Low Energy Fault Tolerant Bounded-Hop Broadcast in Wireless Networks

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ABSTRACT

This paper studies asymmetric power assignments in wireless ad-hoc networks. The temporary and unfixed physical topology of wireless ad-hoc network is determined by the distribution of the wireless nodes as well as the transmission power (range) assignment of each node. We consider the problem of *bounded-hop broadcast* under *k-fault resilience* criterion for linear and planar layout of nodes. The topology which results from our power assignment allows a broadcast operation from a wireless node r to any other node in at most h hops and is k-fault resistant.

We develop simple approximation algorithms for the two cases and obtain the following approximation ratios: linear case – $O(k)$; planar case – we first prove a factor of $O(k^3)$, which is later decreased to $O(k^2)$ by a finer analysis. Finally we show a trivial power assignment with a cost $O(h)$ times the optimum. To the best of our knowledge these are the first non-trivial results for this problem.

Categories and Subject Descriptors

C.2.1 [**Computer-Communication Networks**]: Network Architecture and Design—*Wireless communications*; G.2.2 [**Discrete Mathematics**]: Graph Theory—*Network Problems*

General Terms

Algorithms, Design, Reliability, Theory

Keywords

Wireless Ad-Hoc Networks, Bounded-Hop Broadcast, Fault Tolerance, Energy Consumption

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1. INTRODUCTION

A wireless ad-hoc network consists of several transceivers (stations), communicating by radio. Each transceiver t is assigned a transmission power $p(t)$, which gives it some transmission range, denoted by r_t. This is customary to assume that the minimal transmission power required to transmit to a distance d is d^α, where the *distance-power gradient* α is usually taken to be in the interval $[2, 4]$ (see [31]). Thus, a transceiver t receives transmissions from s if $p(s) \geq d(s,t)^\alpha$, where $d(s,t)$ is the Euclidean distance between s and t. The transmission possibilities resulting from a power assignment induce a communication graph. Research efforts have focused on finding power assignments, for which the induced communication graph satisfies a certain topology property, while minimizing the total cost. *Broadcasting* is one of possible topologies. For a special station r, called the root, we want to establish a transmission graph where there is route from r to every other node in the network. In some cases it is essential to minimize the number of hops from the root transceiver to other nodes in the network. This is called *bounded-hop broadcasting.*

This paper is organized as follows. In the rest of this section we present the model, previous work and briefly describe our results. In Section 3 we address the linear case of the problem. Then in Section 3 we deal with the planar layout of nodes and prove various approximation factors for the problem.

1.1 The Model

We are given a set $\mathcal{T}$ of n transceivers $t_1, t_2, \ldots, t_n$, positioned in $\mathbb{R}^d$, $d \geq 1$. We define the cost of an undirected graph $G(\mathcal{T}) = (\mathcal{T}, \mathcal{E})$ by

$$C_G = \sum_{(s,t)\in\mathcal{E}} c(s,t).$$

With edge costs being $c(s,t) = d(s,t)^\alpha$ for every $(s,t) \in \mathcal{E}$. When each transceiver is assigned a transmission power $p(t) = r_t^\alpha$, an ad-hoc network is created. A power assignment for $\mathcal{T}$ is a vector of transmission powers $\{p(t) \mid t \in \mathcal{T}\}$ and is denoted by $A(\mathcal{T})$ (usually abbreviated to A). The resulting (directed) communication graph is denoted by $H_A = (\mathcal{T}, \mathcal{E}_A)$, where $\mathcal{E}_A$ is the set of directed edges resulting from the power assignment $A(\mathcal{T})$:

$$\mathcal{E}_A = \{(t,s) \mid p(t) \geq d(t,s)^\alpha\}.$$

That is, there is a directed edge from t to s if t has sufficient transmission power to reach s. In this paper we refer to transceivers as nodes. The cost of the power assignment is defined as the sum of all transmission powers:

$$C_A = \sum_{t \in \mathcal{T}} p(t).$$

Throughout this paper we address the linear layout of nodes as well. The transceivers are positioned on a single line in increasing order from right to left (see Figure 1.a). Note that in case of the linear layout if there is a path from t_i to t_j, where $i < j$ then there is a path from t_i to any node t_l, where $i < l < j$.

For some power assignment A and a root node r, we say that a communication graph H_A is a *broadcast* graph rooted at r if for any other node $t \in \mathcal{T}$ there is a path from r to t in H_A. In the case that the maximal number of hops from r to any any node t is limited by some constant h, we say that H_A is an *h-bounded-hop broadcast graph* rooted at r. In this paper we demand that H_A remains a *bounded-hop broadcast graph* even with the removal of up to k nodes (the extraction of a node, removes all edges adjacent to it as well). We refer to k as the *fault resistance* parameter. If H_A holds the above, we say that H_A is a k-fault resistant h-bounded-hop broadcast graph rooted at r. In short we would say H_A is a k-h-broadcast graph. Let us formulate the main problem addressed in this article:

MEkBHB (*Minimum Energy k-Fault Resistant h-Bounded-Hop Broadcast*) — Given a set of nodes $\mathcal{T}$ in $\mathbb{R}^d$ $(d = 1, 2)$, a fault resistance parameter k and a maximal number of allowed hops h, find a power assignment $A(\mathcal{T})$ so that H_A is k-fault resistant h-bounded-hop broadcast graph rooted at r and C_A is minimized.

