph which is obtained by taking P_h and removing the last edge. Then $w(H) = O(w(MST_V))$ and $w(e^*(H)) = O(w(e^*(MST_V)))$.*

We define the power assignment p_3 as follows. For any u_{lk}, $0 \leq l \leq \lfloor (n-1)/k \rfloor$,

$$p_3(u_{lk}) = \max_{1 \leq i \leq \min\{k, n-1-lk\}} d(u_{lk}, u_{lk+i})^2.$$

That is, every node u_{lk}, $0 \leq l \leq \lfloor (n-1)/k \rfloor$, is assigned to reach the next k nodes on the Hamiltonian path constructed in the theorem above. All the other nodes have a power assignment of 0. We can easily obtain the following theorem.

Theorem 5.2. *$c(p_3) = O(k) \cdot c_s^*$, $l(p_3) = \Omega(1/k^2) \cdot l_s^*$, $Len(H_{p_3}, s) = \lfloor (n-1)/k \rfloor$.*

Proof. Note that for any node which is assigned a power, u_{lk} it holds

$$p(u_{lk}) \leq \left(\sum_{i=0}^{\min\{k, n-1-lk\}} d(u_{lk+i}, u_{lk+i+1}) \right)^2 \leq k \sum_{i=0}^{\min\{k, n-1-lk\}} w(u_{lk+i}, u_{lk+i+1}).$$

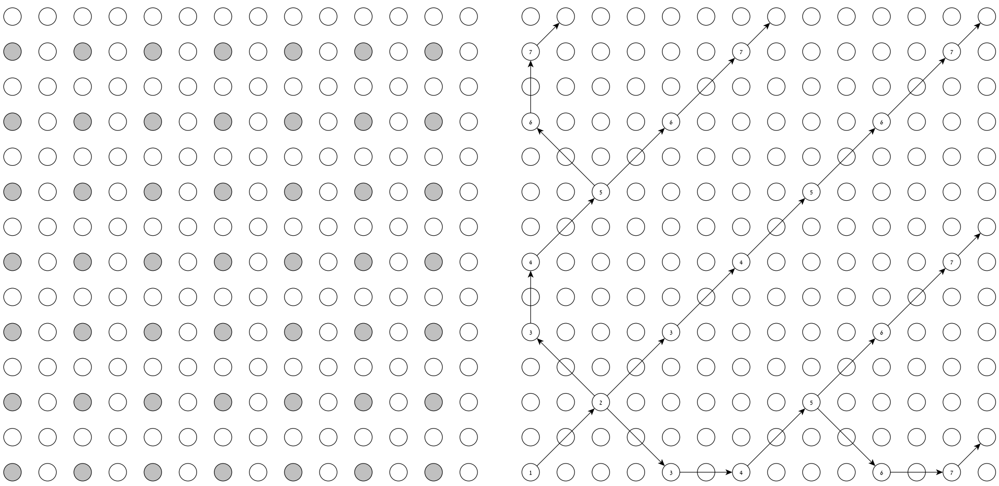
Therefore, $w(e^*(H_{p_3})) \leq k^2 w(e^*(MST_V))$ and $c(p_3) = O(k) w(MST_V)$. This proves the bounds of total cost and lifetime according to Theorems 3.1 and 3.2. Note that there exists a feasible schedule which simply schedules the nodes u_{lk}, $0 \leq l \leq \lfloor (n-1)/k \rfloor$, one after another. Therefore, $Len(H_{p_3}, s) = \lfloor (n-1)/k \rfloor$. □

Finally, note that there exists a class of graphs on which this algorithm provides almost optimal (constant factor approximation) results, both in lifetime and total cost, such as a linear graph with equally distributed nodes.

5.2. The FORK scheme

Our fourth construction is developed for a grid node layout. It is dubbed FORK as the tree resembles a fork (see Figure 2(b)). We assume that the underlying grid has a fixed scheduling requirement m and show that there exists a broadcast tree with scheduling length m, optimal lifetime, and almost optimal power assignment. The algorithm execution is distributed, and partially synchronous (since we assume the scheduling length is fixed).

