Improved Approximation Algorithms for Maximum Lifetime Problems in Wireless Networks

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Abstract

A wireless ad-hoc network consists from a collection of transceivers positioned in the plane. Each transceiver is equipped with a limited battery charge. The battery charge is reduced after each transmission, depending on the transmission distance. One of the major problems in wireless network design is to route network traffic efficiently, so as to maximize the *network lifetime*, i.e., the number of successful transmission rounds. In this paper we consider Rooted Maximum Lifetime Broadcast/Convergecast problems in wireless settings. The instance consists of a directed graph $G = (V, E)$ with edge-weights $\{w(e) : e \in E\}$, node capacities $\{b(v) : v \in V\}$, and a root r. The goal is to find a maximum size collection $\{T_1, \ldots, T_k\}$ of Broadcast/Convergecast trees rooted at r so that $\sum_{i=1}^{k} w(\delta_{T_i}(v)) \leq b(v)$, where $\delta_T(v)$ is the set of edges leaving v in T. In the Single Topology version all the Broadcast/Convergecast trees T_i are identical. We present constant ratio approximation algorithms for various broadcast and convergecast problems, improving the previously best known approximation $\Omega(\lfloor 1/\log n \rfloor)$ by [9]. We also consider a more general Rooted Maximum Lifetime Mixedcast problem, where we are also given $\gamma \geq 0$, and the goal is to find the maximum integer k so that k Broadcast and γk Convergecast rounds can be performed. We also consider the model with partial level aggregation.

Index terms: Minimal energy control, Optimization methods, Ad-hoc networks, Low Power Algorithms and Protocols, Sensor networks.

1 Introduction

Wireless ad-hoc networks received a lot of attention in recent years due to massive use in a large variety of domains, from life threatening situations, such as battlefield or rescue operations, to more civil applications, like environmental data gathering for forecast prediction. The network is composed of nodes located in the plane, communicating by radio. A transmission between two

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nodes is possible if the receiver is within the transmission range of the transmitter. The underlying physical topology of the network depends on the distribution of the nodes as well as the transmission power assignment of each node. Since the nodes have only a limited initial power charge, energy efficiency becomes a crucial factor in wireless networks design.

1.1 The model

The transmission range of node v is determined by the power $p(v)$ assigned to that node. Let $w(v, u)$ denote the minimum transmission power at v so that u can receive transmissions from v. Thus u receives transmissions from v if $p(v) \geq w(v, u)$. It is customary to assume that the minimal transmission power required to transmit to distance d is d^{ϕ}, where the *distance-power gradient* ϕ is usually taken to be between 2 and 4 (see [24]). This implies $w(v, u) = w(u, v) = d(u, v)^{\phi}$, where $d(v, u)$ is the Euclidean distance between v and u. We note however that our algorithms do not use this assumption, and apply for arbitrary (non-negative) transmission powers $w(v, u)$.

There are two possible models: symmetric and asymmetric. In the symmetric model, also referred to as the undirected model, there is an undirected communication link between two nodes $v, u \in V$, if $p(v) \geq w(v, u)$ and $p(u) \geq w(u, v)$, that is if v and u can reach each other. The asymmetric variant allows directed (one way) communication links between two nodes. Krumke et al. [18] argued that the asymmetric version is harder than the symmetric one. This paper addresses the asymmetric model.

Ramanathan and Hain [26] initiated the formal study of controlling the network topology by adjusting the transmission range of the nodes. Increasing the transmission range allows more distant nodes to receive transmissions but leads to faster battery exhaustion, which results in a shorter network lifetime. We are interested in maximizing the network lifetime under two basic transmission protocols – data broadcasting and data gathering (or convergecast). *Broadcasting* is a network task when a source node r wishes to transmit a message to all the other nodes in the network. In *convergecast* there is a destination node r, and all the other nodes wish to transmit a message to it. Here we consider convergecast *with aggregation*, meaning that a node uses aggregation mechanism to encode the data available at that node before forwarding it to the destination. We consider the case of unidirectional antennas, hence the message is transmitted to every node separately. The advantages to use unidirectional antennas over omnidirectional are reduced signal interference, increase in system throughout and improved channel reuse, see [3, 4, 17].

Each node v, has an initial battery charge $b(v)$. The battery charge decreases with each transmission. The network lifetime is the number of transmission rounds performed from network initialization to the first node transmission failure due to battery depletion.

We assume that all the nodes share the same frequency band, and time is divided into equal size slots that are grouped into frames. Thus, the study is conducted in the context of TDMA, see [8]. In TDMA wireless ad-hoc networks, a transmission scenario is valid if and only if it satisfies the following three conditions:

1. A node is not allowed to transmit and receive simultaneously.
2. A node cannot receive from more than one neighboring node at the same time.
3. A node receiving from a neighboring node should be spatially separated from any other transmitter by at least some distance D.

However, if nodes use unique signature sequences (i.e., a joint TDMA/CDMA scheme), then the second and third conditions may be dropped, and the first condition only characterizes a valid transmission scenario. Thus, our MAC layer is based on TDMA scheduling [5, 7, 15], such that collisions and interferences do not occur.