Note that k-fault resistance definition above presumes that there are k-node disjoint paths from r to any node $t \in \mathcal{T}$, which is not directly reached by r. Since if some node u is reached by the root, any $k-1$ node removals will not effect the transmission from r to t. On the other hand, if some node v is node directly reached by r, then there must exist k node disjoint paths from r to v, otherwise a removal of $k-1$ nodes might result in transmission failure from r to v. In this work we assume $\alpha = 2$.

1.2 Previous Work

Topology control in wireless networks is a relatively new field of interest. Nevertheless a wide area of problems has already been studied. Most of the problems are aimed at computing a low energy power assignment that meets global topological constraints. Kirousis et al. [28] introduced the `MinRange(SC)` problem, which is the k-strong connectivity problem for $k = 1$. They proved it to be NP-Hard for the three dimensional Euclidean space for any value of α. The same paper provided a 2-approximation algorithm for the planar case and an exact $O(n^4)$ time algorithm for the one dimensional case. In the planar case, the NP-Hardness of the problem for every α has been proved in [19] and a simple 1.5-approximation algorithm for the case $\alpha = 1$ has been provided in [5]. Ambuhl et al. [4] presented some algorithms for the weighted power assignment, solving it optimally for the broadcast, multi-source broadcast and strong connectivity problems for the linear case (they achieved the same running time for the strong connectivity problem as in [28]). They also presented some approximation algorithms for the multi-dimensional case. An excellent survey covering many variations of the problem is given in [17].

A natural generalization of the strong connectivity requirement is k-strong connectivity. These networks also provide multi-path redundancy for load balancing or transmission fault tolerance. As power-optimal strong connectivity is NP-Hard, so is power-optimal k-strong connectivity. Two versions of the problem arise: symmetric and asymmetric. In the symmetric version for any two nodes $t, s \in T$, $p(t) \geq d(t,s)^\alpha \Leftrightarrow p(s) \geq d(s,t)^\alpha$, that is a node t can reach node s if and only if s can reach node t, we can also refer to it as an undirected model. The asymmetric version allows directed links between two nodes. Krumke et al. [29] argued that the asymmetric version is harder than the symmetric version. A first non trivial result for planar asymmetric k-strong connectivity was presented by Shpungin and Segal in [33]. They derived an approximation factor of $O(k^2)$ for the planar case and some results for the linear case. Carmi et al. [14] improved the approximation ratio to $O(k)$. Another possible connectivity property is k-edge connectivity, which implies that the removal of any k edges results in a disconnected graph. In [13], Calinescu and Wan presented various aspects of symmetric/asymmetric k-strong connectivity and k-edge connectivity. They first proved NP-Hardness of the symmetric two-edge and two-node strong connectivity and then provided a 4-approximation algorithm for both symmetric and asymmetric strong biconnectivity ($k = 2$) and a $2k$-approximation for both symmetric and asymmetric k-edge strong connectivity. Hajiaghayi et al. [25] give two algorithms for symmetric k-strong connectivity, with $O(k \log k)$ and $O(k)$-approximation factors and also some distributed approximation algorithms for $k = 2$ and $k = 3$ in geometric graphs. Jia et al. in [27] present various approximation factors (depending on k) for the symmetric k-strong connectivity, such as $3k$-approximation algorithm for any $k \geq 3$ and 6-approximation for $k = 3$. Segal and Shpungin [32] extend static algorithms for k-connectivity to support dynamic node insert/delete operations. Additional results can be found in [1, 9, 12, 16, 20, 26, 30, 34].

Wieselthier et al. in [36, 37] were the first to study the broadcast problem in wireless ad-hoc networks for the 2-dimensional case and when $\alpha = 2$. In this work, the performances of three heuristics, namely the minimum spanning tree (MST), the shortest path tree (SPT) and the broadcasting incremental power (BIP) have been experimentally compared (one to each other) on the random uniform model without providing theoretical results. The approach taken in [36, 37] is to build a source rooted spanning tree by adjusting transmit powers of nodes, followed by a sweep operation to remove redundant transmissions. Wan et al. in [35] present the first analytical results for this problem by exploring geometric structures of Euclidean MSTs. In particular, they prove that the approximation ratio of MST is between 6 and 12, for BIP it is between $\frac{3}{12}$ and 12 and for SPT it is at least $\frac{n}{2}$, where n is the number of receiving nodes given that there are no obstacles in the network and that the fixed energy cost for electronics is negligible. Cagalj et al. [10] give a proof of NP-Hardness of the minimum-energy broadcast problem in a Euclidean space. Many researchers provided analytic results of the minimum-energy broadcast algorithm based on computing an MST. In [2] Ambuhl et al. proves an approximation factor of 6, which matches the lower bound previously known for this algorithm. Flammini et al. [23] establish improved approximation results on the performance of BIP. Cartigny et al. in [15] develop localized algorithms for minimum-energy broadcasting. Segal and Shpungin [32] develop a general framework for k-fault tolerance in various topology problems and provide an approximation bound of $O(k^2)$ for the k-broadcast problem. Additional references and results may be found in [2, 6, 8].

