

# A SPICE compatible Behavioral Electrical Model of a Heated Tungsten Filament

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**Abstract**— A behavioral, SPICE compatible model of the electrical behavior of a Tungsten filament is proposed and verified experimentally. The model is calibrated by simple measurements on the target filament that include the static V-A relationship and the current transient in response to a voltage step. The model was found to faithfully reproduce the static and dynamic electrical behavior of the tested lamps that include: an automotive H4 lamp, a 500W halogen projector lamp and a filament of a 36W fluorescent lamp.

**Keywords**—component; Tungsten filament, heat capacity, thermal model.

## I. INTRODUCTION

Tungsten filaments are used in devices such as light sources, hot electrodes and heaters. In normal applications, filament temperatures may reach 1000 K and above, and consequently, the hot resistance will be appreciably higher than the cold resistance [1, 2]. This will normally cause a high inrush current at ‘turn on’ if the filament is connected directly to the nominal voltage source. Large inrush currents can be avoided by using a control scheme that limits the current or increases the voltage gradually. A filament model could be a useful tool for the design of such protection systems, especially if the model could be implemented easily in circuit simulators or mathematical modeling programs. Another potential application of a filament model is to explore the dynamic behavior of filaments. This is important when there is a need to reach a predetermined temperature within a given time [3-6] or when the light output has to be controlled in closed loop. In these cases, large-signal and small-signal simulations could be important research and engineering tools. A filament model can thus be useful to study and design incandescent lamp drivers and dimmers, rapid-start electronic ballasts for fluorescent lamps, and in many other applications.

Conventional filament models are difficult to apply since they include a large number of parameters that need to be found either from the literature or by measurement [7-10]. This obstacle was overcome in the present work by developing a

behavioral model that is calibrated by simple electrical measurements on the target filament. The model emulates the electrical behavior and, as a byproduct, provides information on the temperature of the filament. It does not include, however, light output directly. Light emission can be estimated from the filament temperature or by an extra fitting (not carried out in this study).

## II. FILAMENT MODEL

The rate of temperature increase of a filament ( $dT/dt$ ) can be expressed in its simplest form as a function of the electrical power fed to it,  $P_e$ , the heat losses due to thermal conduction and convection,  $P_c$ , and the radiated power  $P_r$ , by:

$$\frac{dT}{dt} = \frac{1}{C} [P_e - (P_c + P_r)] \quad (1)$$

where  $C$  is the heat capacity, and  $P_r$  and  $P_c$  are temperature dependent. The heat capacity is also a function of temperature [11, 12], but using fixed values produced adequate results for our behavioral model.

Note that (1) assumes a one-compartment model in which there is one major mass, the tungsten filament, which absorbs the heat. This is an approximation since in say, a light bulb, one can recognize a number of masses that absorb the heat (the tungsten filament, the gas surrounding it, the filament support leads, and the outer glass shell etc.). Various thermal conduction paths interconnect these thermal storages. When lamp filaments are at normal operating temperatures, the almost all of the dissipated heat escapes through radiation [10, 13]. Conduction through the support leads, however, plays a significant role in the dynamics of filament heating because the ends of the filament are considerably cooler than the middle. A more accurate equivalent circuit of the system would be distributed rather than a lumped network [14]. A simple SPICE model that uses one thermal mass and a linear resistor to represent the heat losses is presented in [15]. A SPICE model with one thermal mass was investigated by the authors at the outset of this work, and it was found that using two thermal masses connected by a thermal resistance as shown in Figures 1-3 produced better results.

This research was supported by THE ISRAEL SCIENCE FOUNDATION (grant No. 113/02) and by the Paul Ivanier Center for Robotics and Production management.

A pair of differential equations can be written to describe the filament model of Fig. 1, which has two thermal masses,  $C_f$  and  $C_s$  that are connected through a thermal resistance  $R_s$ :

$$\frac{dT_f}{dt} = \frac{1}{C_f} \left[ P_e - (P_c + P_r) - \frac{T_f - T_s}{R_s} \right] \quad (2)$$

$$\frac{dT_s}{dt} = \frac{T_f - T_s}{R_s C_s}.$$

From (2) we find for that for steady state conditions:

$$P_e = P_c + P_r. \quad (3)$$

The instantaneous input power to the filament,  $P_e$ , is a function of the input voltage,  $V_{in}$ , and the filament resistance,  $R_h$ , at any given temperature:

$$P_e = \frac{(V_{in})^2}{R_h}. \quad (4)$$

The resistance,  $R_h$ , of a Tungsten filament as a function of its temperate  $T_h$  can be approximated by [1]:

$$R_h = R_c \left( \frac{T_h}{T_c} \right)^{1.2285}. \quad (5)$$

where  $R_c$  is the resistance at the cold temperature  $T_c$  (normally measured at room temperature). In actuality, the exponent in (5) is a function of temperature [16, 17], but using a fixed value produced adequate results in our models.