[4]A Hamiltonian circuit is a cycle in a graph that visits each node exactly once and returns to the starting node.
[5]In the cube of a tree T there are edges between the endpoints of any three- and two-hop paths, in addition to the original one



(a) Cell leader selection: there are $m^2 = 49$ leaders (colored in grey).

(b) FORK scheduling. The numbers inside cell leaders indicate the transmission round when they are active. All the nodes within their transmission range (depicted by an arrow) receive the message in that round.

Figure 2: The FORK scheme with $\sqrt{n} = 14$ and $m = 7$.

To simplify the notation we describe the transmission scheme without using the power assignment notations and then analyze its efficiency. We divide the grid to a smaller $m \times m$ grid with equal size cells, for each cell i, j in the new $m \times m$ grid we select a cell leader, $leader_{i,j}$, the cell leader is the node located at the bottom left corner of any cell (For example, in Figure 2(a) we have $\sqrt{n} = 14$, and $m = 7$; the selected cell leaders are colored in Grey). Without loss of generality, we set the root node r ($leader_{0,0}$) to be the node at the bottom left corner of G (i.e., r has coordinates $(0, 0)$). The progress of the broadcast algorithm is as follows, r transmits the first message with power $(\frac{\sqrt{2n}}{m})^2$, when a cell leader $leader_{i,j}$ receives the message from cell leader $leader_{m,l}$ at the first time, it transmits the message with energy $(\frac{\sqrt{2n}}{m})^2$ if either a.) $i = m+1$ and $j = l + 1$ or b.) $i = 0$ and $j \equiv 2 \bmod 3$ or $j = 0$ and $i \equiv 2 \bmod 3$. If c.) $i = 0$ and $j \equiv 0 \bmod 3$ or $j = 0$ and $i \equiv 0 \bmod 3$ the node transmits the message with energy $(\frac{\sqrt{n}}{m})^2$ note that the fringe cell leaders (cell leaders with either $i = 0$ or $j = 0$) behaves differently and some nodes receives the transmission twice, but without interference in the second time.

A sample scheduling schema is depicted in Figure 2(b), the number inside each cell leader illustrates the message transmission round, the directed arrows illustrate the radius of transmission (i.e., the transmitted message arrives to all the nodes which are at a distance that is less then or equal to the radius). Clearly, the scheduling length of the algorithm is exactly m. To calculate the total power consumption observe that in each half of the grid, there are $\frac{m}{3}$ nodes that initiate a directed transmission line with length $m - 3i$ (where $i \in [0...\frac{m}{3}]$) and $\frac{m}{3}$ nodes that forward their message to an adjacent neighbor. This leads to the following summation:

$$(\frac{\sqrt{2n}}{m})^2 m + 2\sum_{i=1}^{i=\frac{m}{3}}(\frac{\sqrt{2n}}{m})^2(m-3i) + 2\sum_{i=1}^{i=\frac{m}{3}}(\frac{\sqrt{n}}{m})^2 < 2n(\frac{4}{3m} + \frac{1}{3})$$

The next theorem summarizes the properties of FORK.

Theorem 5.3. *The schedule length of FORK is m, the network lifetime is optimal for a schedule of length m and the total power is at most $O(1)$ times the optimal.*

Proof. From the construction, the schedule length is m by definition. According to 3.1 we can derive that the cost of the optimal solution is at least $\Omega(n)$ and therefore the scheme achieves a constant factor approximation for the total cost. Regarding the network lifetime, any optimal broadcast tree which has a schedule length of m has at most m hops and therefore there exists a node which must transmit to distance $\frac{\sqrt{2n}}{m}$ and therefore the network lifetime of FORK is optimal. □