We can also add an additional constraint parameter to the problem, the bounded diameter h of the induced communication graph. For the linear case node disposition, Kirousis et al. [28] develop an optimal power assignment algorithm in $(O(n^4)$ time. In the Euclidean case, [21] obtains constant ratio algorithms for the bounded-hop strong connectivity for well spread instances. Beier et al. [7] discuss the problem of finding a bounded-hop path between pairs of nodes with minimized power consumption. They find an optimal path in $O(hn\log n)$ time. In [11] the authors obtain $(O(\log n), O(\log n))$ bicriteria approximation algorithms for the bounded-hop broadcast, bounded-hop connectivity and bounded-hop symmetric connectivity problems. In their output there are at most $h\log n$ hops with $\log n$ times the optimal cost for h hops. In [3] the authors present an exact algorithm for solving the 2-hop broadcast problem with a running time of $O(n^7)$ as well as a PTAS with a running time of $O(n^\mu)$ where $\mu = O((h^2/\epsilon)^{2^h})$. Funke and Laue [24] provide a PTAS for the h-broadcast algorithm in time linear in n. Additional results for bounded range assignments can be found in [18, 22].

1.3 Our Contribution

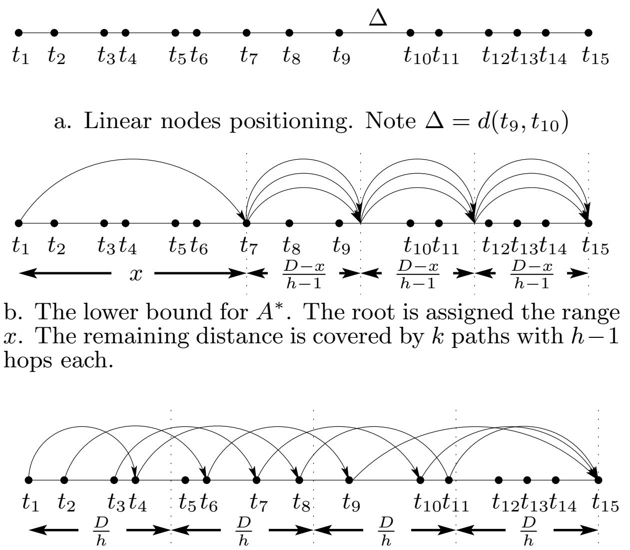
We study the problem of h-bounded broadcasting in conjunction with k-fault resistance (MEkBHB). We first provide a $O(k)$ approximation algorithm for the linear layout of nodes. For the planar case we develop an approximation algorithm, with provable approximation ratio of $O(k^3)$, which is later decreased to $O(k^2)$ by a fine analysis. Finally we show a trivial power assignment with a cost $O(h)$ times the optimum. All our algorithms run in low polynomial time.

2. LINEAR BOUNDED-HOP BROADCAST

Given n nodes positioned on a single line, we assume they are located in an increasing order from left to right (see Figure 1.a). We start by solving a special case of the MEkBHB problem for the linear layout of nodes, when the root node is $r = t_1$, that is from the leftmost node.

2.1 Broadcast from t_1

Let $\Delta = \max_{1\le i<n} d(t_i, t_{i+1})$ be maximal distance between two adjacent nodes and $D = d(t_1, t_n)$ be the distance from the leftmost to rightmost nodes. We assume $\Delta\, k \le \frac{D}{h}$, that is if we divide the line into sections of length D/h then each section will contain at least k nodes. The next Lemma gives the lower bound for the cost of the optimal power assignment A^* under these settings.



a. Linear nodes positioning. Note $\Delta = d(t_9, t_{10})$

b. The lower bound for A^*. The root is assigned the range x. The remaining distance is covered by k paths with $h-1$ hops each.

c. The power assignment L_k^h. The entire distance D is divided into h equal blocks.

Figure 1: Linear k-fault resistant bounded-hop broadcast from t_1 for $k = 3$, $h = 4$.

Lemma 2.1. $C_{A^*} \ge \frac{D^2 k}{k+h-1}$.

Proof. Let t_i be the rightmost node reached by t_1 in A^*. Nodes that are not directly reached by t_1 must be accessible by k-node disjoint paths from it. In case of the linear layout, if there is a path from u to v, then there is a path from u to any node in between. Therefore in A^* there must be k node disjoint traversals to t_n originating in nodes $t_{i-k+1}, \ldots, t_i$, since paths originating in earlier nodes will not be optimal. Let $x = d(t_1, t_i)$. The distance covered by these paths is $D - x$ (see Figure 1.b). Note that C_{A^*} is minimized if all these paths start at t_i and evenly divide the distance $D - x$ into $h - 1$ hops. Given that the root node t_1 is assigned the range x we can bound the optimal power assignment by:

$$C_{A^*} \ge x^2 + \left(\frac{D-x}{h-1}\right)^2 k(h-1).$$

Next we analyze the function $f(x) = x^2 + (\frac{D-x}{h-1})^2 k(h-1)$ to find the lower bound for the cost of the optimal power assignment. For $x = \frac{Dk}{k+h-1}$, the value of function $f(x)$ is minimized. As a result,

$$\begin{aligned} f\left(\frac{Dk}{k+h-1}\right) &= \frac{D^2k^2}{(k+h-1)^2} + \left(\frac{D - \frac{Dk}{k+h-1}}{h-1}\right)^2 k(h-1) \\ &= \frac{D^2k^2}{(k+h-1)^2} + \frac{D^2k(h-1)}{(k+h-1)^2} \\ &= \frac{D^2k(k+h-1)}{(k+h-1)^2} = \frac{D^2k}{k+h-1}, \end{aligned}$$