In this paper, the energy model that we are dealing with accounts for only the transmission power (see [29]) since this can be a good approximation in the case of long-range techniques, although the actual energy consumption is given by a fixed part (receiving power and the power needed to keep the electric circuits on) plus the transmission power component. The authors in [29] also mention the number of techniques for reducing reception cost by reducing the number of packets received by, but not intended for, a node.

Many papers considered *fractional* version of the problem when splitting of data packets into fractional portions is allowed. This versions admits an easy polynomial time algorithm via linear programming, c.f., [2, 10, 14, 16, 23, 32]. As data packets are usually quite small, there are situations where splitting of packets into fractional ones is neither desirable nor practical. We consider a model where data packets are considered as units that cannot be split, i.e., when the packet flows are of *integral* values only. This discrete version was introduced by Sahni and Park [25].

To summarize, our communication network works under the following conditions:

- The links between nodes are directed.
- The nodes use unidirectional antennas.
- TDMA scheduling scheme is applied.
- Data packets cannot be split.
- The cost for message receipt is negligible.

1.2 Formal definition of the problems

In many *Network Design* problems one seeks a subgraph H with prescribed properties that minimizes/maximizes a certain objective function. Such problems are vastly studied in Combinatorial

Optimization and Approximation Algorithms. Some known examples are Max-Flow, Min-Cost k-Flow, Maximum b-Matching, Minimum Spanning/Steiner Tree, and many others. See, e.g., [27, 12].

Definition 1.1 *An* out-arborescence*, or simply an* arborescence*, is a directed tree that has a path from a root r to every node; an* in-arborescence *is a directed tree that has a path from every node to r. For a node v of a graph H, let $\delta_H(v)$ denote the set of edges leaving v in H, and let $\delta_H^{in}(v)$ denote the set of edges entering v in H.*

Formally, we consider variants of the following problem, which is the "wireless" variant of the classic arborescence packing problems. Recall that for an edge $e = (u, v)$, $w(e)$ is the transmission power needed at u so that v can receive transmission from u.

Maximum Network Lifetime Broadcast
Instance: A directed graph $G = (V, E)$ with edge-weights $\{w(e) : e \in E\}$, battery capacities $\{b(v) : v \in V\}$, and a root $r \in V$.
Objective: Find a maximum size collection $\mathcal{T} = \{T_1, \ldots, T_k\}$ of k out-arborescences in G rooted at r that satisfies the *energy constraints*

$$\sum_{i=1}^{k} w(\delta_{T_i}(v)) \leq b(v) \quad \text{for all } v \in V \tag{1}$$

Another two variants of the problem are:

- **Maximum Network Lifetime Convergecast:** here we seek a maximum size collection of in-arborescences that are directed to r.
- **Maximum Network Lifetime Mixedcast:** here we are also given an integer $\gamma \geq 0$, and the goal is to find the maximum integer k so that k Broadcast and γk Convergecast rounds can be performed.

A related problem is minimizing the battery capacity b so that for $b(v) = b$ for all $v \in V$ at least k rounds of communications can be performed. Formally, this problem can be stated as follows:

Minimum Battery k-Broadcast
Instance: A directed graph $G = (V, E)$ with edge-weights $\{w(e) : e \in E\}$, a root $r \in V$, and an integer k.
Objective: Find a minimum battery capacity b so that if $b(v) = b$ for all $v \in V$, then there exists a collection $\mathcal{T} = \{T_1, \ldots, T_k\}$ of k arborescences in G rooted at r so that (1) holds.

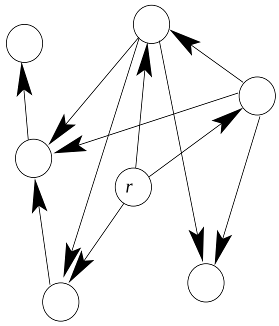
This problem also has the following two variants:

- **Minimum Battery k-Convergecast:** here we seek a collection of k in-arborescences, that are directed to r.
- **Minimum Battery k-Mixedcast:** here we are also given an integer $\gamma \geq 0$, and the goal is to perform k Broadcast and γk Convergecast rounds.

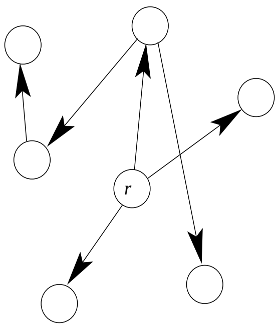
More generally, we characterize the class of **Maximum Network Lifetime** problems as follows. In these problems, every node v has a limited battery capacity $b(v)$, and a transmission energy $w(v, u)$ to any other node u is known. In transmission round i, we choose a subnetwork H_i with given properties, and every node transmits one message to each one of its neighbors in H_i; in many applications, each H_i is an arborescence (see [9]). The goal is to maximize the *lifetime* of the network, that is to find a maximum length feasible sequence $H_1, H_2, \ldots, H_k$ of subnetworks; feasibility means that every graph H_i satisfies the required properties, and that for every node v the total transmission energy during all rounds is at most $b(v)$. This is the *Multiple Topology* version of the problem. In the *Single Topology* variant, all the networks H_i are identical, c.f., [9] for more details. Figure 1 shows the difference in obtained solution for single and multiple topology variants in the case of broadcast. We note that in [21] was given a constant ratio approximation algorithm for the case when each H_i should be an st-path, for given two nodes $s, t \in V$. Here we consider **Rooted Maximum Network Lifetime** problems, when each H_i is an arborescence rooted from/to the root r.

