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Energy and Lifetime Efficient Connectivity in Wireless Ad-Hoc Networks

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The temporary and unfixed physical topology of a wireless ad-hoc network is determined by the distribution of the wireless nodes as well as the transmission power (range) assignment of each node. This paper studies asymmetric power assignments for which the induced communication graph is k-strongly connected, while minimizing the total energy assigned and maximizing the network lifetime.

We show that our power assignment algorithm from [9] achieves a bicriteria approximation ratio of $(O(k), O(k \log n \sqrt[k]{n\varphi(n)}))$ with high probability, where $\varphi(n)$ is any function with $\lim_{n\to\infty} \varphi(n) = \infty$, for the minimal total cost/maximal network lifetime problem in the plane, respectively, in the case of arbitrary battery charges. The same algorithm provides an $(O(k), O(1))$-approximation in the case of uniform batteries. To the best of our knowledge, this is the first attempt to provide a bicriteria approximation ratio for the total power assignment cost and the network lifetime under the k-fault resilience criterion. In addition, we study the special cases where the nodes are located on a torus or along a line. In the linear case with uniform batteries, we show that our algorithm from [33] obtains an $(O(1), 1)$-approximation. For the toral case, we suggest an assignment which is simultaneously the optimum in terms of each of the criteria.

Keywords:

1 INTRODUCTION

A wireless ad-hoc network consists of several transceivers (nodes) located in the plane, communicating by radio. The underlying physical topology of the network depends on the distribution of the wireless nodes (location) as well as the transmission power (range) assignment of each node.

In addition to the general case, there are several special cases of nodes layout. When the wireless nodes are positioned on a line, e.g., antennas along a highway, the network is called a *linear network*. Another layout we consider in this paper is when the nodes are located on a grid in the torus [14,18–20]. In this type of a network, there are no edge effects (as opposed to a grid in the plane).

The transmission range r_t of node t is determined by the power assigned to that node, denoted by $p(t)$. It is customary to assume that the minimal transmission power required to transmit to distance d is d^α, where the *distance-power gradient* α is usually taken to be in the interval [2, 4] (see [29]). Thus, node t receives transmissions from s if $p(s) \geq d(s,t)^\alpha$, where $d(s,t)$ is the Euclidean distance between s and t. In this work we assume $\alpha = 2$ for simplicity, although our results can be easily extended to any constant α.

There are two possible models: symmetric and asymmetric. In the symmetric settings, also referred to as the undirected model, there is an undirected communication link between two nodes $u, v \in T$ if $p(u) \geq d(u,v)^\alpha$ and $p(v) \geq d(v,u)^\alpha$, that is node u can reach node v and node v can reach node u. The asymmetric variant allows directed (one-way) communication links between two nodes. Krumke *et al.* [26] argued that the asymmetric version is harder than the symmetric one. This paper addresses the asymmetric model.

The most fundamental problem in wireless ad-hoc networks is to find a power assignment which induces a communication graph that satisfies some topology property. Two natural optimization objectives arise, *minimizing the total energy consumption* and *maximizing the network lifetime*.

This paper is organized as follows. In the rest of this section, we present the model, discuss previous work and state our results. In Sections 2, 3, 4 we derive a bicriteria approximation factor for the linear, planar and toral layout of nodes. Section 5 presents some simulation results, and finally, and in Section 6 we conclude.

1.1 The model

We are given a set $\mathcal{T}$ of n transceivers $t_1, t_2, \ldots, t_n$, positioned in the plane or on the torus, $\mathbb{T}^2$. A power assignment for $\mathcal{T}$ is a vector of transmission powers $A = (p(t))_{t\in\mathcal{T}}$. The transmission possibilities resulting from a power assignment induce a directed communication graph $H_A = (\mathcal{T}, \mathcal{E}_A)$, where $\mathcal{E}_A = \{(s,t) \mid p(s) \geq d(s,t)^\alpha\}$ is the set of directed edges. The cost of the power assignment is given by $C_A = \sum_{t\in\mathcal{T}} p(t)$. For each node $t \in \mathcal{T}$, let $N_t \subseteq \mathcal{T}$ be the set of k nodes closest to t, and let $r_t^* = \max_{t' \in N_t} d(t, t')$. That is, r_t^* is the minimum range so that t has at least k outgoing edges in H_A.

We assume that all the nodes share the same frequency band, and time is divided into equal size slots that are grouped into frames. Thus, the study is conducted in the context of TDMA. In TDMA wireless ad-hoc networks, a transmission scenario is valid if and only if it satisfies the following three conditions:

1. A node is not allowed to transmit and receive simultaneously.
2. A node cannot receive from more than one neighboring node at the same time.
3. A node receiving from a neighboring node should be spatially separated from any other transmitter by at least some distance D.

However, if nodes use unique signature sequences (i.e., a joint TDMA/CDMA scheme), then the second and third conditions may be dropped, and the first condition only characterizes a valid transmission scenario. Thus, our MAC layer is based on TDMA scheduling [15,16,34], such that collisions and interferences do not occur.

The wireless distribution of communication nodes presumes that each transceiver does not have unlimited power supply, but rather has to rely on its own energy sources. Each node t has some initial battery charge b_t, which is sufficient for a limited amount of time, depending on the power assigned to t. It is common to take the lifetime of a wireless node t to be $l_t = b_t / {r_t}^\alpha$. Let $\mathcal{B} = (b_t)_{t \in \mathcal{T}}$ be a vector of initial battery charges. The network lifetime is defined as the time it takes the first node to run out of its battery charge. For a given power assignment A, the network lifetime of the induced communication graph with respect to the initial battery charges $\mathcal{B}$ is

$$l_A(\mathcal{B}) = \min_{t \in \mathcal{T}} l_t = \min_{t \in \mathcal{T}} \frac{b_t}{{r_t}^\alpha}.$$

In the special case where all initial battery charges are equal, that is $b_t = b$ for all $t \in T$, we say that $\mathcal{B}$ is *uniform*. Otherwise, $\mathcal{B}$ is *arbitrary*.

For a power assignment A, the communication graph H_A is *strongly connected* if, for any two nodes $s, t \in \mathcal{T}$, there exists a directed path from s to t in H_A. In this paper, we require that H_A remains strongly connected even if any set of at most $k - 1$ nodes is deleted.[1] We refer to k as the *fault resistance* parameter. If H_A satisfies this requirement, H_A is a *k-fault resistant strongly connected* graph. In short, H_A is k-strongly connected.

This paper addresses the *minimum energy-maximum lifetime k-fault resistant strong connectivity* problem (MEMLkSC).

[1]The deletion of a node removes all edges adjacent to it as well.

Problem 1 (MEMLkSC).

Input:	*A set of nodes $\mathcal{T}$ in $\mathbb{R}^2$ or $\mathbb{T}^2$, a vector of initial battery charges $\mathcal{B}$ and a constant $k \geq 1$.*
Output:	*A power assignment A, so that H_A is a k-fault resistant strongly connected graph.*
Objective:	*Minimize C_A and maximize $l_A(\mathcal{B})$.*

1.2 Previous work

In [32], a formal study of controlling the network topology by adjusting the transmission power of the nodes was initiated. Most of the problems are aimed at computing a low energy power assignment that meets global topological constraints. In this paper we focus on the topological property of strong connectivity (all-to-all). This property is extremely useful in certain applications of wireless networks (e.g., a battlefield or rescue operation).

To the best of our knowledge, no non-trivial results are known for both the total power assignment cost and network lifetime under the k-fault resilience criterion. In what follows, we present previous results separately for estimating the cost of the power assignment and for maximizing the network lifetime.

Total energy consumption. Kirousis *et al.* [25] were the first to study the strong connectivity problem while minimizing the total energy consumption. They proved it to be NP-hard for the 3-dimensional Euclidean space for any value of α. For the planar case, they provided a 2-approximation algorithm. The NP-hardness for the 2-dimensional Euclidean space for any value of α was proved in [12], and a simple 1.5-approximation algorithm for the case $\alpha = 1$ has been provided in [3]. Further results may be found in [2,5,11].