which completes our proof. ■

Next we describe our algorithm. We divide the line into h blocks of length D/h each. Note that due to our assumption, there are at least k nodes in each block. Let t_{i_j} be the rightmost node in the j^{th} block. We assign power as follows. The root is assigned $p(t_1) = d(t_1, t_{i_1})^2$ to reach t_{i_1}. In blocks 1 to $h-2$, each of the k rightmost nodes is assigned with enough power to reach the k rightmost nodes of the next block. In block $h-1$, all k rightmost nodes are assigned with enough power to reach t_n and nodes in the last block are assigned zero power (see Figure 1.c). Formally for $1 \leq l \leq k$,

$$\begin{aligned} p\left(t_{i_j-k+l}\right) &= d(t_{i_j-k+l}, t_{i_{j+1}-k+l})^2, \text{ for } 1 \leq j < h-1 \\ p\left(t_{i_{h-1}-k+l}\right) &= d(t_{i_{h-1}-k+l}, t_n)^2. \end{aligned}$$

Let L_k^h be the resulting power assignment. Easy to see that the the induced (directed) communication graph $H_{L_k^h}$ is k-fault resistant h-bounded-hop broadcast graph rooted at t_1. It is sufficient to show the existence of k-node disjoint paths from t_1 to t_n. These paths can be described as,

$$y_l = \left(t_1, t_{i_1-k+l}, t_{i_2-k+l}, \ldots, t_{i_{h-1}-k+l}, t_n\right)\right), \ 1 \leq l \leq k.$$

Lemma 2.2. $C_{L_k^h} \leq \frac{D^2}{h^2}\left(4k(h-1)+1\right).$

Proof. The root is assigned a transmission range of at most D/h. A total of $k(h-1)$ nodes are assigned a transmission range of at most $D/h + k\Delta$ each. Recall our assumption $\Delta\, k \leq \frac{D}{h}$. Therefore,

$$\begin{aligned} C_{L_k^h} &\leq \left(\frac{D}{h}\right)^2 + k(h-1)\left(\frac{D}{h} + k\Delta\right)^2 \\ &\leq \left(\frac{D}{h}\right)^2 + k(h-1)\left(\frac{2D}{h}\right)^2 = \frac{D^2}{h^2}\left(4k(h-1)+1\right). \end{aligned}$$

This completes our proof. ■

Finally we can easily derive our main Theorem.

Theorem 2.3. $C_{L_k^h} \in O(k)C_{A^*}$.

Proof. According to Lemmas 2.1 and 2.2 we have $C_{A^*} \geq \frac{D^2 k}{k+h-1}$ and $C_{L_k^h} \leq \frac{D^2}{h^2}\left(4k(h-1)+1\right)$. Also, since $k \geq 1$ and $h \geq 2$ then $k+h-1 \leq kh$. Therefore,

$$\begin{aligned} \frac{C_{L_k^h}}{C_{A^*}} &\leq \frac{D^2}{h^2}\left(4k(h-1)+1\right)\frac{k+h-1}{D^2 k} \\ &\leq \frac{4k(h-1)+1}{h} \leq 4k+1. \end{aligned}$$

We conclude $C_{L_k^h} \in O(k)C_{A^*}$. ■

2.2 Broadcast from any t_i

The generalization of our algorithm to broadcast from any node t_i, $1 \leq i \leq n$ is very simple. We use the described algorithm to obtain two power assignments A_L and A_R; the former is a power assignment for k-h-broadcast from t_i to nodes to the left of it – namely $t_1, \ldots, t_{i-1}$; the latter is for nodes to the right of it – namely $t_{i+1}, \ldots, t_n$. Next we combine the two power assignments. Let $p_L(t)$ and $p_R(t)$ be the power assigned to t in A_L and A_R respectively. We define the power assignment $L(i)_k^h$ as follows, $p(t) = \max\{p_L(t), p_R(t)\}$.

Clearly the induced communication graph H_A is k-fault resistant and h-bounded-hop broadcast from t_i. Let $A(i)^*$ be the optimal power assignment for k-h-broadcast from t_i. Since solving the problem for a subset of adjacent nodes will produce a cheaper solution, we can use the bound in Theorem 2.3 and conclude $C_{A_L} \in O(k)C_{A(i)^*}$ and $C_{A_R} \in O(k)C_{A(i)^*}$. Therefore $C_{L(i)_k^h} \in O(k)C_{A(i)^*}$. Easy to see that after sorting the nodes the running time of the algorithm is $O(n \log n)$.

3. PLANAR BOUNDED-HOP BROADCAST

The general idea for a power assignment which induces a k-fault resistant bounded-hop broadcast graph in the plane s to first obtain a bounded-hop broadcast graph and then make it k-fault resistant. In [24] the authors present a PTAS algorithm for the MEkBHB problem with fault resistance parameter $k = 1$. We use this construction as a basis for k-fault resistant bounded-hop broadcast. We first explain the technique used for k-fault resistant strong connectivity suggested in [14], that we will use later to obtain k-fault resistant broadcast tree.

3.1 Planar k-strong connectivity

Let $\mathcal{T}$ be a set of n points in the plane (representing n transceivers). For each node $t \in \mathcal{T}$, let $N_t \subseteq \mathcal{T}$ be a set of k-closest nodes to t, and put $r_t^* = \max_{t' \in N_t} d(t, t')$. We now describe the power assignment algorithm. Compute a minimum spanning tree MST of the Euclidean graph induced by $\mathcal{T}$. Assign to each node $t \in \mathcal{T}$ a power $p(t) = (r_t^*)^2$. As a result, each node can reach its k-closest neighbours. Denote this initial range assignment by A'. For each edge $e = (t, s)$ of MST, increase the power of the nodes in $N_t \cup N_s$ (if necessary), such that each node $t' \in N_t$ can reach all nodes in N_s, and vice versa. Let A_k denote the resulting power assignment.