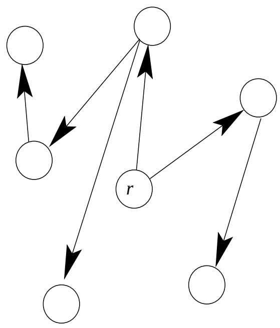
In a more general setting, we might also be given a *cost-function* c on the edges, which can be distinct from the *weight-function* w, and an integer k, and wish to find a feasible sequence $H_1, H_2, \ldots, H_k$ of k communication subnetworks while minimizing their total cost $\sum_{i=1}^{k} c(H_i)$. We call this variant the **Min-Cost Network k-Lifetime** problem, and **Min-Cost Rooted Network k-Lifetime** problem in the Broadcast/Convergecast/Mixedcast cases.

While the Convergecast problems considered in this paper assume aggregation, we also deal with the problem **Partial Level Aggregation Convergecast** where we want to find a tree T of G directed towards the root node r that satisfies the energy constraints $\sum w(\delta_T(v)) \leq \frac{b(v)}{level(v)}$ for all $v \in V$, where $level(v)$ is the length of the longest path between v and its descendant in T. This problem arises in the context of sensor networks consisting of a number of sensors distributed randomly in a geographical region. A sensor v is said to be *correlated* to a set of sensors $K \subset V$ if the data measured by v can be inferred/computed from the data measured by the sensors of K within an acceptable error bound as defined by the application, see [13]. Partial Level Aggregation Convergecast problem corresponds to the case where the information gathering process (towards the root) defines correlated sensors located at the same level of the convergecast tree and having the same parent (normally, this corresponds to the same proximity geographic area). In this way, the root of the tree needs to know only one piece of information transmitted by one of the correlated



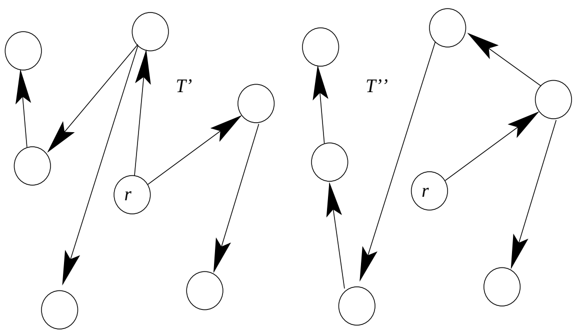
a) G=(V,E), b(v) = 6, w(e) =1.





b) Single Topology Broadcast Tree. Lifetime=2, since r can be activated only twice.

c) Single Topology Optimal Broadcast Tree. Lifetime=3.



d) Multiple Topology Optimal Broadast Trees: T’ and T’’ Lifetime=4, by running twice T’ and twice T’’.

Figure 1: (a) Initial input network $G = (V, E)$, with all batteries equal to 6 and weight of each edge is 1; (b) some broadcast tree having lifetime of 2 since for every activation root r spends 3 units of energy; (c) optimal (not unique) broadcast tree having lifetime 3; (d) multiple broadcast trees may lead to an increase in lifetime (in this case, 4), although the lifetime of each one of them is only 3, since every tree T' and T'' has a vertex with 2 outcoming edges.

sensors.

1.3 Previous work

The authors in [22] show that for Broadcast, the problem is NP-Hard in the case of Single Source/Single Topology and has a polynomial solution for fractional version in the case of Single Source/Multiple Topology. They also show that it is NP-Hard in both of these cases for multicast. Segal [28] improved the running time of the solution for the Broadcast protocol and also showed an optimal polynomial time algorithm for the integral version of Single Topology Convergecast; the latter algorithm simply finds, using binary search, the largest integer k so that the graph $G - \{(u,v) \in E : w(u,v) > b(v)/k\}$ contains an arborescence directed to the root. For Multiple Topology Convergecast fractional version Kalpakis et al. [16] does have a polynomial solution in $O(n^{15}\log n)$ time. To counter the slowness of the algorithm, Stanford and Tongngam [30] proposed a $(1-\varepsilon)$-approximation in $O(n^3 \frac{1}{\varepsilon} \log_{1+\varepsilon} n)$ time based on the algorithm of Garg and Könemann [11] for packing linear programs. Elkin et al. [9] gave an $\Omega(\lfloor 1/\log n \rfloor)$-approximation for the discrete version of Multiple Topology Convergecast problem. Regarding the case without aggregation, some partial results were given in [1], [19] and [31]. The paper [31] considered the conditional aggregation where data from one node can be compressed in the presence of data from other nodes. Liang and Liu [19] present a number of heuristics for different types of aggregation problems. Buragohain et al. [1] proved the hardness of optimal routing tree problem for non-aggregate queries.

1.4 Our contributions

Let k^* denote the optimal value of a Maximum Network Lifetime problem at hand. We observe that the Broadcast version is APX-hard even for unit weights; (however, Convergcast version with unit weights is polynomially solvable). In fact, for Broadcast, even determining whether $k^* \geq 1$ is NP-complete [9]. Hence it seems that an approximation of the type $\lfloor k^*/\rho \rfloor$ is the best one can expect. We provide polynomial time algorithms with constant ρ for Rooted Maximum Network Lifetime Broadcast/Convergecast/Mixedcast; in particular, this improves the ratio $\rho = O(\log n)$ established in [9] for the Broadcast case. Our main result can be stated as follows.