A natural extension to the topology problems above is to impose the constraint of fault resistance. The benefits of a k-fault resistant topology is the multi-path redundancy for load balancing and higher transmission reliability. As power-optimal strong connectivity is NP-hard, so is power-optimal k-strong connectivity. The best approximation result up to date for planar asymmetric k-strong connectivity is due to Carmi *et al.* [9], with an approximation ratio of $O(k)$. Another possible connectivity property is k-edge connectivity, which implies that the removal of any k edges results in a disconnected graph. In [7], Calinescu and Wan presented various aspects of symmetric/asymmetric k-strong connectivity for nodes and edges. Hajiaghayi *et al.* [21] gave an algorithm for symmetric k-strong connectivity with $O(k)$-approximation factor in geometric graphs. Jia *et al.* [23] present various approximation factors (depending on k) for the symmetric k-strong connectivity. Additional results can be found in [1,4,6,13,22,27].

Network lifetime. In the case of uniform battery charges, maximizing the network lifetime is equivalent to minimizing the maximal power assigned to any node. The first to study this problem were Ramanathan and Hain [32], who provided an optimal polynomial time algorithm for this problem under the strong connectivity property. A general approach, which leads to polynomial time algorithms for any monotone[2] property, was developed in [27]. In [5], a PTAS for the problem under various network tasks was developed by devising an LP formulation for the problem. For additional results, see [10,24].

Moscibroda and Wattenhofer [28] tackle the problem for data gathering applications by maximizing the time the network is clustered by a dominating set. They give approximation algorithms with an approximation ratio of $O(\log n)$ for the cases of both uniform and arbitrary battery charges. In the case of uniform batteries, they also add the k-fault resilience criterion. A similar problem, maximizing the residual energy of a node after a broadcast operation, the so-called *critical energy problem*, was studied in [30], for which an optimal polynomial time algorithm was designed. Some results for efficient sensor scheduling and sensing range assignment to maximize the network lifetime can be found in [8].

1.3 NP-Hardness

The MEMLkSC problem has a double optimization objective—total power cost and network lifetime. These objectives are quite similar in the sense that they aim to be energy conserving. Let us introduce notations for the optimal value of each of them for a given set of nodes $\mathcal{T}$ and initial battery charges $\mathcal{B}$:

$$C^* = \min\{C_A : H_A \text{ is } k\text{-strongly connected}\},$$
$$l^* = \max\{l_A(\mathcal{B}) : H_A \text{ is } k\text{-strongly connected}\}.$$

Unfortunately, the optima in the two problems are typically obtained at distinct points. For example, Figure 1 depicts a case where the two objectives cannot be achieved simultaneously even for uniform initial battery charges $\mathcal{B}_U$. There are 5 nodes with distances $d(a,b) = d(d,e) = 10$, and $d(b,c) = d(c,d) = 3$. The minimal cost power assignment A_1, for 1-strong connectivity, is to assign $p(c) = 13^2$, $p(b) = p(d) = 3^2$, and $p(a) = p(e) = 10^2$, with $C^* = C_{A_1} = 387$ and $l_{A_1}(\mathcal{B}_U) = b/169$. While the maximal network lifetime for 1-strong connectivity is obtained in A_2, which assigns $p(c) = 3^2$ and $p(a) = p(b) = p(d) = p(e) = 10^2$, with $l^* = l_{A_2}(\mathcal{B}_U) = b/100$, but $C^* = C_{A_2} = 409$. This fact encourages us to

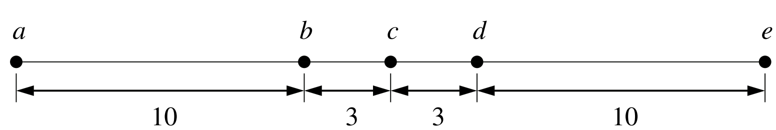


FIGURE 1
Reaching both optimal values in one power assignments is impossible.

[2]A property is *monotone* if it continues to hold when the powers of some nodes are increased and the other remain unchanged.

seek a solution which approximates both. A power assignment A is a *bircreteria (γ, λ)-approximation* to the MEMLkSC problem if $C_A \leq \gamma C^*$ and $l_A(\mathcal{B}) \geq l^*/\lambda$.

The NP-Hardness of the MEMLkSC problem easily follows from the fact that the problem of finding a power assignment with minimum cost that induces a 1-strongly connected graph is NP-Hard [12].

1.4 Our contribution

We study the MEMLkSC problem for the one-, two-, and three-dimensional (toral) cases. We analyze the power assignment algorithms for k-strong connectivity in [9,33] and show that, in addition to admitting provable approximation bounds for the total energy consumption, they also produce a good approximation factor for the maximal network lifetime in the linear and planar cases, respectively. We also show a power assignment which optimizes both the total cost and lifetime in the case of a torus network. In particular, our main contributions are:

1. For the linear layout of nodes (one-dimensional case), we show that the algorithm described in [33] results in an optimal network lifetime in the case of uniform $\mathcal{B}$, which guarantees an $(O(1), 1)$ bicriteria approximation for the MEMLkSC problem.
2. In the planar (two-dimensional) case, we show that the algorithm described in [9] achieves a bicriteria approximation of $(O(k), O(k \log n \sqrt[k]{n\varphi(n)}))$ with high probability, where $\varphi(n)$ is any function with $\lim_{n\to\infty} \varphi(n) = \infty$, for the MEMLkSC problem with arbitrary initial battery charges. For uniform initial battery charges, the same algorithm is an $(O(k), O(1))$-approximation.
3. If the nodes are positioned on a torus (three-dimensional), we construct an optimal power assignment to the MEMLkSC problem for arbitrary battery charges in terms of total power cost and network lifetime.
4. We provide simulation results that compare the network lifetime of the power assignment in [9] to the optimal network lifetime in [32].

2 LINEAR NODE LAYOUT

In this section we address the linear layout of nodes $\mathcal{T}$ and uniform initial battery charges $\mathcal{B}_U$. We show that the power assignment proposed in [33] results in an optimal network lifetime for linear k-strong connectivity.

2.1 Linear k-strong connectivity ([33])

Put $d_i = d(t_i, t_{i+1})$, for $1 \leq i \leq n-1$, and

$$d^L_{i,k} = d(t_i, t_{\max\{1,i-k\}}),\, d^R_{i,k} = d(t_i, t_{\min\{n,i+k\}}).$$

Now, for each i, assign $p(t_i) = \max\{(d^L_{i,k})^2, (d^R_{i,k})^2\}$. Let $A_{L,k}$ denote the resulting power assignment. It was shown in [33] that the cost of this power assignment is within a constant factor from the optimum.

Easy to see that there are k node-disjoint paths between any two nodes t_i, t_j, in both directions, for $1 \leq i < j \leq n$. For example, the l-th path from t_i to t_j, $1 \leq l \leq k$, can be described as $P_l = (t_i, t_{i+l}, t_{i+l+k}, t_{i+l+2k}, \ldots, t_j)$. Note that, if node t_i reaches t_j in one hop, there is no need to have k node-disjoint paths from t_i to t_j, since any failure of a node other than t_i, t_j does not interrupt transmission from t_i to t_j.

2.2 Analysis

In our analysis of the network lifetime $l_{A_{L,k}}(\mathcal{B}_U)$ we use the following observation from [33].

Observation 2.1 ([33]). *For a linear layout of nodes $\mathcal{T}$, let A_L be a power assignment so that H_A is a k-strongly connected line. Then for each node $t_i \in \mathcal{T}$ there are at least* $\min\{i-1, k\}(\min\{n-i, k\})$ *nodes to its left (right) with sufficient range assignment to reach t_i in one hop.*

The following theorem shows that the network lifetime of the power assignments $A_{L,k}$ is optimal.

Theorem 2.2. *For a linear layout of nodes $\mathcal{T}$, and uniform initial battery charges $\mathcal{B}_U$, the power assignment $A_{L,k}$ results in an optimal network lifetime, that is $l_{A_{L,k}}(\mathcal{B}_U) = l^*$.*

Proof. From Lemma 2.1 it easily follows that

$$l^* \leq \min_{1 \leq i \leq n} \frac{b}{(d^L_{i,k})^2} \quad \text{and} \quad l^* \leq \min_{1 \leq i \leq n} \frac{b}{(d^R_{i,k})^2},$$

since t_i has to be reachable in one hop by at least $\min\{i-1, k\}$ $(\min\{n-i, k\})$ nodes to its left (right). Therefore, $l_{A_{L,k}}(\mathcal{B}_U) \leq l^*$. □

3 PLANAR NODE LAYOUT

This section addresses the planar layout of nodes $\mathcal{T}$. We analyze the network lifetime of a power assignment developed in [9].