The idea is rather simple, we want to construct k-node disjoint paths along the edges of the MST. Think about each N_t as large intersections containing k intersection points, and that there are k symmetric links between N_t and N_s iff (t, s) is an edge in the MST. The range assignment of each node t should be at least r_t^* (otherwise k-strong connectivity is impossible), and in addition sufficient enough to create the intersections mentioned above. The following Lemma can be easily proved (see Figure 2).

Lemma 3.1. *Given two nodes $t, s \in \mathcal{T}$, let $r_{t'}^{t,s}$ be the range node $t' \in N_t$ has to be assigned in order to reach any node in N_s including s. Then, $r_{t'}^{t,s} < r_t^* + d(t, s) + r_s^*$.*

We provide a proof sketch for the following Theorem from [14]. Let A_k^* be an optimal power assignment for the k-connectivity problem.

Theorem 3.2 (Carmi et al. [14]). $C_{A_k} \in O(k)C_{A_k^*}$

Proof sketch. Note that the range assignment of each node t in A_k must satisfy the following two conditions; (a) it *must* reach at least k other nodes (b) it *must* satisfy the demands of other nodes, for which it is one of their k-closest neighbors (i.e. all those nodes s so that $t \in N_s$).

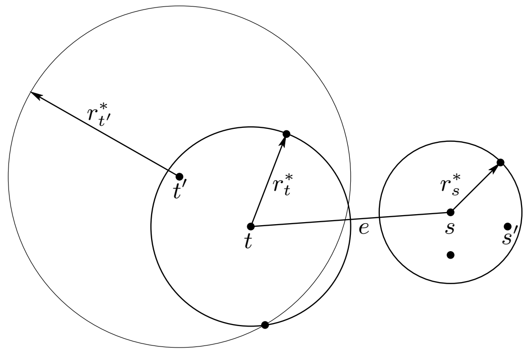


Figure 2: Node t' is assigned a range of at most $r^*_t + d(t,s) + r^*_s$ to reach all $s' \in N_s$.

For an edge $e = (t,s)$ in the MST we denote $r^e_{t'} = r^{t,s}_{t'}$. Therefore each node t' is assigned a transmission range which is the maximum between $r^*_{t'}$, and its *obligation* to some node t. Where $t' \in N_t$ and there is an edge $e \in$ MST so that $r^e_{t'} > r^*_{t'}$ (see Figure 2). Therefore,

$$C_{A_k} = \sum_{t' \in \mathcal{T}} p(t') \leq \sum_{t' \in \mathcal{T}} \max\left\{ r^*_{t'}, \max_{e \in \text{MST}} \{r^e_{t'}\} \right\}.$$

From Lemma 3.1 and the fact that geometrical MST has a bounded degree of 6,

$$C_{A_k} \in O(k) \left(\sum_{t \in \mathcal{T}} (r^*_t)^2 + \sum_{e \in \text{MST}} |e|^2 \right).$$

According to [28] $C_{\text{MST}} \leq C_{A^*_1} \leq C_{A^*_k}$. We can conclude, $C_{A_k} \in O(k)\left(C_{A^*_k} + C_{\text{MST}}\right) = O(k) C_{A^*_k}$. ■

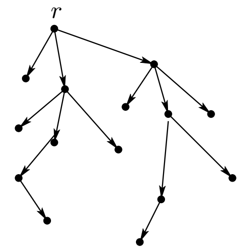
3.2 The algorithm

Given a set of nodes $\mathcal{T}$ and a root node r, we wish to construct a power assignment A^h_k, so that the induced communication graph $H_{A^h_k}$ is k-h-broadcast rooted at r. As before, for each node $t \in \mathcal{T}$, let $N_t \subseteq \mathcal{T}$ be a set of k-closest nodes to t, and put $r^*_t = \max_{t' \in N_t} d(t,t')$. Let A_h be a power assignment constructed in [24] for some constant h, so that H_{A_h} is a 1-h-broadcast graph. We are ready to describe the power assignment algorithm.

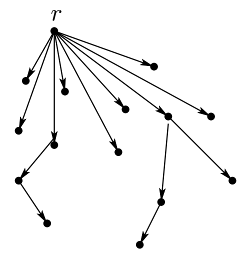
We start by constructing a directed spanning tree of H_{A_h} by running a BFS from the root node r. Denote the resulting tree by BHT$'$ and by *level-i* nodes to be the nodes at distance i from the root. Clearly for each node $t \in \mathcal{T}$ there is a unique directed path of at most h hops from r to t in the BHT$'$. Note that the power assignment $A_{\text{BHT}'}$ required to induce this tree has a cost $C_{A_{\text{BHT}'}} \leq C_{A_h}$. Next we decrease the depth of BHT$'$ to be $h-1$ by adding a directed edge from r to every level-2 node and remove all edges between level-1 nodes and level-2 nodes (see Figure 3). Call this tree BHT and by A_{BHT} the power assignment required to induce this tree. Easy to see that $C_{A_{\text{BHT}}} \leq 2C_{A_{BHT'}} \leq 2C_{A_h}$ by using the following observation.