6. Distributed implementation

The construction of Hamiltonian circuit can be done very efficiently in a distributed fashion using standard leader election techniques proposed by Awerbuch [3]. Once the leader in the tree is found using $O(n)$ time with $O(n \log n)$ messages, the distributed algorithm for finding Hamiltonian circuit behaves exactly as the centralized description below. The algorithm is applied to a tree T and an edge $e = (u, v)$ of T. Removing the edge e divides the tree into two subtrees T' and T''. In each subtree the algorithm selects an arbitrary edge $e' = (u, w)$ (for T') and $e'' = (x, v)$ (for T''), and recursively computes a Hamiltonian cycle of T' and T'' that includes the edge e' and e'', respectively. The circuit consists of the cycles in T' and T'' without two edges e' and e''. The two resulting paths are glued together using e and the edge connecting other endpoints of two edges e' and e''. This can be done by the convergecast process through the nodes towards the leader. The same algorithm in [3] can be used in order to define power assignments GBT (with a consequent distributed BFS algorithms after leader election), MST, and FORK (after the leader is found, it sets the root node and follows the sequential algorithm scheme). All in all, the total message complexity of every algorithm is bounded by $O(n \log n)$ messages and $O(n)$ time.

7. Numerical results

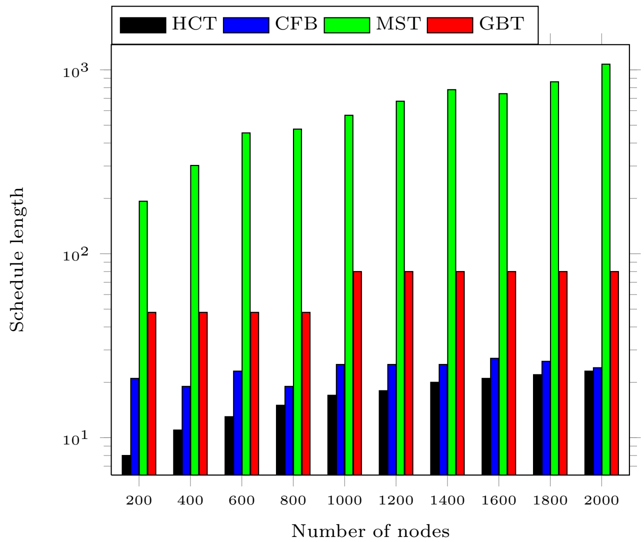


Figure 3: Schedule length in random networks. Comparing HCT ($k = \sqrt{n}$), CFB, MST, and GBT.

We have performed extensive simulations to evaluate the performance of the proposed schemes for random and static networks. For random networks, we compared all algorithms discussed in this paper, grid based (GBT), Minimum spanning tree (MST), and Hamiltonian circuit based (HCT), against a well known collision free broadcast (CFB) algorithm by Gandhi et el. [13]. For static networks, we compared the performance of the FORK scheme to CFB. In our experiments, we measure the *schedule length*, *total energy consumption*, and the *conflict set size*.

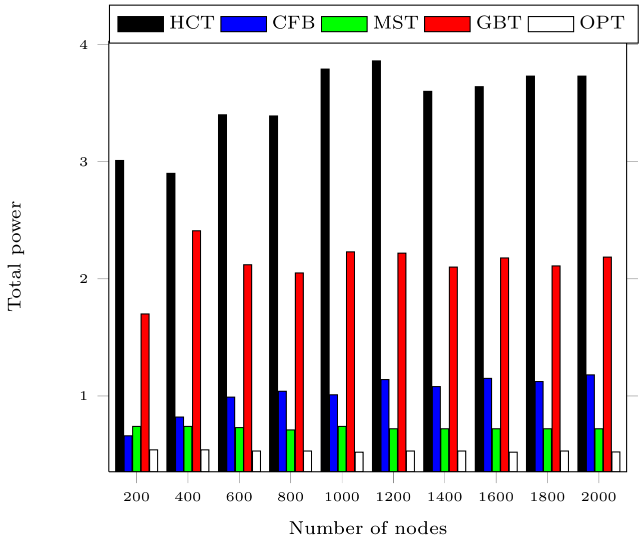


Figure 4: Total energy consumption in random networks. Comparing HCT ($k = \sqrt{n}$), CFB, MST, GBT, and OPT (which is the optimal lower bound).