3.1 Planar k-strong connectivity ([9])

Compute a minimum spanning tree MST of the Euclidean graph induced by $\mathcal{T}$. Assign to each node $t \in \mathcal{T}$ the range r^*_t. Denote this initial range assignment by A'. For each edge $e = (t, s)$ of MST, increase the range of the nodes in $N_t \cup N_s$ (if necessary), so that each node $t' \in N_t$ will reach all nodes in $N_s \cup \{s\}$, and vice versa. Let A_k denote the resulting power assignment. The cost of the power assignment has an approximation ratio of $O(k)$ times the optimum and can be computed in $O(n \log n)$ time.

It is rather simple to show that H_{A_k} is k-strongly connected. We construct k node-disjoint paths along the edges of the MST. Think of each N_t as a large intersection, which contains k intersection points. For every edge (t, s) in the MST, all nodes in $N_s \cup \{s\}$ are made reachable in one hope from the nodes in N_t. The range assignment of each node t must be at least r_t^* (otherwise k-strong connectivity is impossible), and in addition sufficient enough to create the intersections mentioned above.

3.2 Analysis

We start by showing a constant factor approximation in the case of uniform initial battery charges $\mathcal{B}_U$. Then we present our analysis for the general case, $\mathcal{B}_A$.

3.2.1 Uniform battery charges

First we assume that the initial battery charges $\mathcal{B}_U$ are uniform, that is $b_t = b$ for all $t \in \mathcal{T}$. In our analysis we need the following lemma from [9].

Lemma 3.1 ([9]). *Given an* MST *edge (t, s), let $r_{t'}^{t;s}$ be the range which node $t' \in N_t$ has to be assigned in order to reach all nodes in $N_s \cup \{s\}$. Then*

$$r_{t'}^{t;s} < r_t^* + d(t, s) + r_s^*.$$

Note that the inequality holds if t' is replaced by any $s' \in N_s$. For a given edge $e = (t, s)$, we denote $|e| = d(t, s)$. The following two lemmas provide upper bounds for the optimal network lifetime l^*.

Lemma 3.2. $l^* \leq \min_{t \in \mathcal{T}} \frac{b}{r_t^{*2}}$.

Proof. For a graph to be k-strongly connected, each node has to have at least k neighbors. Thus, each node $t \in \mathcal{T}$ has to be assigned $p(t) \geq r_t^{*2}$. □

Lemma 3.3. $l^* \leq \min_{e \in \text{MST}} \frac{b}{|e|^2}$.

Proof. Obviously, the network lifetime decreases as the fault resistance factor increases. That is, the higher the value of k, the lower the maximal possible network lifetime. This is the case since higher fault resistance requires larger range assignments. Consider the maximal possible lifetime of a 1-strongly connected graph. Let A_{MST} be a power assignment in which each node is assigned to reach its neighbors in the MST. It is easy to see that $H_{A_{\text{MST}}}$ is 1-strongly connected. Let $e_l = (u, v)$ be the longest edge in the MST, so that

$$l_{A_{\text{MST}}}(\mathcal{B}_U) = \min_{e \in \text{MST}} \frac{b}{|e|^2} = \frac{b}{|e_l|^2}. \tag{1}$$

Suppose by contradiction that there exists some power assignment A' such that the corresponding network lifetime $l_{A'}(\mathcal{B}_U) > l_{\text{MST}}(\mathcal{B}_U)$ and $H_{A'}$ is a 1-strongly connected graph. Consider the cut (S, T) induced by e_l in MST. Since the graph $H_{A'}$ is 1-strongly connected, there exists an edge $e' = (s', t') \in \mathcal{E}_{A'}$ such that $s' \in S$ and $t' \in T$. Clearly, e' is not in the MST. From (1) and the fact that $l' \leq \frac{b}{|e'|^2}$, we obtain $\frac{b}{|e_l|^2} < \frac{b}{|e'|^2}$. This means that the MST is not a minimum spanning tree of the Euclidean graph induced by $\mathcal{T}$, which is a contradiction. □

Let $r^*_{\max} = \max_{t \in \mathcal{T}} r^*_t$ and let $e^* \in$ MST be the longest edge of the MST, so that $|e^*| = \max_{e \in \text{MST}} |e|$. We are ready to state our main result.

Theorem 3.4. *For the planar k-strong connectivity power assignment A_k and uniform initial battery charges $\mathcal{B}_U$, it holds $l_{A_k}(\mathcal{B}_U) \geq l^*/9$.*

Proof. According to the range assignment algorithm, each node t is initially assigned a power $p(t) = r^{*2}_t$ so as to have at least k neighbors. Then the power of each node is increased if needed according to some MST edge. We distinguish between two cases:

Case 1: $p(t) = r^{*2}_t$. If the power of node t does not increase, then obviously $p(t) \leq r^{*2}$.

Case 2: $p(t) > r^{*2}_t$. Then the power is increased due to some MST edge $e = (u, v)$. According to Lemma 3.1:

$$p(t) = (r^{u,v}_t)^2 < (r^*_u + |e| + r^*_v)^2.$$

Consider two possibilities:

(a) If $|e| \leq \max\{r^*_u, r^*_v\}$, then

$$p(t) < 9(\max\{r^*_u, r^*_v\})^2 \leq 9 r^{*2}_{\max}.$$

(b) If $|e| > \max\{r^*_u, r^*_v\}$, then

$$p(t) < 9|e|^2 \leq 9|e^*|^2.$$

We have shown that the power assignment of each node t is $O((\max\{r^*_{\max}, |e^*|\})^2)$. According to Lemmas 3.2 and 3.3, for every $t \in \mathcal{T}$ we have $l_t \geq l^*/9$, and therefore $l_{A_k}(\mathcal{B}_U) \geq l^*/9$. □

3.2.2 Arbitrary battery charges

In the case of arbitrary battery charges we use a probabilistic approach, assuming that nodes are placed uniformly in the unit square, and analyze the lifetime for a sufficiently large number of nodes n.

Recall that the range increase of some node t' is at most $r^*_t + d(t, s) + r^*_s$, where $t' \in N_t$ and $e = (t, s)$ is some MST edge adjacent to t. Unfortunately, in the case of arbitrary initial battery charges $\mathcal{B}_A$ we cannot use previous bounds on maximal network lifetime (Lemmas 3.2 and 3.3). In the following lemma we state a much more general bound on the maximal network lifetime.

Lemma 3.5. $l^* \leq \min_{t \in \mathcal{T}} \frac{b_t}{r_t^{*2}}$.

We omit the proof; it is similar to the one in Lemma 3.2, but with varying initial battery charges. The difference between the bounds given by Lemmas 3.2 and 3.5 is crucial, since we cannot use them in a similar manner as we did in the proof of Theorem 3.4. For example, node t' can be assigned a range of $O(r_t^*)$, which will result in an arbitrarily small network lifetime, as depicted in Figure 2. We counter that by proving in Lemma 3.6 that the ratio between r_t^* and r_s^* for any two nodes is bounded with high probability under uniform distribution of nodes in the plane for $k < \frac{n}{(1+\gamma)\log n}$, where γ is any positive constant. Let $r_{\max}^* = \max_{t \in \mathcal{T}} r_t^*$ and $r_{\min}^* = \min_{t \in \mathcal{T}} r_t^*$.

Lemma 3.6. *For a set of n points $\mathcal{T}$ placed uniformly in the unit square*

$$\lim_{n \to \infty} \Pr\left[\frac{r_{\max}^*}{r_{\min}^*} = O\left(\sqrt{k \log n \sqrt[k]{n\varphi(n)}}\right)\right] = 1,$$

where $\varphi(n)$ is any function with $\lim_{n\to\infty} \varphi(n) = \infty$.

To prove the lemma, we will need Lemmas 3.9 and 3.10 below. Before we get there, let us point out an additional difficulty, due to the fact that we cannot use the bound projected by the longest edge in the MST (Lemma 3.3). To cope with that, we use the following lemma.

Lemma 3.7 (Penrose [31]). *For n points placed uniformly in the unit square, let M_n (respectively, M'_n) be the longest edge-length of the nearest neighbor graph (respectively, the minimum spanning tree) on these points. Then,* $\lim_{n\to\infty} \Pr[M_n = M'_n] = 1$.

Since $M_n \leq r_{\max}^*$ for any value of k, and $e^* = M'_n$, we can easily derive the following conclusion.