Observation 3.3. *For any $x_1, x_2, \ldots, x_m \in \mathbb{N}$ it holds*

$$\left(\sum_{i=1}^{m} x_i \right)^2 \leq m \sum_{i=1}^{m} {x_i}^2.$$



a. The initial depth of the BHT$'$ is $h = 4$.



b. The depth of the obtained BHT is $h - 1 = 3$.

Figure 3: Depth decrease of BHT$'$.

Similar to the case of strong connectivity, we would like to create k-node disjoint paths along the edges of BHT, from r to any node other $t \in \mathcal{T}$. We start by assigning the root node r with a power $p(r) = (r^*_r)^2$, so that it can reach its k-closest neighbors. Next, for each directed edge $e = (t,s)$ (from t to s) in BHT we increase the power (if required) of each node $t' \in N_t$ so it could reach all nodes in $N_s \cup \{s\}$. Let A^h_k denote the resulting power assignment.

It is easy to see that the resulting (directed) communication graph $H_{A^h_k}$ is k-fault resistant h-bounded-hop broadcast rooted at r. That is, for any node $t \in \mathcal{T}$ there are k-node disjoint paths from r to t, that "follow" the path from r to t in BHT. And each of these paths has at most h hops.

3.3 Analysis

In order to analyze the cost of the power assignment A^h_k we need to take a closer look at the power increase stage of each node. All nodes (except for the root) start with no power and it is increased if required. The power of $t' \in N_t$ is increased only to satisfy the demand of some outgoing edge $e = (t,s)$ from t in BHT, that is to reach any node in $N_s \cup \{s\}$. Since node t' can be a member in many sets of k-closest neighbors, it might be required to increase its power many times, but eventually its power will be *dominated* by some outgoing edge $e^{t'} = (t_i, s)$, where $t' \in N_{t_i}$. Recall that for an edge $e = (t,s)$ we denote by $r^e_{t'} = r^{t,s}_{t'}$ the range node t' has to be assigned to reach s and all the k-closest neighbors of s.

To simplify the notation, for any node t'_i, denote by $e_i = (t(i), s(i))$ the edge which dominates the power assignment of t', where $t(i) \neq s(i)$ are some nodes in $\mathcal{T}$. Note that is possible that $t(i) = t(j)$ for $i \neq j$. The root node might not have a dominating edge since its initial power is greater than 0. However, we will assume it has one and later show that it does not influence our analysis at all.

LEMMA 3.4. $C_{A_k^h} \in O(k)\left(\sum_{t\in\mathcal{T}} (r_t^*)^2 + C_{A_h}\right)$.

PROOF. From Lemma 3.1 we have the following inequality: $r_{t'}^{t(i),s(i)} < r_{t(i)}^* + d(t(i), s(i)) + r_{s(i)}^*$. From Observation 3.3, $p(t_i') \le 3\left(\left(r_{t(i)}^*\right)^2 + d(t(i),s(i))^2 + \left(r_{s(i)}^*\right)^2\right)$.
Let $p'(t)$ be the power node t is assigned in A_{BHT}. Then $p'(t(i)) \le d(t(i),s(i))^2$. We can write,

$$C_{A_k^h} = \sum_{i=1}^{n} p(t_i') \le 3\sum_{i=1}^{n}\left(\left(r_{t(i)}^*\right)^2 + p'(t(i)) + \left(r_{s(i)}^*\right)^2\right).$$

For any node $t \in \mathcal{T}$, only for $t_i' \in N_t$ we have $t(i) = t$. For any node $s \in \mathcal{T}$, let $e_s = (t_s, s)$ be an incoming edge of s in BHT. Then only for $t_i' \in N_{t_s}$ we have $s(i) = s$. As a result, $\sum_{i=1}^n \left(r_{t(i)}^*\right)^2 \le k\sum_{t\in\mathcal{T}}(r_t^*)^2$, $\sum_{i=1}^n \left(r_{s(i)}^*\right)^2 \le k\sum_{t\in\mathcal{T}}(r_t^*)^2$ and $\sum_{i=1}^n p'(t(i)) \le k\sum_{t\in\mathcal{T}} p'(t) = k\,C_{A_{\mathrm{BHT}}} \le 2k\,C_{A_h}$. Therefore, $C_{A_k^h} \in O(k)\left(\sum_{t\in\mathcal{T}}(r_t^*)^2 + C_{A_h}\right)$. Note that if the root has a dominating edge it does not affect the analysis. ■

Paper [24] proves that given power assignment algorithm which produces the A_h assignment is PTAS and therefore it holds $C_{A_h} \le (1+\epsilon)C_{A_1^{h*}} \le C_{A_k^{h*}}$, where $A_1^{h^*}$ and $A_k^{h^*}$ are the optimal power assignments that induce a 1-h-broadcast and k-h-broadcast communication graphs respectively. In the next two Lemmas, which prove two different approximation rations of our algorithm, the cost of C_{A_h} is negligible in relation to $\sum_{t\in\mathcal{T}}(r_t^*)^2$.

THEOREM 3.5. $C_{A_k^h} \in O(k^3)C_{A_k^{h*}}$.

PROOF. Let every node $t \in \mathcal{T}$ be assigned a transmission range r_t^*. Call this power assignment A_1. The induced communication graphs holds the following property, *every node has at least k neighbors*. We also claim that A_1 is an optimal power assignment that induces a communication graph with such a property.