7.1. Random networks

In each simulation, the nodes were randomly, independently and uniformly distributed in a unit square. For each network size, 3 trials were performed. For HCT, we fixed the value of k to $\sqrt{n}$, which yields good results both in total power consumption and scheduling length. In order to balance the power consumption and scheduling length for the CFB algorithm, we initialized the transmission range of each node to 0.1. The schedule length of the algorithms is plotted in Figure 3. The results show that GBT, HCT, and CFB compared to MST, have very good performance. In addition, the schedule length of HCT remains well below its performance bound ($\approx$ 50 percepts better). This result can be attributed to the fact that in a random distribution, the structure of the Hamiltonian circuit is not a perfect oval, and it has many “shortcuts”. Therefore, as the network becomes more dense, many nodes receive the transmission before the anticipated round. Overall, the results shows that for scheduling length, HCT is superior to all other algorithms and is a good candidate for practical implementation if the network distribution is random. Furthermore, since the higher k is set the shorter the schedule length is. In a distributed implementation, this value can be calibrated by the nodes distributively to improve the schedule length even further.

The percentage of nodes that received the broadcast message as a function of the round number was computed for HCT, CFB, MST, and GBT. The results are plotted in Figure 5. We can observe that the GBT has the poorest performance. The somewhat slower performance of GBT can be attributed to the fact that it has stricter geometrical constraints (transmissions divided into grid cells), and thus cannot take the advantage of “shortcuts”, where a broadcast message skips several levels in a single transmission.

Another interesting observation is that in each transmission round of HCT, around $\frac{100}{totalRounds}\%$ new nodes received the transmission. This makes HCT a good candidate to tasks that do not require all nodes to receive the transmission. In such scenarios, a node that receives the transmission will terminate the transmission if the cover threshold has been reached. Overall, both CFB and HCT has good convergence rate, with a slight advantage to HCT.

The total power consumption of the nodes is plotted in Figure 4. The four power assignments show a constant approximation ratio over the optimal lower bound (OPT). We can learn that the MST is the most energy constrained, followed by GBT, CFB and HCT. In can be seen that the ratio between the power consumption of different power assignments to the optimum is at most ≈ 8. Surprisingly, even though we picked a very high value of $k = \sqrt{n}$ for HCT, the energy consumption is far below the theoretical bound of $\sqrt{n} \cdot w(MST_V)$. This can be explained since the average transmission radius per node is less

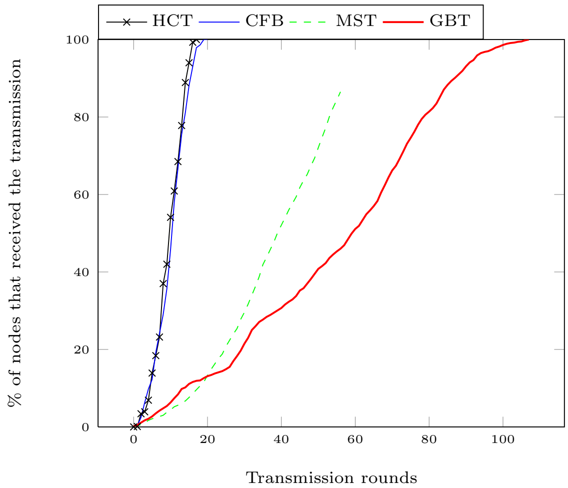
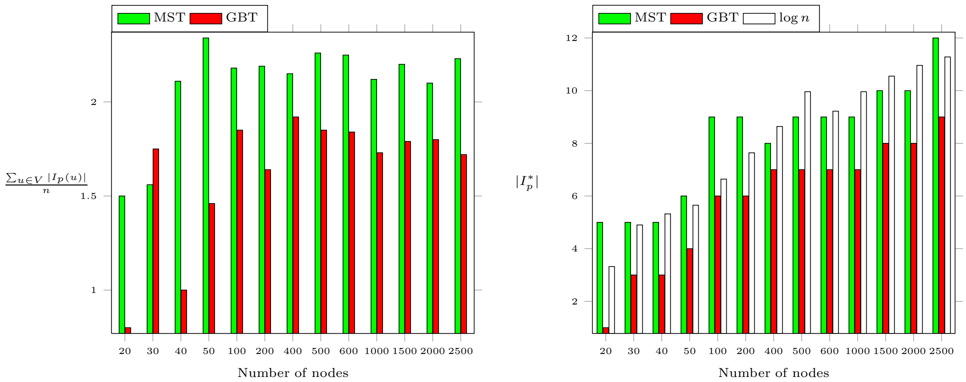