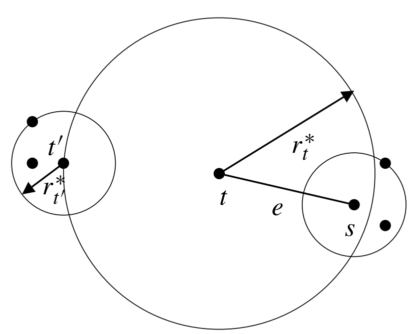


FIGURE 2
Node $t' \in \mathcal{T}$ is assigned a range of $O(r_{t'}^*)$ to reach all nodes in N_s. The lifetime of t' is decreased by a factor of $r_t^*/r_{t'}^*$, which may be arbitrarily large.

Conclusion 3.8. *For a set of n points, placed uniformly in the unit square,*

$$\lim_{n\to\infty} \Pr[|e^*| \le r^*_{\max}] = 1.$$

Lemma 3.9. *For a set of n points, placed uniformly in the unit square,*

$$\lim_{n\to\infty} \Pr\left[r^*_{\max} > 2\sqrt{\frac{(k+1)\log n}{\pi(n-1)}}\right] = 0.$$

Proof. Let $\varepsilon = 2\sqrt{\frac{(k+1)\log n}{\pi(n-1)}}$. Since

$$\Pr[r^*_{\max} > \varepsilon] = \Pr\left(\bigcup_{t\in\mathcal{T}} [r^*_t > \varepsilon]\right) \le \sum_{t\in\mathcal{T}} \Pr[r^*_t > \varepsilon],$$

we have

$$\limsup_{n\to\infty} \Pr[r^*_{\max} > \varepsilon] \le \limsup_{n\to\infty} \sum_{t\in\mathcal{T}} \Pr[r^*_t > \varepsilon].$$

We will prove that $\lim_{n\to\infty} \sum_{t\in\mathcal{T}} \Pr[r^*_t > \varepsilon] = 0$. For any node t, the probability that there are at most $k-1$ out of the other $n-1$ nodes within a distance ε from t is maximal when the point t is a corner of the unit square. Therefore,

$$\begin{aligned}
\sum_{t\in\mathcal{T}} \Pr[r^*_t > \varepsilon] &\le n \sum_{i=0}^{k-1} \binom{n-1}{i} \left(\frac{\pi\varepsilon^2}{4}\right)^i \left(1 - \frac{\pi\varepsilon^2}{4}\right)^{n-1-i} \\
&\le n \sum_{i=0}^{k-1} \frac{(1-\frac{1}{4}\pi\varepsilon^2)^{n-1}}{i!} \left(\frac{\frac{1}{4}\pi\varepsilon^2(n-1)}{1-\frac{1}{4}\pi\varepsilon^2}\right)^i \\
&= n\left(1 - \frac{(k+1)\log n}{n-1}\right)^{n-1} \sum_{i=0}^{k-1} \frac{1}{i!} \left(\frac{(k+1)\log n}{1 - \frac{(k+1)\log n}{n-1}}\right)^i \\
&\le \frac{n}{e^{(k+1)\log n}} \sum_{i=0}^{k-1} \frac{1}{i!} \left(\frac{(k+1)\log n}{1 - \frac{(k+1)\log n}{n-1}}\right)^i \\
&= \frac{1}{n^k} \sum_{i=0}^{k-1} \frac{1}{i!} \left(\frac{(k+1)\log n}{1 - \frac{(k+1)\log n}{n-1}}\right)^i \\
&\le \frac{k}{n^k(k-1)!} \left(\frac{(k+1)\log n}{1 - \frac{(k+1)\log n}{n-1}}\right)^{k-1} \\
&\le \frac{k}{n(k-1)!} \left(\frac{(k+1)\log n}{n-1-(k+1)\log n}\right)^{k-1}
\end{aligned}$$

Clearly $\lim_{n\to\infty}\sum_{t\in\mathcal{T}}\Pr[r_t^* > \varepsilon] = 0$ for $k < \frac{n}{(1+\gamma)\log n}$, where γ is any positive constant. $\square$

Lemma 3.10. *For a set of n points, placed uniformly in the unit square*

$$\lim_{n\to\infty}\Pr\left[r_{\min}^* < \sqrt{\frac{1}{2\pi(n-1)\sqrt[k]{n\varphi(n)}}}\right] = 0,$$

where $\varphi(n)$ is any function with $\lim_{n\to\infty}\varphi(n) = \infty$.

Proof. Let $\delta = \sqrt{\frac{1}{2\pi(n-1)\sqrt[k]{n\varphi(n)}}}$. Similarly to the previous lemma,

$$\Pr[r_{\min}^* < \delta] = \Pr\left(\bigcup_{t\in\mathcal{T}}[r_t^* < \delta]\right) \le \sum_{t\in\mathcal{T}}\Pr[r_t^* < \delta],$$

so that

$$\limsup_{n\to\infty}\Pr[r_{\min}^* < \delta] \le \limsup_{n\to\infty}\sum_{t\in\mathcal{T}}\Pr[r_t^* < \delta].$$

We will prove that $\lim_{n\to\infty}\sum_{t\in\mathcal{T}}\Pr[r_t^* < \delta] = 0$. For any node t, the probability that there are at least k out of the other $n-1$ nodes within a distance δ from t is maximal when the point t is at a distance of at least δ from the boundary of the unit square. Let $a_i = \binom{n-1}{i}(\pi\delta^2)^i(1-\pi\delta^2)^{n-1-i}$, for $0 \le i \le n-1$. It is easy to verify that, for $\delta \le \sqrt{\frac{1}{2\pi(n-1)}}$, we have $a_{i+1} \le a_i/2$ for each i. Therefore,

$$\begin{aligned}\sum_{t\in\mathcal{T}}\Pr[r_t^* < \delta] &\le n\sum_{i=k}^{n-1}\binom{n-1}{i}(\pi\delta^2)^i(1-\pi\delta^2)^{n-1-i}\\ &\le 2n\binom{n-1}{k}(\pi\delta^2)^k(1-\pi\delta^2)^{n-1-k}\\ &\le \frac{2n(n-1)!}{k!(n-1-k)!}(\pi\delta^2)^k \le \frac{2n(n-1)^k}{k!}(\pi\delta^2)^k \qquad \square\end{aligned}$$

Lemma 3.6 follows easily from Lemmas 3.9 and 3.10. We are ready to state our main result.

Theorem 3.11. *For a set of n points, placed uniformly in the unit square, arbitrary initial battery charges $\mathcal{B}$ and a planar k-strong connectivity power assignment A_k, the network lifetime $l_{A_k}(\mathcal{B}_A)$ is at most $O(k\log n\sqrt[k]{n\varphi(n)})$ times worse than l^* with high probability, where $\varphi(n)$ is any function with* $\lim_{n\to\infty}\varphi(n) = \infty$.

Proof. The proof resembles the proof of Theorem 3.4. If the power of node t is increased due to some MST edge $e = (u, v)$, then according to Lemma 3.1

it holds $p(t) = (r_t^{u,v})^2 < (r_u^* + |e| + r_v^*)^2$. In conjunction with Conclusion 3.8, we have $p(t) \le 9r_{\max}^{*2}$ with high probability. By Lemma 3.5, $l^* \le \min_{t\in\mathcal{T}} b_t/r_t^{*2}$. Finally, from Lemma 3.6, for any node t,

$$(r_{\max}^*/r_t^*)^2 \le (r_{\max}^*/r_{\min}^*)^2 = O(k \log n \sqrt[k]{n\varphi(n)})$$

with high probability. This rests our proof. □

4 TORUS NODES LAYOUT

Here we assume that the nodes are positioned on a unit torus and have arbitrary initial battery charges $\mathcal{B}_A$. We also assume that the number of nodes is sufficiently large, so that flattening the torus produces only a negligible difference in the distance between any two nodes. In other words, we assume that the close environment of each node is a flat grid.

We claim that assigning each node with enough power to reach its k-closest neighbors is sufficient to induce a k-strongly connected communication graph, and the power assignment is optimal in terms of lifetime and total power.

4.1 Optimal power assignment

Let $A_\mathcal{T}$ be a power assignment, so that $p(t) = (r_t^*)^2$ for every $t \in \mathcal{T}$. It is easy to see that $A_\mathcal{T}$ is indeed optimal in terms of both network lifetime and total power, since, in any k-strongly connected graph, each node has enough power to reach at least k other nodes. Since on a unit torus the range to the k closest nodes is the same for all nodes, we may denote $r^* = r_t^*$, for any $t \in \mathcal{T}$.