Theorem 1.1 *The* Rooted Maximum Network Lifetime *problem admits a polynomial time algorithm that finds a solution of value $\ell \geq \lfloor k^*/\beta^2 \rfloor$ where:*

- *In the Single Topology case: $\beta = 1$ for Convergecast, $\beta = 5$ for Broadcast and $\beta = 6$ for Mixedcast.*
- *In the Multiple Topology case: $\beta = 4$ for Convergecast, $\beta = 6$ for Broadcast, and $\beta = 10$ for Mixedcast.*

Furthermore, for Min-Cost Rooted Network Lifetime *the algorithm computes* $\ell \geq \lfloor k/\beta^2 \rfloor$ *arborescences so that their total cost is at most the total cost of* k *cost-optimal arborescences.*

Recall that for Broadcast, even checking whether $k^* \geq 1$ is NP-complete [9], and thus an approximation of the form $\lfloor k^*/\rho \rfloor$ is the best one can expect. However, as Single Topology Convergecast is in P [28], for Multiple Topology Convergecast checking whether $k^* \geq 1$ can be done in polynomial time. This implies:

Corollary 1.2 *Multiple Topology version of* Rooted Maximum Lifetime Convergecast *problem admits a* $1/31$*-approximation algorithm.*

Proof: We return the better solution among two algorithms. The first algorithm is as in Theorem 1.1 that returns a solution of size $\ell \geq \lfloor k^*/16 \rfloor$. The second algorithm is the one that checks whether $k^* \geq 1$; if so, it returns a single feasible arborescence, otherwise it declares that no feasible arborescence exists (namely, returns $\mathcal{T} = \emptyset$).

Now, if $k^* = 0$ then the returned solution is optimal. If $1 \leq k^* \leq 31$, then we return a single arborescence. Finally, if $k^* \geq 32$, then we return at least $\ell \geq \lfloor k^*/16 \rfloor$ arborescences. Following this, the worst case is for $k^* = 31$ where we return a single arborescence. Thus, the approximation ratio is $1/31$. $\square$

The following table summarizes our ratios for variants of Rooted Maximum Network Lifetime problem.

Single Topology			**Multiple Topology**		
Convergecast	*Broadcast*	*Mixedcast*	*Convergecast*	*Broadcast*	*Mixedcast*
k^*	$\lfloor k^*/25 \rfloor$	$\lfloor k^*/36 \rfloor$	$\max\{\lfloor k^*/16 \rfloor, 1\}$	$\lfloor k^*/36 \rfloor$	$\lfloor k^*/100 \rfloor$

Table 1: Summary of the ratios for variants of Rooted Maximum Network Lifetime problem. Except the polynomial solvability of the Single Topology Convergecast that has been proved in [28], the other ratios are proved in this paper.

Theorem 1.3 Minimum Battery Rooted k-Broadcast/Convergecast/Mixedcast *problem admits a* β*-approximation algorithm, where* β *is as in Theorem 1.1.*

For Partial Level Aggregation Convergecast we gave a number of optimal and approximate solutions and discuss the possibilities to extend them to more general case.

This paper is organized as follows. In Section 2 we briefly survey the results from [20] for the Weighted Degree Constrained Network Design problems, which are the basis for our algorithms. Theorems 1.1 and 1.3 are proved in Section 3. In Section 4 we consider the model with partial level aggregation mechanism. Finally, some conclusions are presented in Section 5.

2 Weighted Degree Constrained Network Design

In what follows we present a number of observations from a graph theory that lie in the basis of our algorithms. Our proof of Theorems 1.1 and 1.3 is based on a recent result due to Nutov [20] for directed Weighted Degree Constrained Network Design problems with intersecting supermodular demands. The purpose of this section is to present this result (see Theorem 2.1 at the end of this section). In Degree Constrained Network Design problems (without weights) one seeks a subgraph H of a given graph G that satisfies both prescribed connectivity requirements and degree constraints. One such type of problems are the Matching/Edge-Cover problems, which are solvable in polynomial time, c.f., [27]. For other degree constrained problems, checking whether there exists a feasible solution is NP-complete. Hence one considers approximation algorithms when the degree constraints are relaxed.

The connectivity requirements can be specified by a set function f on V, as follows.

Definition 2.1 *For an edge set of a graph H and node set S let $\delta_H(S)$ ($\delta_H^{in}(S)$) denote the set of edges in H leaving (entering) S. Given a set-function f on subsets of V and a graph H on V, we say that H is* f-connected *if*

$$|\delta_H^{in}(S)| \geq f(S) \quad \textit{for all } S \subseteq V. \tag{2}$$

Several types of f are considered in the literature, among them the following known one:

Definition 2.2 *A set function f on V is* intersecting supermodular *if any $X, Y \subseteq V$ with $X \cap Y \neq \emptyset$ satisfy the* supermodularity condition

$$f(X) + f(Y) \leq f(X \cap Y) + f(X \cup Y)\ . \tag{3}$$

Nutov [20] considered network design problems with *weighted-degree* constraints. The *weighted degree* of a node v in a graph H with edge-weights $w(e)$ is $w(\delta_H(v)) = \sum_{e \in \delta_H(v)} w(e)$.