An approximation algorithm for a power assignment A_k^{n-1} which induces a k-$(n-1)$-broadcast communication graph (there is no constant bound on the path length) is given in Segal and Shpungin [32] and they prove that $C_{A_k^{n-1}} \in O(k^2)C_{A_k^{n-1*}}$, where $A_k^{n-1^*}$ is the optimal power assignment for the k-fault resistant unbounded broadcast.

The induced communication graph $H_{A_k^{n-1}}$ maintains the property that *every node has at least k neighbors*. Therefore we can conclude that $C_{A_1} \le C_{A_k^{n-1}}$. Easy to see that $C_{A_k^{n-1*}} \le C_{A_k^{h*}}$, since forcing a constant maximal number of hops increases the power assignment. As a result,

$$\sum_{t\in\mathcal{T}}(r_t^*)^2 = C_{A_1} \in O(k^2)C_{A_k^{h*}}.$$

In conjunction with Lemma 3.4 we conclude

$$C_{A_k^h} \in O(k)\left(\sum_{t\in\mathcal{T}}(r_t^*)^2 + C_{A_h}\right) = O(k^3)C_{A_k^{h*}}.$$

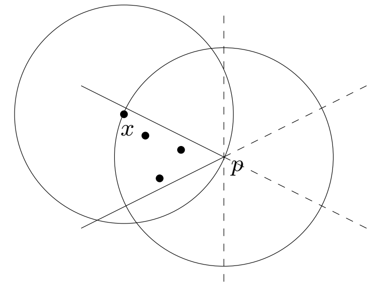


Figure 4: Point p covering for $k = 2$.

■

Next we present a more delicate analysis in order to improve the approximation ratio bound.

THEOREM 3.6. $C_{A_k^h} \in O(k^2)C_{A_k^{h*}}$.

We will need the following two Lemmas to prove Theorem 3.6.

LEMMA 3.7. *Any single point p in the plane can not be covered by more than $6k+6$ disks, so that each disk does not cover more than k centers of other disks.*

PROOF. We divide the plane into 6 equal sectors of $\pi/3$ and show that at most $k+1$ disks that cover p can reside in each sector so that each disk does not cover more than k centers of other disks. Suppose, by contrary, that one of the sectors contains more than $k+1$ disks. In Figure 4 there is a sector β with 4 disk centers ($k = 2$). Let x be the center of a disk in β at largest distance most from p. Let D_p be a disk centered at p with a radius $d(p,x)$. Clearly, all the other disk centers in the sector β are covered by D_p and there are at least $k+1$ of them. Since we have chosen a sector β of $\pi/3$, all these disk centers belong to $D_x \cap D_p$ and therefore are covered by D_x. Contradiction. ■

LEMMA 3.8. *Let $\mathcal{S}$ be a set of $m > k$ nodes in the plane positioned inside disk D of radius r. Let A^* be the optimal power assignment so that each node can reach at least k nodes in $\mathcal{S}$. Then $C_{A^*} \in O(k)r^2$.*

PROOF. Let $p^*(s_i)$ be the power node $s_i \in \mathcal{S}$ is assigned in A^*. Let D_i be a disk centered at node s_i as a result of the power assignment (i.e. the disk is centered at s_i and has a radius of $\sqrt{p^*(s_i)}$). Since A^* is an optimal power assignment, then each disk can cover at most k centers of other nodes. Due to Lemma 3.7 each point inside disk D can be covered by at most $O(k)$ disks, therefore

$$\bigcup_{i=1}^{m} area(D_i) \in O(k)area(D) \Rightarrow \sum_{i=1}^{m} p^*(s_i) \in O(k)r^2.$$

■

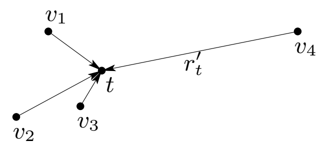


Figure 5: Node t is reached by 4 nodes. $R_t = \{v_1, v_2, v_3, v_4\}$ and $r'_t = d(t, v_4)$.

PROOF OF THEOREM 3.6. We construct a power assignment in which each node has at least k neighbors and bound its cost in a similar manner as in Theorem 3.5. Recall that A_k^{n-1*} is the optimal power assignment for the k-fault resistant unbounded broadcast rooted at r. Note that in the induced communication graph $H_{A_k^{n-1*}}$, the root reaches at least k nodes and any other node $t \neq r \in \mathcal{T}$ is either reached by r or by *at least k different nodes* (otherwise we do not have k-broadcast). Let R_t be a set of all nodes that reach t by a single hop in $H_{A_k^{n-1*}}$. For all nodes t reached by r we define $R_t = \{r\}$. Let r'_t is the range node t has to be assigned in order to reach all nodes in R_t (see Figure 5).

Let A_2 be a power assignment defined as follows. To avoid confusion, let x_r be the radius assigned to r in order to have k neighbors. Assign each node $t \in \mathcal{T}$, $t \neq r$ with a power $p_2(t) = (r'_t)^2$ and $p_2(r) = (x_r)^2$. In H_{A_2} the root reaches at least k nodes and any other node reaches either the root or at least k different nodes. Let $f(t) \in R_t$, be the most distant node reached by $t \neq r$ in H_{A_2}, that is $d(t, f(t)) = r'_t$ (if there are more than one, choose one arbitrarily). Call this node the *target* of t.