4.2 *k*-strong connectivity

We argue that the induced communication graph $H_{A_\mathcal{T}}$ is indeed k-strongly connected. To prove that, we need to show the existence of k node-disjoint paths from any $s \in \mathcal{T}$ to any $t \in \mathcal{T}$.

On a unit torus, it is usually the case that each node has more than k neighbors in $H_{A_\mathcal{T}}$. Let k' be the number of these neighbors. We shall construct k' node-disjoint paths from s to t. Note that k' is divisible by 4. We refer to the points, where these k' nodes around s are located, as *origin points*. Similarly, the closest k' nodes to t are *destination points* (see Figure 3). It is easy to see that each of the k' node-disjoint paths from s to t starts at one of the nodes in the origin points and ends at one of the destination points.

It turns out to be non-trivial to find k' node-disjoint paths from s to t. We address the following alternative formulation of the problem. Suppose there are k' (chess-like) pieces located at the origin points. We need to navigate each piece through the grid points in a way that no point is used by more than one piece, while a single piece move is from one grid point to another within a distance of at most r^* (Figure 4).

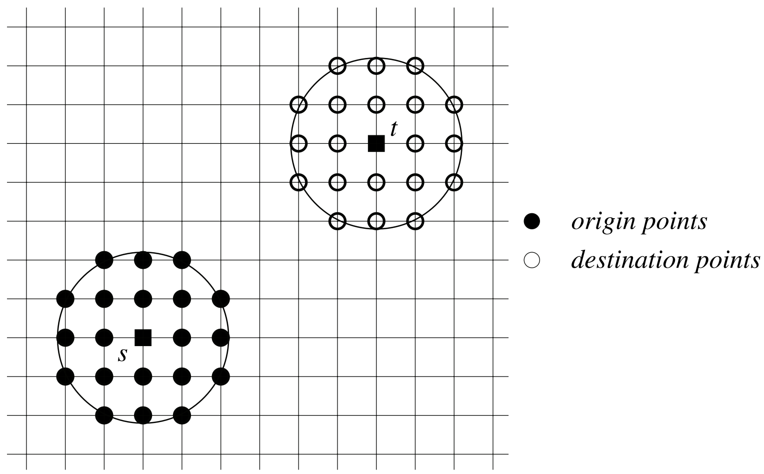
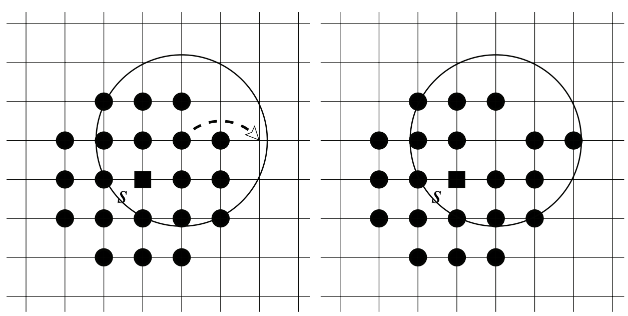


FIGURE 3
Origin and destination points, $k' = 20$.



(a) Before the move (b) After the move

FIGURE 4
The traveled distance is 2, while $r^* = \sqrt{5}$.

The pieces will be distinguished according to their initial location into two classes—*axis* pieces and *inner* pieces. There are four groups of axis points and four groups of inner points (see Figure 5). The pieces are usually moved in groups, each having its own movement pattern. Each group is navigated along a route, which depends on the relative location of s and t. Let $l = \lfloor r^* \rfloor$ be the number of points in an axis group. Next we discuss the movement patterns of each group type.

Axis group movement. Each axis group can move either horizontally or vertically. The move is performed by moving the last piece to the head of the group. For example, in Figure 6(a), a horizontal axis makes one move right, and in Figure 6(b) it then performs a left turn of 90°. The distance traveled by

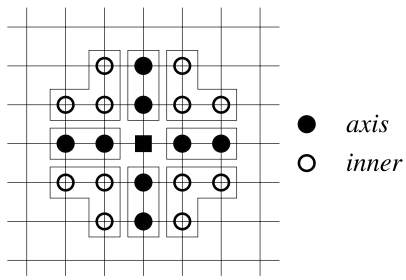
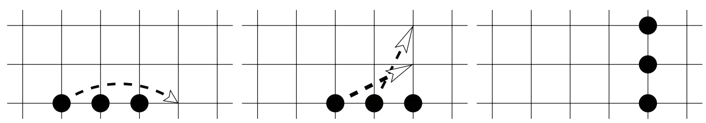
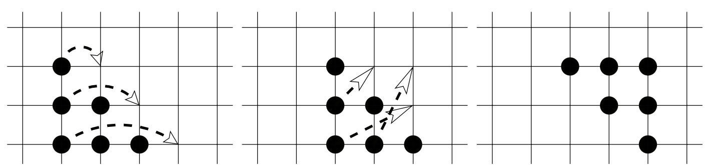


FIGURE 5
Axis and inner groups.



(a) Horizontal move right (b) A 90° left turn (c) Final position

FIGURE 6
Axis movement to the right and a left turn.



(a) Horizontal move right (b) A 90° left turn (c) Final position

FIGURE 7
Inner group movement.

a piece is at most l, and therefore the move is legal. The final position is given in Figure 6(c).

Inner group movement. A group of inner pieces can move horizontally and vertically. While moving horizontally (vertically, resp.), each row (column, resp.) moves in a similar way to a horizontal (vertical, resp.) axis group. The turn is then performed on each row (column, resp.) in the same way as for an axis group (Figure 7).

We now demonstrate the navigation of the k' pieces at the origin points to the destination points. Let x_s, y_s and x_t, y_t be the x, y-coordinates of the nodes s and t, respectively. Without loss of generality, $x_s \leq x_t$ and $y_s \leq y_t$. We distinguish between several cases:

Case 1. $x_t > x_s + 2l$, $y_t > y_s + 2l$.

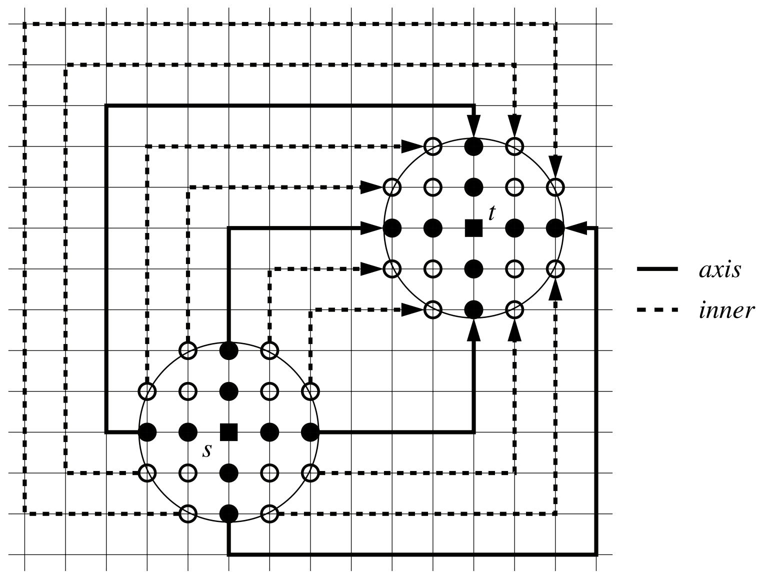


FIGURE 8
Case 1 navigation for $r^* = \sqrt{5}$.

This case is very simple. The sets of origin and destination points cannot possibly intersect. The routes of the axis and inner groups are depicted in Figure 8.

Case 2. $x_s + l < x_t < x_s \leq 2l,\ y_s + l < y_t$.

In this case, the destination points are entirely in the first quadrant relative to s. First we consider a situation where the sets of origin and destination points do not intersect.

The routes remain the same as in the previous case, except for the inner group of pieces in the first quadrant of the origin points (Figure 9). The first quadrant is still navigated to the third quadrant of the destination points, but in a different way. These pieces are navigated in a 2-step process as described below.