Directed Weighted Degree Constrained Network (DWDCN)
Instance: A directed graph $G = (V, E)$ with edge-weights $\{w(e) : e \in E\}$, a set-function f on V, and degree bounds $\{b(v) : v \in V\}$.
Objective: Find an f-connected subgraph H of G that satisfies the *weighted degree constraints*

$$w(\delta_H(v)) \leq b(v) \quad \text{for all } v \in V \tag{4}$$

The function f is usually not given explicitly, but is assumed to admit an evaluation oracle (or other relevant oracles). Since for many functions f, DWDCN is NP-complete, one considers approximation algorithms. A β-approximation algorithm for DWDCN either computes an f-connected subgraph H of G that satisfies $w(\delta_H(v)) \leq \beta \cdot b(v)$ for all $v \in V$, or correctly determines that the problem has no feasible solution. Note that even if the problem does not have a feasible solution, the algorithm may still return a subgraph that violates the degree constraints (4) by a factor of β.

For an edge set I, let $x(I) = \sum_{e \in I} x(e)$. A natural LP-relaxation for DWDCN is obtained by considering the following polytope P_f:

$$\begin{array}{rcllr} x(\delta_G^{in}(S)) & \geq & f(S) & \text{for all } S \subset V & \text{(Cut Constraints)} \\ \sum_{e \in \delta_G(v)} x(e)w(e) & \leq & b(v) & \text{for all } v \in V & \text{(Weighted Degree Constraints)} \\ 0 \leq x(e) & \leq & 1 & \text{for all } e \in E & \end{array}$$

Similarly, we may consider the version of DWDCN where the Cut Constraints are on edges leaving S, namely when we require $x(\delta_G(S)) \geq f(S)$ for all $S \subset V$. In the *min-cost version* of the problem, we are also given *edge-costs* $\{c(e) : e \in E\}$ and require that H has low cost. In this case, the LP-relaxation seeks to minimize $c \cdot x$ over the polytope P_f.

Let us fix parameters α and β as follows:

- Single Topology Convergecast, or DWDCN with Cut Constraints on the edges entering S and $0,1$-valued f:
  $\alpha = \beta = 1$.

- Single Topology Broadcast, or DWDCN with Cut Constraints on the edges leaving S and $0,1$-valued f:
  $\alpha = 2$ and $\beta = 5$.

- Multiple Topology Convergecast, or DWDCN with Cut Constraints on the edges entering S:
  $\alpha = 1$ and $\beta = 4$.

- Multiple Topology Broadcast, or DWDCN with Cut Constraints on the edges leaving S:
  $\alpha = 3$ and $\beta = 6$.

Theorem 2.1 ([20]) DWDCN *with intersecting supermodular f admits a polynomial time algorithm that computes an f-connected graph H so that the weighted degree of every $v \in H$ is at most $\beta \cdot b(v)$. Furthermore, in the min-cost version of the problem, $c(H) \leq \alpha \cdot \tau^*$ where $\tau^* = \min\{c \cdot x : x \in P_f\}$.*

3 Proof of Theorems 1.1 and 1.3

Now we are ready to prove Theorems 1.1 and 1.3 based on Theorem 2.1. We start with some definitions. A graph H is *k-edge-outconnected from r* if it has k-edge-disjoint paths from r to any other node; H is *k-edge-inconnected from r* if it has k-edge-disjoint paths from every node to r.

The DWDCN problem includes as a special case the Weighted Degree Constrained k-Outconnected Subgraph problem, by setting $f(S) = k$ for all $\emptyset \neq S \subseteq V \setminus \{r\}$, and $f(S) = 0$ otherwise. For $k = 1$ we get the Weighted Degree Constrained Arborescence problem. Note that our problem are equivalent to the problem of packing a maximum number k^* of edge-disjoint arborescences rooted at r such that their union H satisfies (4). By Edmond's Theorem [6], this is equivalent to requiring that H is k^*-edge-outconnected from r (or k^*-edge in-connected to r, in the case of Convergecast) and satisfies (4). This gives the following problem:

Weighted-Degree Constrained Maximum k-Outconnected Subgraph
Instance: A directed graph $G = (V, E)$ with edge-weights $\{w(e) : e \in E\}$, degree bounds $\{b(v) : v \in V\}$, and a root $r \in V$.
Objective: Find a k-edge-outconnected from r spanning subgraph H of G that satisfies the weighted degree constraints (4) so that k is maximized.

In the **Weighted-Degree Constrained Maximum k-Inconnected Subgraph** problem, we require from H to be k-edge-inconnected to r.

A natural LP-relaxation for the Weighted-Degree Constrained Maximum k-Outconnected Subgraph problem seeks to find the maximum integer k so that the following polytope P_k is non-empty:

$$\begin{array}{rcll}
x(\delta_G^{in}(S)) & \geq & k & \text{for all } \emptyset \neq S \subseteq V \setminus \{r\} \qquad \text{(Cut Constraints)} \\
\sum_{e \in \delta_G(v)} x(e)w(e) & \leq & b(v) & \text{for all } v \in V \qquad \text{(Weighted Degree Constraints)} \\
0 \leq x(e) & \leq & 1 & \text{for all } e \in E
\end{array}$$

Namely, here we have $P_k = P_f$ for

$$f(S) = \begin{cases} k & \text{if } \emptyset \neq S \subseteq V \setminus \{r\} \\ 0 & \text{otherwise} \end{cases}$$

In the Convergecast case the Cut Constraints are on edges leaving S, namely we have $x(\delta_G(S)) \geq f(S)$ for all $S \subset V$, with f as defined above. In both cases, the function f is intersecting supermodular, hence Theorem 2.1 applies.