A heat balance of equation such as (2) can be emulated by an equivalent electrical circuit such as Fig. 1 in which currents represent power, capacitances represent heat capacity, and resistances represent thermal resistances. The voltage source in Fig. 1 represents an infinite heatsink to the ambient temperature,  $T_a$ , which is assumed to be 300 K.  $C_f$ ,  $C_s$  and  $R_s$  form a simple distributed thermal model to represent the filament and its immediate surroundings. It should be noted that, unlike an electrical resistor, no power is dissipated by the thermal resistance in this electro-thermal analogy.

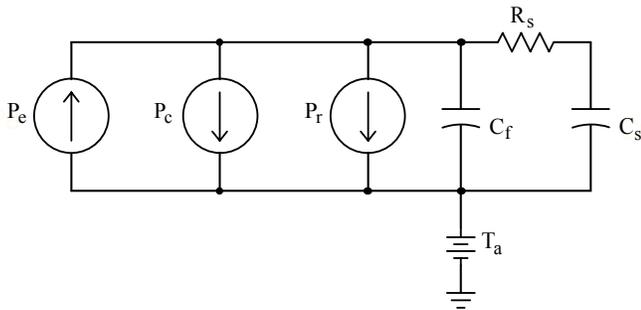


Figure 1. Proposed filament thermal model.

Tungsten filaments used in lamps are supported at each end with leads that also provide electrical connections to the filament. The leads conduct heat away from the filament near each end, and consequently, the middle of the filament is the hottest portion. Several complex schemes have been proposed to deal with this situation [6, 13, 14], but the simple C-R-C distributed network of Figs. 1-3 produces surprisingly accurate results. As explained below, the values for the distributed thermal network can be obtained by an optimization routine in which the values of these components are varied to minimize the error between measured data and the results of a series of simulations.

Using (3-5) the relationships among the sum of the conducted and radiated powers,  $(P_c+P_r)$ , and the filament temperature and resistance can be obtained from a set of static measurements on the filament. In these measurements, the filament is exposed to a range of voltages, from near zero to an appropriate maximum value, and the input current is measured at each point under steady state conditions.

A function based on the steady state measurements that reproduces  $(P_c+P_r)$  as a function of  $R_h$  can be used on the fly to determine the numerical values of the sum of the currents from the corresponding current sources in the model of Fig. 1. Namely, it is assumed that for any instance and given filament temperature  $T_f$ , the total filament losses can be combined into a conducted plus radiated term,  $(P_c+P_r)$ , which is equal to the value measured under steady state conditions. The difference between the input power,  $P_e$ , and the dissipated power,  $(P_c+P_r)$ , flows into or out of the C-R-C thermal mass network.

A filament circuit model is shown in Fig. 2. It includes the thermal mass network of Fig. 1, and four calculation blocks. The top two blocks implement (5) and (4). The third block implements a function to compute  $(P_c+P_r)$  from  $R_h$ . It may be implemented as an interpolated table or with a curve-fitted equation. The bottom block calculates the input current from the input voltage and  $R_h$ . The voltage source marked  $T_a$  represents the ambient temperature above which the filament temperature rises.

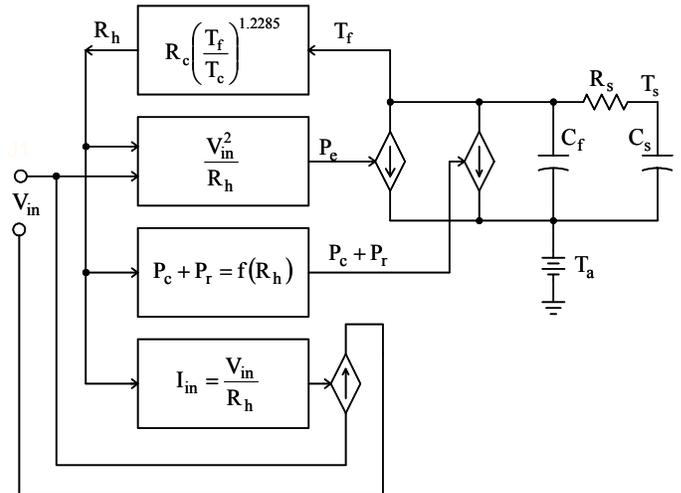


Figure 2. Proposed filament circuit model.