The first step, called *trimming*, is performed when the sets of origin points and destination points have different projections on the x-axis. The leftmost column and bottom row of inner pieces are moved to the corresponding top row and rightmost column, respectively. This process repeats until the remaining sets of points have the same projections on the x-axis. For example, in Figure 10 we are able to trim two columns and two rows. The 4 columns of remaining pieces are positioned exactly below the remaining 4 columns of destination points.

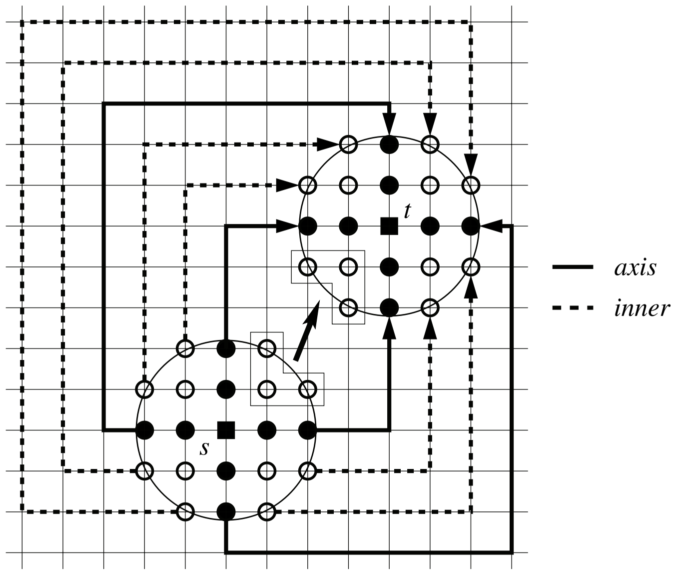


FIGURE 9
Case 2 navigation for $r^* = \sqrt{5}$ and $l = 2$.

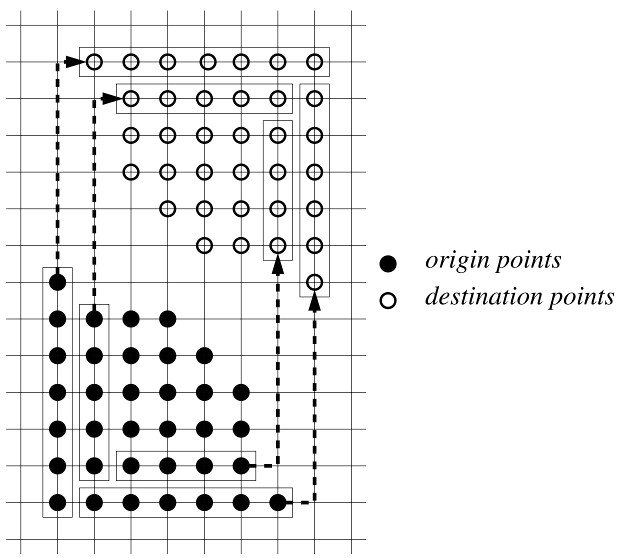


FIGURE 10
Trimming.

The second step is to navigate the remaining pieces. Note that the number of pieces in the leftmost column is the same as the number of destination points in the rightmost column, the number of pieces in the second column from the left is the same as the number of destination points in the second column from

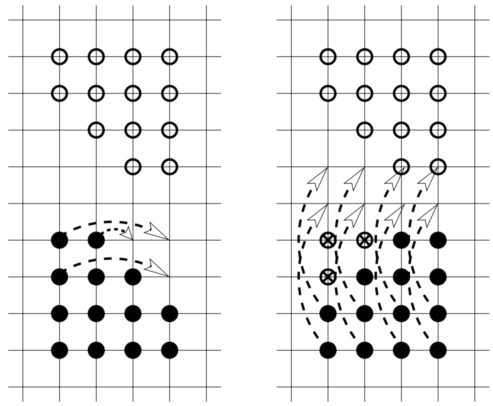


FIGURE 11
Balance and movement up (cont. from Figure 10).

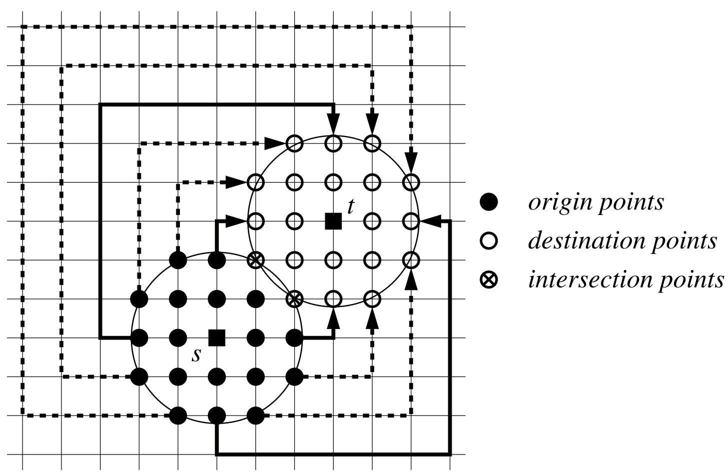


FIGURE 12
Case 2-intersection.

the right, and so on. We start by balancing the columns of pieces to match in size the columns of destination points above them. Let m be the number of columns (numbered from left to right), and let a_j, $1 \leq j \leq m$, be the initial number of pieces in the j-th column. We move $a_j - a_{m-j+1}$ pieces from j-th column to the $(m - j + 1)$-st column[3], $1 \leq j \leq \lfloor m/2 \rfloor$. For example, in Figure 11(a), two nodes from the leftmost column move to the rightmost

[3]It holds $a_j \geq a_{j+1}$ for any $1 \leq j \leq m - 1$.

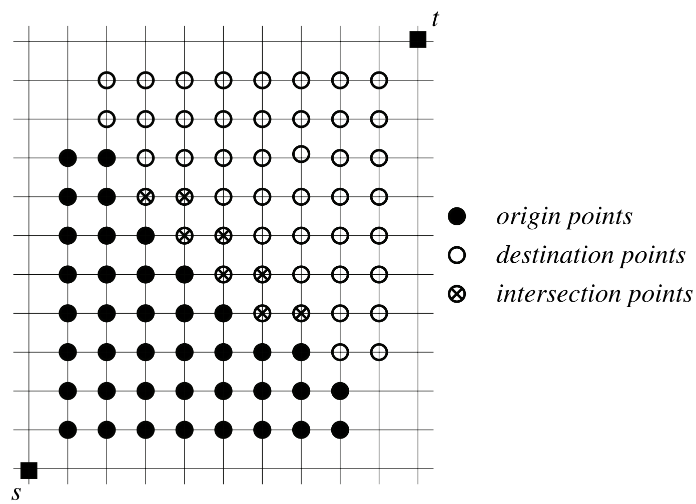


FIGURE 13
Case 2-intersection.

column and one node from the second column moves to column 3. Finally, each column moves up as before, the only difference being that the origin points where the relocated pieces have resided are skipped (Figure 11(b)). By the end of this step (shown only in part in Figure 11), all pieces have arrived at their final destination.

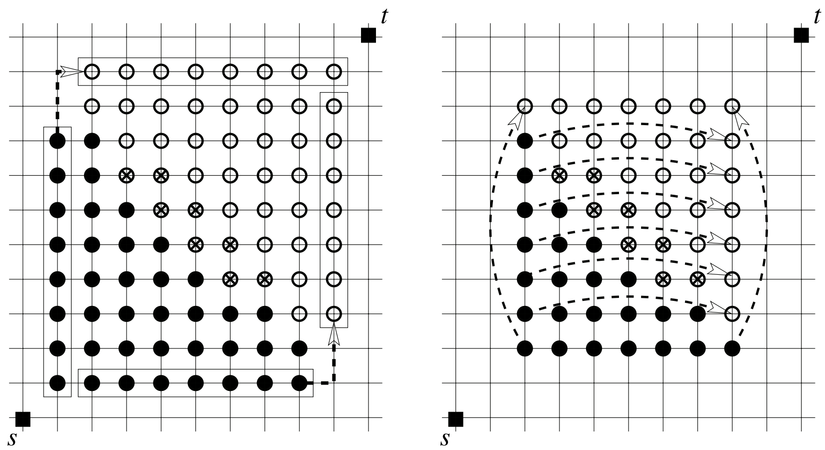
If there is an intersection between the sets of origin and destination points (Figure 12), the routes remain the same and we still navigate the first quadrant of origin points into the third quadrant of destination points, but in a different way. Note that we do not move the pieces in the intersection points, as they have already reached their destination.