The Weighted-Degree Constrained Maximum k-Outconnected/k-Inconnected Subgraph problem is related to our problem via binary search. Note that the maximum number k^* of arborescences is in the range $0, \ldots, nq$ where

$$q = \max_{v \in V} \frac{b(v)}{\min\{w(e) : e \in \delta_G(v), w(e) > 0\}} .$$

Indeed, if G contains an arborescence of weight 0, then k^* is infinite. Otherwise, every arborescence contains a node v that uses an edge $e \in \delta_G(v)$ with $w(e) > 0$. As there are n nodes, this implies the bound $k^* \leq nq$. As an edge of G may be used several times, we add nq copies of each edge of G. Equivalently, we may assign to every edge capacity nq, and consider the corresponding "capacitated" problems; this will give a polynomial algorithm, rather than a pseudo-polynomial one. For simplicity of exposition, we will present the algorithm in terms of multigraphs, but it can be easily adjusted to capacited graphs.

As has been mentioned, for the Convergecast case checking whether $k^* \geq 1$ can be done in polynomial time [28]. Now we observe that Theorem 2.1 implies Theorem 1.3, as well as a "pseudo-approximation" algorithm for Weighted-Degree Constrained Maximum k-Outconnected/k-Inconnected Subgraph:

Corollary 3.1 Weighted-Degree Constrained Maximum k-Outconnected/k-Inconnected Subgraph *problems admit a polynomial time algorithm that either correctly establishes that the polytope P_k is empty, or finds a k-outconnected subgraph H that violates the energy constraints by a factor at most β, namely*

$$\sum_{e \in \delta_H(v)} w(e) \leq \beta \cdot b(v) \quad \text{for all } v \in V \ . \tag{5}$$

The above corollary immediately implies Theorem 1.3. We show how to derive from it also Theorem 1.1. For Broadcast, the algorithm from Theorem 1.1 is as follows:

1. Set $b'(v) \leftarrow b(v)/\beta$ for all $v \in V$, where β is as in Corollary 3.1.
2. Find the maximum integer ℓ so that for bounds $b'(v)$ the algorithm as in Corollary 3.1 returns an ℓ-outconnected from r spanning subgraph H of G so that $\sum_{e \in \delta_H(v)} w(e) \leq \beta \cdot b'(v)$ for all $v \in V$.

In the Convergecast case H is ℓ-inconnected to r. In the Mixedcast case H is the union of an ℓ-inconnected to r and $\gamma\ell$-outconnected from r spanning subgraphs denoted by H_{out} and H_{in}, each computed using the corresponding parameters β_{out} and β_{in}.

The maximum integer ℓ at Step 2 of the algorithm can be found using binary search in the range $0 \leq \ell \leq nq$, so the running time is indeed polynomial. Also, the computed solution H is feasible since $\sum_{e \in \delta_H(v)} w(e) \leq \beta \cdot b'(v) = b(v)$ for all $v \in V$.

For the approximation ratio, all we need to prove is that if the original instance admits a k-outconnected/k-inconnected subgraph, then the new instance with weighted-degree bounds $b(v)/\beta$ admits an ℓ-outconnected/ℓ-inconnected spanning subgraph with $\ell = \lfloor k/\beta^2 \rfloor$; in the min-cost version we also need that H has low cost. This is achieved via the following lemma.

Lemma 3.2 *Let H_k be a k-outconnected from r (k-inconnected to r) directed graph with costs $c(e)$ and weights $w(e)$ on the edges. Then for any integer $0 \leq \ell \leq k$ the graph H_k contains an ℓ-outconnected from r (an ℓ-inconnected to r) spanning subgraph H_ℓ so that $w(\delta_{H_\ell}(v)) \leq w(\delta_{H_k}(v)) \cdot (\beta\ell/k)$ for all $v \in V$. Furthermore, $c(H_\ell) \leq c(H_k) \cdot (\alpha\ell/k)$ in the min-cost version.*

Proof: Consider the Weighted Degree Constrained ℓ-Outconnected Subgraph (Weighted Degree Constrained ℓ-Inconnected Subgraph) problem on H_k with degree bounds $b(v) = w(\delta_{H_k}(v)) \cdot (\ell/k)$. Clearly, $x(e) = \ell/k$ for every $e \in F$ is a feasible solution of cost $c(H_k) \cdot (\ell/k)$ to the LP-relaxation $\min\{c \cdot x : x \in P_\ell\}$. By Corollary 3.1, our algorithm computes a subgraph H_ℓ as required. $\square$

Substituting $\ell = \lfloor k/\beta^2 \rfloor$ in Lemma 3.2 and observing that $\alpha \cdot \lfloor k/\beta^2 \rfloor / k \leq 1$ in all cases, we obtain:

Corollary 3.3 *Let H be a k-outconnected from r (k-inconnected to r) directed graph with edge weights $w(e)$. Then H contains a subgraph H' so that H' is $\lfloor k/\beta^2 \rfloor$-outconnected from r ($\lfloor k/\beta^2 \rfloor$-inconnected to r), $c(H') \leq c(H)$, and $w(\delta_{H'}(v)) \leq w(\delta_H(v))/\beta$ for all $v \in V$.*

Except the Mixedcast part, Theorem 1.1 is easily deduced from Corollaries 3.1 and 3.3. For Mixedcast, note that $\beta = \beta_{out} + \beta_{in}$, where β_{out} and β_{in} are the parameters in Theorem 1.1 for Broadcast and Convergecast, respectively. From Lemma 3.2, we obtain that if H_k is k-outconnected from r and γk-inconnected to r, then for any $\ell \leq k$, H_k contains two spanning subgraphs: H_{out} that is ℓ-outconnected from r and H_{in} that is $\gamma\ell$-inconnected to r satisfying:

$$\begin{aligned} w(\delta_{H_{out}}(v)) + w(\delta_{H_{in}}(v)) &\leq w(\delta_H(v)) \cdot \beta_{out} \cdot (\ell/k) + w(\delta_H(v)) \cdot \beta_{in} \cdot (\gamma\ell/\gamma k) \\ &= w(\delta_H(v)) \cdot (\ell/k) \cdot (\beta_{out} + \beta_{in}) = w(\delta_H(v)) \cdot (\beta\ell/k) \ . \end{aligned}$$

Then, similarly to Corollary 3.3, we deduce that if H is k-outconnected from r and γk-inconnected to r, then H contains a subgraph H' so that H' is $\lfloor k/\beta^2 \rfloor$-outconnected from r and $\lfloor \gamma k/\beta^2 \rfloor$-inconnected to r, $c(H') \leq c(H)$, and $w(\delta_{H'}(v)) \leq w(\delta_H(v))/\beta$ for all $v \in V$.

4 Partial Level Aggregation Convergecast

This section is devoted to the partial level aggregation convergecast problem that rises in the context of correlated sensors. In the problem of Partial Level Aggregation Convergecast we want to find a tree T of G directed towards the root node r that satisfies the energy constraints $\sum w(\delta_T(v)) \leq \frac{b(v)}{level(v)}$ for all $v \in V$, where $level(v)$ is the length of the longest path between v and its descendant in T. We consider the following cases.

- **Uniform initial batteries.** In this case, we note that the optimal solution is achieved by the tree of minimal depth (in regard to tree's root). We can find such tree by choosing every vertex to serve as the root, building the Breadth First Search tree starting at the chosen vertex, and picking up the tree of minimal depth. The total time complexity of the proposed algorithm is $O(|V|(|V| + |E|))$. Notice, that the problem becomes NP-complete when we are aiming to find the tree of maximal depth (HAMILTONIAN PATH).
- **Arbitrary initial batteries.** The simplest way to do is to use the above mentioned algorithm and to obtain $B_{\max}/B_{\min}$ approximate solution, where $B_{\max} = \max_{v \in V} b(v)$ and $B_{\min} =$

$\min_{v \in V} b(v)$. We mention that for the case of complete graph, the optimal solution is the star, rooted at the node with minimal battery charge.

We can slightly change the definition of the problem in order to introduce the notion of weighted edges. To reflect this change, we transform the energy constraint to be

$$\sum w(\delta_T(v)) \leq \frac{b(v)}{w(e_1) + w(e_2) + \ldots + w(e_h)},$$

where $w(e_i)$ is the weight of the edge e_i located on the most energy consumed path between v and its descendant in T. For arbitrary initial batteries and arbitrary edge weights we will use the construction based on Hamiltonian circuit and presented at Elkin et al. [9] where G is the complete graph. The authors at [9] have shown how to construct a spanning tree T of G that has a bounded hop-diameter of $O(n/\rho + \log \rho)$ with $w(e_T^*) = O(\rho^2 w(e^*))$, where e_T^* and e^* are the longest edges in T and the MST of G, respectively. The tightness of the tradeoff has been also established in [9]. Following this, the best approximation factor that can be given for this problem is $\Omega(n)$ which is achieved by $\rho = 1$.

5 Conclusions

In this paper we consider a number of broadcast and convergecast problems in wireless settings under the criterion of maximizing the lifetime of the underlying wireless backbone. For unidirectional antennas, we present constant factor approximate solutions improving previously known results as well as extending them and dealing with different aggregation cases. One of the main open questions is obtaining a non-trivial approximation solutions for the case of omnidirectional antennas.

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References

[1] D. C. Buragohain, D. Agrawal and S. Suri. Power aware routing for sensor databases. In *INFOCOM*, pages 1747–1757, 2005.

[2] J. Chang and L. Tassiulas. Maximum lifetime routing in wireless sensor networks. *IEEE/ACM Transaction on Networking*, 12(4):609–619, 2004.

[3] S. Chatterjee, S. Roy, S. Bandyopadhyay, T. Ueda, H. Iwai, and S. Obana. A power aware routing strategy for ad hoc networks with directional antenna optimizing control traffic and power consumption. In *IWDC*, pages 275–280, 2005.

[4] H. Dai, K.-W. Ng, and M.-Y. Wu. A busy-tone based mac scheme for wireless ad hoc networks using directional antennas. In *GLOBECOM*, pages 4969–4973, 2007.