The parameters of the filament model of Fig. 2 include: a table of the static data or a fitted equation (power as a function of filament resistance), the cold resistance of the filament,  $R_c$ , and the temperature,  $T_c$ , at which it was measured, the ambient temperature,  $T_a$ , and the values of the components of the thermal mass network  $C_f$ ,  $R_s$ , and  $C_s$ . The values of the thermal network components are derived from a dynamic measurement as detailed in the Experimental section below. In the present model, we assume that the heat capacity of the filament is temperature independent, although it varies somewhat with temperature.

The circuit model of Fig. 2 can be easily translated to computer based mathematical packages such as MATLAB, Mathcad or Mathematica, and to electronic circuit simulators such as PSpice or Saber. In this work, a PSpice (Cadence, USA) implementation is demonstrated. Fig. 3 shows a PSpice realization of Fig. 2 that utilizes controlled voltage and current sources to perform the required calculations. The input voltage is realized with a piecewise-linear voltage-time ETABLE voltage source having values that were selected to approximate the measured input voltage waveform of a particular experiment.

### III. EXPERIMENTAL SETUP AND MODEL FITTING

The simulation model was verified by checking it against experimental measurements for several types of lamp filaments. This was done by first fitting the model for each filament to one set of measurements, and then testing the model with a different set of measurements conducted under different experimental conditions.

*Model Setup - Static Fitting:* The lamp power as a function of its resistance is measured and inserted into a table element, GTABLE-G3. This was accomplished by exposing the lamp to a range of voltages from nearly zero to at least the nominal value, and measuring the steady-state lamp current at each voltage. The experimental setup is shown in Fig. 4. The value

of the cold resistance,  $R_c$ , is determined with measurements taken with a low applied voltage.

*Model Setup - Dynamic Fitting:* The lamp is subjected to a voltage step by turning on switch S of Fig. 4, and the lamp current and lamp voltage are recorded during the transient state. The lamp voltage information is used to provide the table data for ETABLE E2, which is used in the simulation to replicate the actual voltage that was applied to the lamp. This voltage differs from the source voltage due to the voltage drops across the resistances of the interconnections and the MOSFET switch. The data of this single dynamic experiment was then used to determine optimal values of the components in the thermal network ( $C_f$ - $R_s$ - $C_s$ ) by using the PSpice optimization tool [18]. This add on package allows the selection of components values to meet a specific goal function. The initial data that are fed to the optimizer include an expression of the goal function, additional constrains, if any, and initial values of the components to be optimized.

The optimization routine was set to search for the values of  $C_f$ - $R_s$ - $C_s$  such that it will minimize the squared error between the measured and simulated currents.

The lamp voltage for the simulations runs of the fitting procedure was an approximation of the experimental lamp voltage generated by an ETABLE E2 that contained about 15 manually-selected data points.

An alternative method of determining the optimal values of the components in the thermal network was implemented in Mathcad, which can directly read the experimental data files. Equation (2) was implemented with a Runge-Kutta differential equation solver that used the experimental voltage data as a driving function. The differential equation solver was called by a minerr solve block that minimized sum of the squared differences between the measured and calculated data points. Selecting the Nonlinear Conjugate Gradient solver option for the minerr solve block produced the best results.