We now examine the intersecting quadrants in more detail. In Figure 13 we see two intersecting quadrants with $r^* = \sqrt{68}$. The trimming, balancing and moving-up is performed in a similar way as before for columns/rows without intersecting points (Figure 14).

Finally, on the remaining columns, with intersecting points, we perform an operation of *crossing*. The first type of crossing is for rows which do not contain intersection points. We simply move the whole row to a matching row in the set of destination points (Figure 15(a)).

As for rows with intersection points, we perform another type of crossing (Figure 15(b)), which is iterative, and continues until there are no pieces to move. In each iteration the pieces from the bottom row and the leftmost column are moved to the destination top row and rightmost column, respectively.

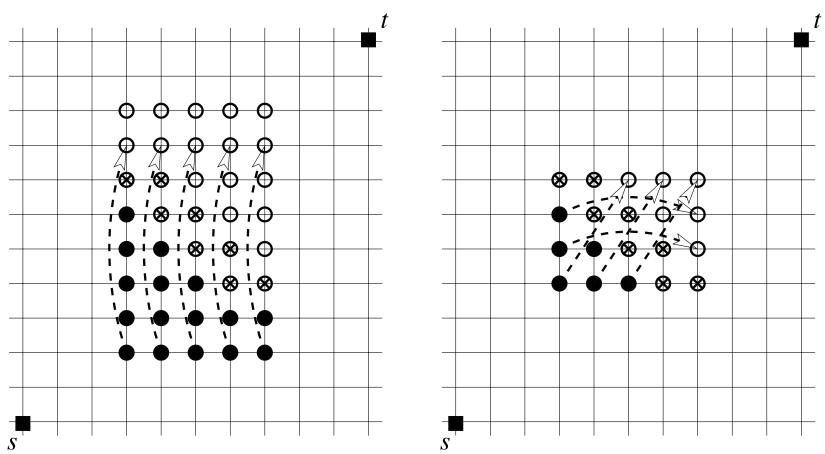
Case 3. $x_s + l < x_t,\ y_s \leq y_t < y_s + l.$



(a) Trimming

(b) Balancing and moving up

FIGURE 14
Trimming, balancing and moving up of intersecting quadrants (case 2).



(a) Crossing of rows without intersection points

(b) Crossing of rows with intersection points

FIGURE 15
Case 2, intersection-crossing.

In this last case, the y-coordinate of some of the points in the destination set is smaller than y_s. Again, we start from a non-intersecting position of the sets of origin and destination points.

The navigation we propose uses the toroidal properties of the space. We route the second and third quadrants (with all three adjacent axes) leftward

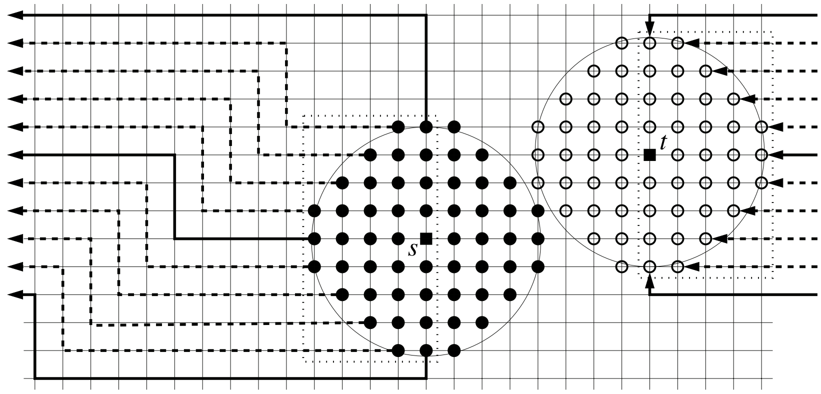


FIGURE 16
Case 3 navigation for $r^* = \sqrt{17}$ and $l = 4$.

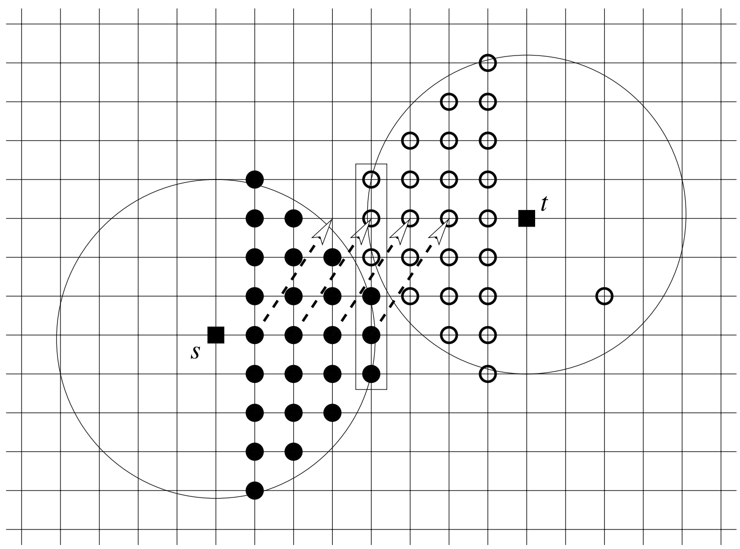


FIGURE 17
Case 3-moving the remaining axis group (cont. from Figure 16).

(with a small vertical adjustment) until they emerge from the right and arrive directly at the destination quadrants 1 and 4 and the adjacent axes. This navigation is demonstrated in Figure 16. The routed pieces and their final destinations are outlined by a dotted rectangle. The navigation of the remaining inner groups and one axis group is described below.

The pieces in the remaining axis group are moved to the row of the destination axis, just to the right of the rightmost origin point in that row (Figure 17). This move is possible since s itself reaches the rightmost point in any row of the set of origin points and $y_t < y_s + l$. From there, it is a basic movement right until all the axis group pieces are in place.

Next, the pieces in the first and fourth quadrants of the set of origin points are navigated to the third and second quadrants in the set of destination points, respectively. Note that the projections on the x-axis of the third destination quadrant and the first origin quadrant might intersect (outlined by a solid rectangle in Figure 17). Pieces that fall under this intersection are balanced and moved up as before.

The rest of the pieces in these quadrants are navigated using the toroidal properties of the space (see an example in Figure 18). The pieces in the first quadrant are moved up until they emerge from the bottom. Then they turn right and then left again, to match the free spots in the third quadrant of the

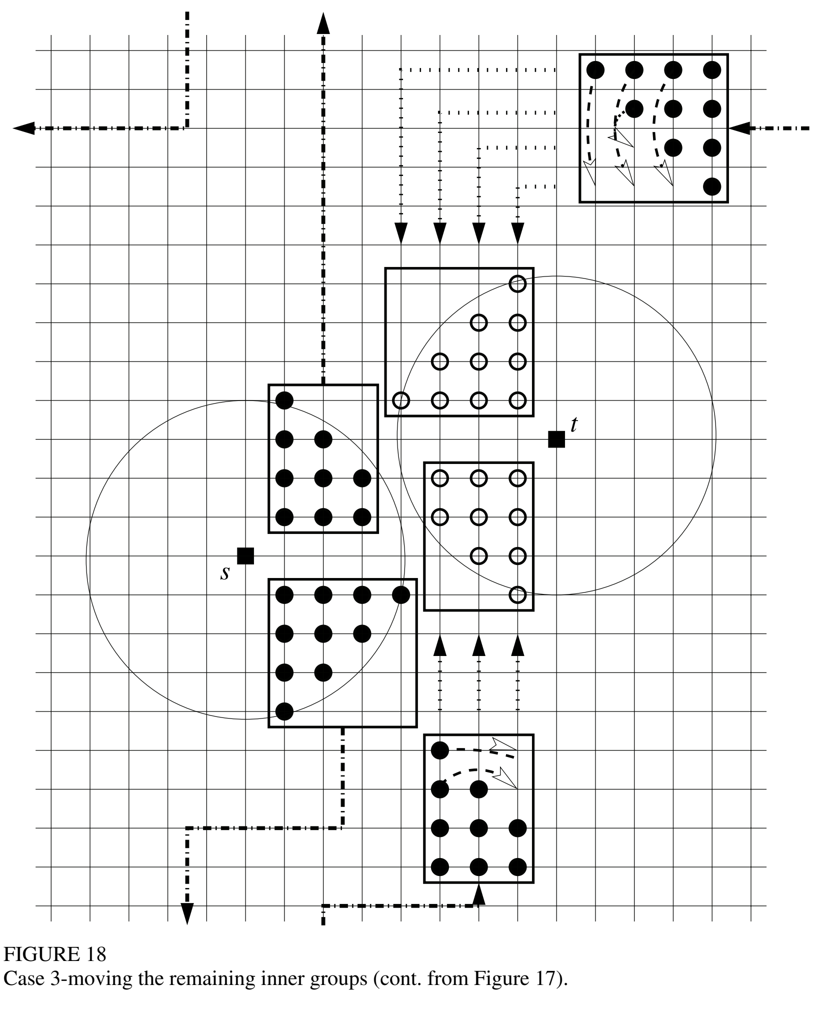


FIGURE 18
Case 3-moving the remaining inner groups (cont. from Figure 17).