[5] B. Deb and B. Nath. On the node-scheduling approach to topology control in ad hoc networks. In *ACM MOBIHOC*, pages 14–26, 2005.

[6] J. Edmonds. Matroid intersection. *Annals of discrete Mathematics*, 4:185–204, 1979.

[7] T. A. ElBatt and A. Ephremides. Joint scheduling and power control for wireless ad-hoc networks. In *IEEE INFOCOM*, pages 976–984, 2002.

[8] T. A. ElBatt and A. Ephremides. Joint scheduling and power control for wireless ad hoc networks. *IEEE Transactions on Wireless Communications*, 3(1):74–85, 2004.

[9] M. Elkin, Y. Lando, Z. Nutov, M. Segal, and H. Shpungin. Novel algorithms for the network lifetime problem in wireless settings. In *ADHOC-NOW*, pages 425–438, 2008.

[10] P. Floréen, P. Kaski, J. Kohonen, and P. Orponen. Exact and approximate balanced data gathering in energy-constrained sensor networks. *Theor. Comp. Sci.*, 344(1):30–46, 2005.

[11] N. Garg and J. Könemann. Faster and simpler algorithms for multicommodity flow and other fractional packing problems. In *FOCS'98*, pages 300–309, 1998.

[12] T. F. Gonzalez, editor. *Handbook on Approximation Algorithms and Metaheuristics*. Chapman & Hall/CRC, 2007.

[13] H. Gupta, V. Navda, S. R. Das, and V. Chowdhary. Efficient gathering of correlated data in sensor networks. In *MobiHoc '05: Proceedings of the 6th ACM international symposium on Mobile ad hoc networking and computing*, pages 402–413, New York, NY, USA, 2005. ACM.

[14] B. Hong and V. K. Prasanna. Maximum lifetime data sensing and extraction in energy constrained networked sensor systems. *J. Parallel and Distributed Computing*, 66(4):566–577, 2006.

[15] G. D. N. J. E. Wieselthier and A. Ephremides. Algorithms for energy-effcient multicasting in static ad hoc wireless networks. *ACM MONET*, 6(3):251–263, 2001.

[16] K. Kalpakis, K. Dasgupta, and P. Namjoshi. Efficient algorithms for maximum lifetime data gathering and aggregation in wireless sensor networks. *Computer Networks*, 42(6):697–716, 2003.

[17] Y.-B. Ko, V. Shankarkumar, and N. H. Vaidya. Medium access control protocols using directional antennas in ad hoc networks. In *INFOCOM*, pages 13–21, 2000.

[18] S. O. Krumke, R. Liu, E. L. Lloyd, M. V. Marathe, R. Ramanathan, and S. S. Ravi. Topology control problems under symmetric and asymmetric power thresholds. In *AdHoc-NOW'03*, pages 187–198, 2003.

[19] W. Liang and Y. Liu. Online data gathering for maximizing network lifetime in sensor networks. *IEEE Transactions on Mobile Computing*, 6(1):2–11, 2007.

[20] Z. Nutov. Approximating directed weighted-degree constrained networks. In *APPROX*, pages 219–232, 2008.

[21] Z. Nutov. Approximating maximum integral flows in wireless sensor networks via weighted-degree constrained k-flows. In *DIALM-POMC*, pages 29–34, 2008.

[22] A. Orda and B.-A. Yassour. Maximum-lifetime routing algorithms for networks with omnidirectional and directional antennas. In *MobiHoc'05*, pages 426–437, 2005.

[23] F. Ordonez and B. Krishnamachari. Optimal information extraction in energy-limited wireless sensor networks. *IEEE Journal on Selected Areas in Communications*, 22(6):1121–1129, 2004.

[24] K. Pahlavan and A. H. Levesque. *Wireless information networks*. Wiley-Interscience, 1995.

[25] J. Park and S. Sahni. Maximum lifetime broadcasting in wireless networks. *IEEE Transactions on Computers*, 54(9):1081–1090, 2005.

[26] R. Ramanathan and R. Hain. Topology control of multihop wireless networks using transmit power adjustment. In *INFOCOM'00*, pages 404–413, 2000.

[27] A. Schrijver. *Combinatorial Optimization Polyhedra and Efficiency*. Springer-Verlag Berlin Heidelberg New York, 2004.

[28] M. Segal. Fast algorithm for multicast and data gathering in wireless networks. *Information Processing Letters*, 107(1):29–33, 2008.

[29] W.-Z. Song, X.-Y. Li, O. Frieder, and W. Wang. Localized topology control for unicast and broadcast in wireless ad hoc networks. *IEEE Trans. Parallel Distrib. Syst.*, 17(4):321–334, 2006.

[30] J. Stanford and S. Tongngam. Approximation algorithm for maximum lifetime in wireless sensor networks with data aggregation. In *SNPD'06*, pages 273–277, 2006.

[31] P. von Rickenbach and R. Wattenhofer. Gathering correlated data in sensor networks. In *DIALM-POMC*, pages 60–66, 2004.

[32] Y. Xue, Y. Cui, and K. Nahrstedt. Maximizing lifetime for data aggregation in wireless sensor networks. *Mobile Networks and Applications*, 10:853–864, 2005.