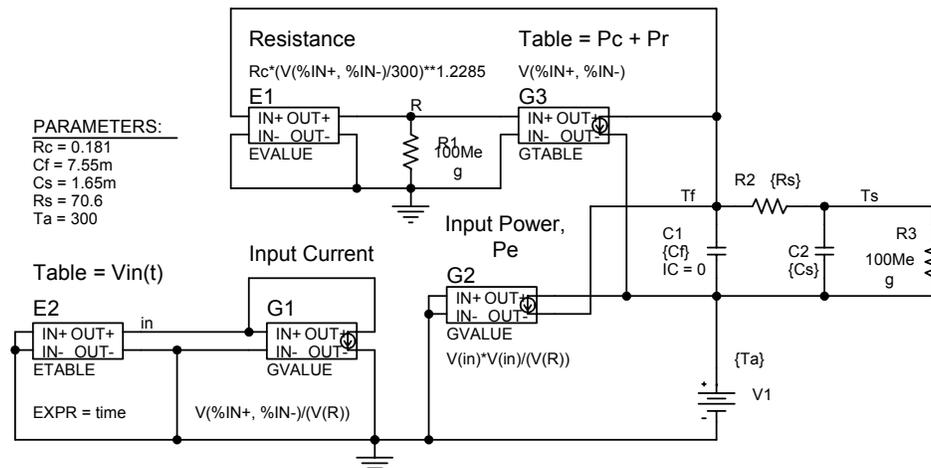


Figure 3. Cadence/ORCAD (Version 9.2) implementation of the proposed behavioral model of a Tungsten filament.

#### IV. MODEL VERIFICATION

As one would expect, very good agreement was found between simulation results and the experimental data used for calibrating the SPICE model. For example, Fig. 5 shows the measured and simulated filament current change of the 60 W filament of an H4 automotive lamp when subjected to a 0V to 12V step in the lamp voltage.

It should be noted that, in the simulation, the lamp was subjected to a close approximation of the voltage as measured in the experiment (Fig. 6). By this, the parasitic voltage drops of the interconnecting cables and the MOSFET are taken into account. Verification of the model was carried out by comparing the model response to experimental data that is different from the data used for calibrating the model.

Fig. 7 shows, for example, the response of the H4 lamp to a 0V to 7V voltage step. Fig. 8 shows the response of a 500W projector lamp to a 170V to 210V voltage step. The model of this lamp was fitted by a 0V to 60V voltage step. The response of a fluorescent lamp filament to a 2V to 5V voltage step is shown in Fig. 8. In this case, the model was calibrated by a 0V to 6V voltage step. The good agreement between the experimental and simulated results of the independent experiments subsequent to the calibration experiments demonstrates the strength of the model. The values of the fitted  $C_f$ - $R_s$ - $C_s$  parameters for the three lamp filaments tested in this study are given in Table 1.

#### V. DISCUSSION AND CONCLUSIONS

The electrical model of a Tungsten filament proposed in this study is based on a behavioral model that can be easily calibrated by simple measurements on the target filament. The measurements include the static V-A relationship and the current transient in response to a voltage step. The model was found to faithfully reproduce the electrical static and dynamic behavior of the lamps. The slight discrepancies between the simulated and experimental responses may be due to the fact that the model neglects a number of the physical properties of the Tungsten filament. For example, the model neglects the fact that the specific heat capacity of Tungsten is temperature dependent and that for a large temperature span the change could be significant, in the order of 20% per a 1000°K temperature change. Another source of error is that the resistance of the leads inside of a light bulb could significantly affect the accuracy of the cold resistance measurement for low-resistance filaments [19]. The lead resistance of the H4 lamp, for example, was 14% of the cold resistance of the filament.

In order to get a better understanding of the possible physical meaning of the components in the thermal network, the 60 W filament of an H4 lamp was weighed, and found to have a mass of 39 mg. The specific heat of tungsten at 2000 K

is  $172 \text{ mJ g}^{-1} \text{ K}^{-1}$  [19], giving a thermal mass of 6.7 mJ/K. The electrical capacitances expressed in mF given in Table 1 correspond to thermal masses expressed in mJ/K, so the total thermal mass of the H4 network is 9.26 mJ/K, which is 38% greater than the computed thermal mass. Since both thermal masses arrive at the same steady-state temperature, the thermal network could be viewed as an approximation of a distributed network that represents the filament and its immediate surroundings.

Notwithstanding the approximate nature of the model, it could be useful in the design of a wide variety of power electronics systems that drive filaments, such as incandescent lamp dimmers, and electronic ballasts for fluorescent lamps.

TABLE I. FITTED VALUES OF THE  $C_f$ ,  $R_s$ ,  $C_s$  NETWORK COMPONENTS VALUES OF THE EXPERIMENTAL FILAMENT TYPES.