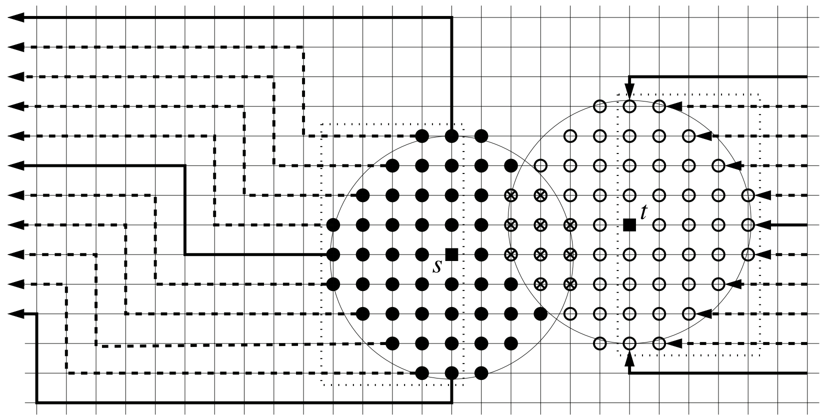


FIGURE 19
Case 3-intersection.

destination points. It is then balanced and moved up as before. The pieces in the third quadrant of the set of origin points are moved down until they emerge from the top. Unfortunately, we cannot take a left turn similarly to case of the first quadrant, since we then step on the trail of the pieces of the first quadrant. Instead, we take a right turn and emerge from the right. From there we need balancing and then a final movement to the destination points.

Finally, suppose that the sets of origin and destination points intersect (Figure 19). The navigation is exactly the same as in the non-intersecting case. This is due to the fact that the number of intersection points in the first quadrant of the origin set of points is the same as the number of intersection points in the third quadrant of the destination set. The equality also holds for intersection points in the fourth and second quadrants of the set of origin and destination points, respectively.

We can now conclude the following Theorem.

Theorem 4.1. *H_{A_T} is k-strongly connected.*

5 SIMULATIONS

Our simulations study the approximation ratio of the network lifetime resulting from the power assignment in Section 3. We compare the network lifetime $l_{A_k}(\mathcal{B}_U)$ to the optimal lifetime l^*, which results from the algorithm in [32].

5.1 Optimal network lifetime

The algorithm in [32] is aimed at minimizing the maximal power assigned to some node, that is minimizing $\max_{t\in\mathcal{T}} p(t)$, while maintaining k-strong connectivity. Note that there are at most $O(n^2)$ different ranges that can be

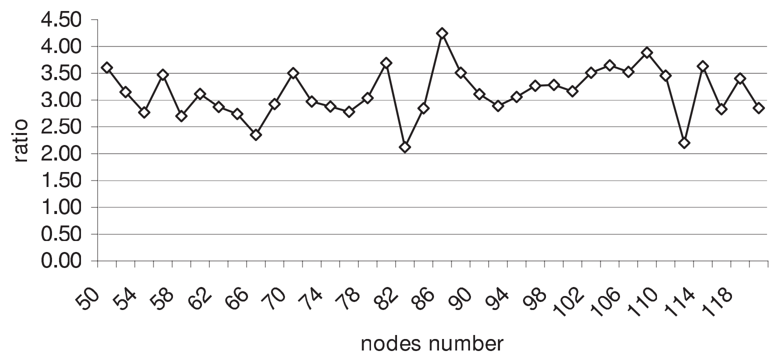


FIGURE 20
The lifetime approximation ratio for $k = 2$.

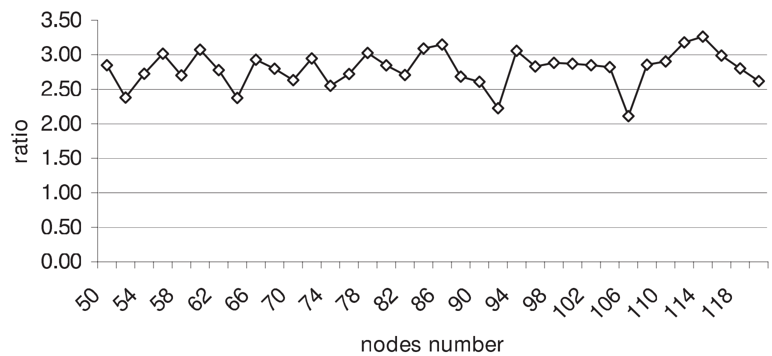


FIGURE 21
The lifetime approximation ratio for $k = 6$.

assigned. Since we look for maximizing the network lifetime, we can simply go over these ranges, starting from the smallest one, until we obtain a k-strongly connected graph. The smallest range r^* which induces a k-strongly connected graph is also the upper bound for the network lifetime, that is $l^* = b/r^*$.

We performed a binary search over the sorted collection of ranges; for each range r we assigned $p(t) = r^2$ for all $t \in \mathcal{T}$, and tested each induced graph for k-strong connectivity. The k-strong connectivity test was done by a reduction to a network flow problem according to Menger's theorem [17].

5.2 Results

We computed the ratio l^*/l_{A_k} for uniform random distribution of nodes on a unit square. We tested the network lifetime of k-strongly connected networks

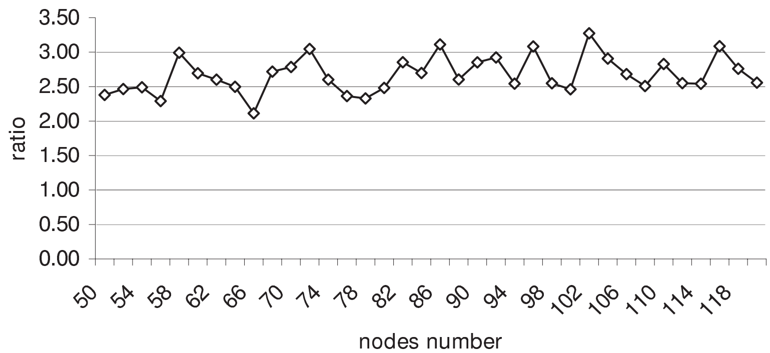


FIGURE 22
The lifetime approximation ratio for $k = 10$.

for various values of n and k. The simulations have been carried out for values of n ranging from 50 to 120 with steps of 2, and k being an integer value in the interval [2, 11]. For each pair of values n and k, we took the average of 5 tries.

The ratio ranged from 1.97 to 4.25 with an average value of 2.77. This is better than the theoretical upper bound of 9. Figures 20, 21, and 22 show the approximation ratio as a function of n, for $k = 2$, $k = 6$, and $k = 10$, respectively.

6 CONCLUSIONS AND FUTURE WORK

In this paper we studied the problem of inducing a k-strongly connected communication graph in a wireless ad-hoc network, with two optimization objectives: minimizing the total energy consumption and maximizing the network lifetime. The communication graph is induced by adjusting the transmission powers of the wireless nodes.

We addressed three different node layouts: linear, planar and toral. For each, we proposed a power assignment which obtains a good upper bound on the total energy and analyzed the lifetime of the induced network. To the best of our knowledge, this is the first attempt to provide a bicriteria approximation ratio for the total power assignment cost and the network lifetime under the k-fault resilience criterion.

We also conducted simulations to test the performance of the induced network in terms of lifetime.

One possible future direction is to consider a layout of nodes on a grid in the plane. The difference from the case studied in this work will be the edge effects. In addition, we believe that it is possible to improve the network lifetime analysis in the case of arbitrary battery charges and planar layout of nodes. It could also be of interest to derive a tighter upper bound on the network lifetime, for uniform batteries, to match the simulation results.

ACKNOWLEDGEMENT

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