Figure 5: Message dissemination pace in a network with $n = 1000$. Comparing HCT ($k = \sqrt{n}$), CFB, MST, and GBT.

than $w(e^*(MST_V))$ (the weight of the longest edge in MST). Practically, if a power consumption is limited, selecting MST will yield good results for both measures, and its distributed implementation is fairly easy [12].



(a) Average conflict set size

(b) Maximum conflict set size

Figure 6: Studying the conflict sets in GBT and MST.

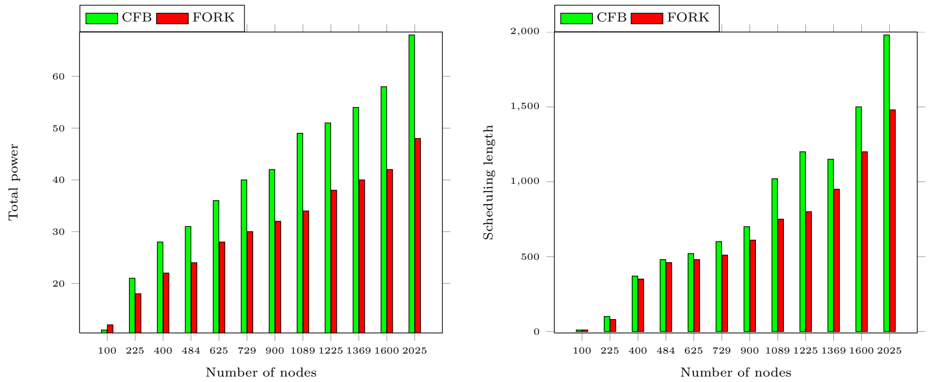
We also measured the average and maximum sizes of the conflict sets for the GBT and MST schemes for random networks with varying number of nodes. Interestingly, the average conflict set size for both schemes is consistently close to 2 for both schemes (Figure 6(a)).

The maximum conflict set size is depicted in Figure 6(b). In the case of MST, the maximum conflict set size has a growth rate similar to $\log n$, as expected according to Lemma 4.8. For GBT, on the other hand, the upper bound is around 8, due to the fact that interference mostly originates in adjacent cells, and occasionally two cells away.

7.2. Grid networks

In each simulation, we placed n nodes on a $\sqrt{n} \times \sqrt{n}$ grid. The communication topology was constructed by assigning each node a $\sqrt{2}$ transmission radius. We compared the results of FORK and CFB [13] algorithms.

The schedule length is plotted in Figure 7(a) and the power consumption is plotted in Figure 7(b). For FORK the simulation results support the theoretical finding, while in CFB the actual scheduling length is even better than expected ($\approx \sqrt{2n}$). Since CFB does not consider energy conservation as an optimization



(a) Schedule length of FORK (with $m = \sqrt{n}$) and CFB

(b) Total cost of FORK (with $m = \sqrt{n}$) and CFB

Figure 7: FORK vs. CFB: schedule length and total power consumption.

criterion, its energy consumption is much higher ($\approx 30\%$ for all measurements). In addition, FORK has better scheduling and cost performance since the algorithm takes into account the special structure of the grid and the limited transmission radius of other nodes.