Let Z_u be a set of nodes that share the same target node u, that is for every $t \in Z_u,\ f(t) = u$. Note that Z_r is a set of all nodes reached by r in a single hop in $H_{A_k^{n-1*}}$. This is because for every such node t we define $R_t = \{r\}$. Let $C_{A_2(Z_u)}$ be the total power assigned to nodes of Z_u in A_2. Easy to see that sets Z_u are disjoint and that $\bigcup_{u\in\mathcal{T}} Z_u = \mathcal{T}\backslash\{r\}$. Note that for any $t \in Z_u$ it holds $p_2(t) \leq p(u)$, where $p(u)$ is the power u is assigned in A_k^{n-1*}. The cost of A_2 is given by

$$\begin{aligned} C_{H_{A_2}} &= \sum_{t\neq r\in\mathcal{T}} p_2(t) + (x_r)^2 \\ &= \sum_{u\in\mathcal{T}}\sum_{t\in Z_u} p_2(t) + (x_r)^2 = \sum_{u\in\mathcal{T}} C_{A_2(Z_u)} + (x_r)^2 . \end{aligned}$$

We cannot bound the cost of H_2 just yet. Moreover, not every node necessarily has k neighbors (nodes reached by r in A_k^{n-1*} might have less than k neighbors). We would like to bound the costs $C_{A_2(Z_u)}$ and ensure that all nodes have at least k neighbors. Let A_3 be a power assignment defined as follows. For every Z_u we consider two cases:

Case 1: If $|Z_u| < k$ then $C_{A_2(Z_u)} \leq k\,p(u)$, so for every $t \in Z_u$ we assign $p_3(t) = p_2(t)$ (remains as in A_2).

Case 2: If $|Z_u| \geq k$ then there are at least $k+1$ nodes inside of a disk created by the power assignment of u. The nodes $Z_u \cup \{u\}$ are inside of a disk centered at u with radius $\sqrt{p(u)}$. For every node $t \in Z_u$ assign $p_3(t)$ to be the minimal power required for t to reach its k closest neighbors in Z_u. According to Lemma 3.8, $\sum_{t\in Z_u} p_3(t) \in O(k)p(u)$. The root is assigned as before with a power to reach its k closest neighbors $p_3(r) = p_2(r) = (x_r)^2$.

Let $C_{A_3(Z_u)}$ be the total power assigned to nodes of Z_u in A_3. If we combine the two cases we obtain for every $u \in \mathcal{T}$, $C_{A_3(Z_u)} \in O(k)\,p(u)$. Therefore we can conclude,

$$C_{A_3} = \sum_{u\in\mathcal{T}} C_{A_3(Z_u)} + (x_r)^2 \in O(k)\sum_{u\in\mathcal{T}} p(u) = O(k) C_{H_{A_k^{n-1*}}} .$$

We claim every node in H_{A_3} has at least k neighbors. Easy to see that the root has k neighbors. Note that $|Z_r| \geq k$, because the root necessarily has at least k neighbors in A_k^{n-1*}. For any other node t:

Case 1: If $|Z_u| < k$ then $u \neq r$ and necessarily $|R_t| \geq k$. Also $p_3(t) = p_2(t)$ and therefore node t reaches all the nodes in R_t.

Case 2: If $|Z_u| \geq k$ then it is assigned to reach k different nodes in Z_u.

We have obtained a power assignment A_3 so that the induced communication graph H_{A_3} satisfies the property that *every node has at least k neighbors*. As a result, $C_{A_1} \leq C_{A_3}$. We showed that $C_{A_3} \in O(k)C_{H_{A_k^{n-1*}}}$. Similarly to the proof of Theorem 3.5 and in conjunction with Lemma 3.4 we conclude that

$$C_{A_k^h} \in O(k)\left(\sum_{t\in\mathcal{T}} (r_t^*)^2 + C_{A_h}\right) = O(k^2) C_{A_k^{h*}}$$

■

It takes linear time to construct H_{A_h}. Easy to see that our algorithm runs in a total of $O(n^2)$ time.

A simple $O(h)$ approximation for very high fault resistance - Instead of forming k-node disjoint paths from r to any other node, we could simply assign the root with enough power to reach all nodes in a single hop. Clearly such a power assignment is very fault resistant since the transmission between r and any other node does not rely on relay nodes.

LEMMA 3.9. *Let t be the most distant node from r. Let A_r be a power assignment where the root is assigned $p(r) = d(r,t)^2$. Then $C_{A_r} \in O(h)C_{A_k^{h*}}$.*

PROOF. There is a path $y = (r = u_1, u_2, \ldots, u_{l+1} = t)$ of at most $l \leq h$ hops from r to t in H_{A_h}. Let r_u be the range assigned to node u in A_h. Clearly $p(r) \leq \left(\sum_{i=1}^{l} r_{u_i}\right)^2$ and $\sum_{i=1}^{l} (r_{u_i})^2 \leq C_{A_h}$. According to Observation 3.3 we have $p(r) \leq \left(\sum_{i=1}^{l} r_{u_i}\right)^2 \leq l\left(\sum_{i=1}^{l} r_{u_i}\right)^2 \leq hC_{A_h}$. We conclude $C_{A_r} \in O(h)C_{A_h}$ and as a result $C_{A_r} \in O(h)C_{A_k^{h*}}$. ■

4. CONCLUSIONS

In this paper we showed numerous results for the fault tolerant bounded hop broadcast topology problem in wireless networks. A possible interesting direction would be to improve the analysis of the approximation ratios obtained in this work or show a ratio which is k,h-dependent. It might be also of interest to develop a distributed approach for such a problem.

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