8. Concluding remarks and future work

In this paper we studied the interference-free broadcast problem in wireless ad hoc networks. We considered both the random and non-random scenarios, for which we developed four energy efficiency broadcast schemes that have a good schedule length. For the best of our knowledge, this is the first work to look at the three measures (total energy, network lifetime, schedule length) *simultaneously.*

8.1. Results discussion

We developed four broadcast schemes and analyzed their energy efficiency and schedule length. As noted in the introduction section, energy efficiency can be evaluated through two measures: network lifetime and total energy consumption. These two measures are not completely independent, and thus optimizing either of them improves the overall performance. To the best of our knowledge, this is the first paper to consider these two objectives objectives simultaneously combined with schedule length parameter. We supply a number of solutions that involves a balancing between network lifetime, total cost and schedule length. In particular, for random network deployments, we obtain a constant approximation results for network lifetime and cost, while the schedule length has $O(\sqrt{n/\log n})$ approximation ratio (for GBT scheme) and $O(\log n \times h(MST))$ approximation (for MST scheme). In practice, the MST scheme performs better, although theoretically it may lead to worser schedule length. For arbitrary and grid nodes deployments we obtain different tradeoffs for the above mentioned measures. This is mostly, due to the fact, that the network lifetime (which heavily depends on the length of the longest link in the network) can be made worse for particular cases. In fact, we discovered, that applying an MST scheme to deterministic nodes deployments can lead, in some cases, to better practical results.

8.2. Applicability of the results in more general wireless settings

In this work we have made two simplifying assumptions regarding the wireless channel: that the path loss coefficient is $\alpha = 2$, and the protocol interference model. We now discuss the implications of relaxing those two assumptions.

Interestingly, some of the bounds hold for any constant path loss coefficient, while other can be generalized. In particular, it is possible to show that for any constant α:

- **GBT** (Section 4.1) – the asymptotic guarantee for schedule length remains unchanged as it is independent of α. The upper bound of network lifetime in Corollary 4.2 becomes $O((\log n/n)^\alpha)$ due to the change in the weight function. In terms of total cost, Theorem 4.1 is generalized to $c_s^* = \Omega(1/n^{\alpha/2-1})$ [34]. As a result, the approximation factor for the cost in Lemma 4.4 becomes $O(\log^{\alpha/2-1} n)$ instead of $O(1)$, while for network lifetime it remains unchanged.

- **MST** (Section 4.2) – the bounds are not affected by the value of α (in particular, Theorems 3.1 and 3.2), and thus all the analytical results are valid.

- **Single hop** (Section 4.3) – the asymptotic guarantees in Theorem 4.10 for total cost and network lifetime are generalized to $O(n^{\alpha/2-1})$ and $\Omega((\log n/n)^\alpha)$, instead of $O(1)$ and $O(\log n/n)$, respectively due to the change in Theorem 4.1 and Corollary 4.2. The schedule length remains 1.

- **HCT** (Section 5.1) – the bounds in Theorem 5.2 become $O(k^{\alpha-1})$ and $\Omega(1/k^\alpha)$ (due to Hölder's inequality), instead of $O(k)$ and $O(1/k^2)$ for the total cost and network lifetime, respectively. The schedule length remains unchanged.

- **FORK** (Section 5.2) – the cost of any spanning tree with scheduling length m is bounded by $\frac{n^{\frac{\alpha}{2}}}{m}(m^2 - 1)$. Since the transmission cost of FORK is bounded by $\frac{n^{\frac{\alpha}{2}}}{m}(m^2)$, the constant approximation ratio for the total power holds. The analysis of the schedule length and network lifetime remains unchanged.

Although we have considered the protocol interference model, our schemes can be extended to a more general, SINR model, where a transmission is successful if the signal is strong enough compared to the interference (as a result of simultaneous transmissions). We note that the Single hop (Section 4.3) and HCT (Section 5.1) schemes are not affected by the wireless model as they involve a single transmission at any given time. For GBT and MST we could use some of the known techniques for dividing the nodes into interference/transmission regions based on the transmit powers (e.g. [17]) – note that the maximum transmission power in both schemes (GBT and MST) is upper bounded by $O((\log n/n)^\alpha)$, which allows us to use these techniques. Note that this will affect the schedule length, although not drastically. Finally, for FORK we can use a similar approach of area division, which will add a constant factor to the schedule length.

8.3. Future research

A possible future research direction would be to improve the asymptotic bounds which we derived throughout the paper. Specifically, it may happen for MST based scheme since we obtained a practical evidence that it may behave good in practice. It would also be of interest to consider additional interference models, such as the SINR model.

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