Filament type	$C_f$	$R_s$	$C_s$
H4	7.60mF	72.0 $\Omega$	1.66mF
OSRAM 36W fluorescent	1.986mF	199 $\Omega$	2.63mF
Halogen 500W	47.6mF	49.9 $\Omega$	143mF

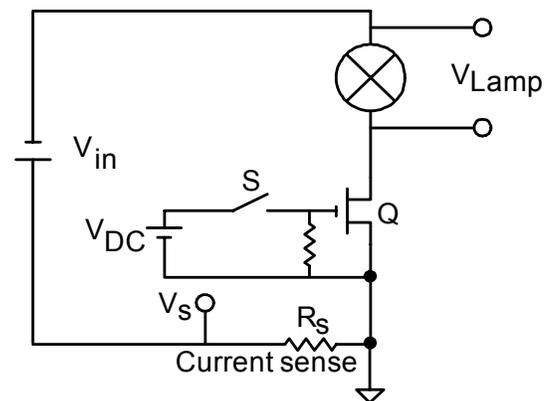


Figure 4. Lamp characteristics experimental setup.

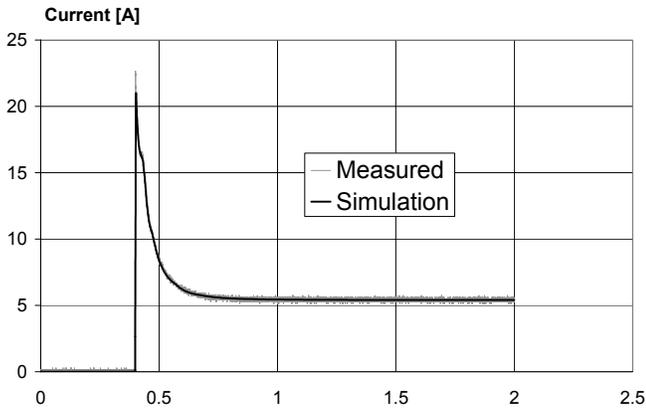


Figure 5. Measured and simulation current responses of an H4 lamp to a 0V to 12V voltage step

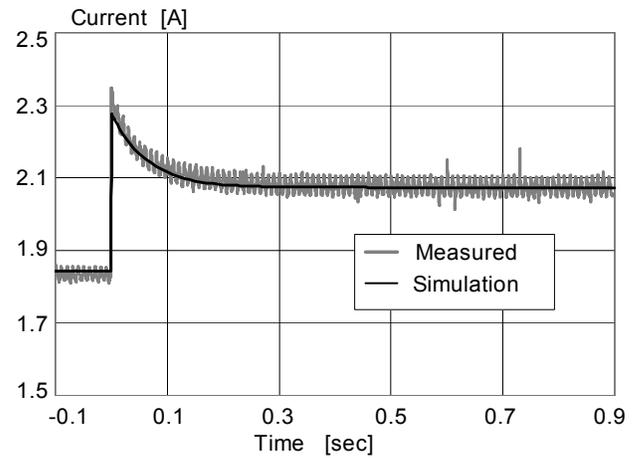


Figure 8. Measured and simulated current responses of a 500W projector lamp to a 170V to 210V step. Model was fitted by a 0V to 60V voltage step.

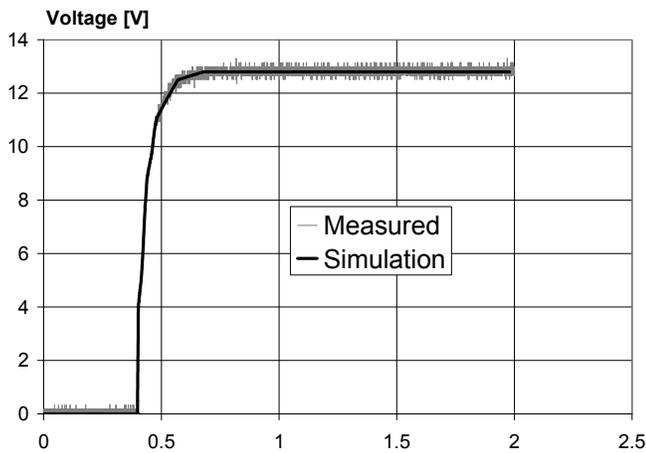


Figure 6. Experimental and reconstructed 0V to 12V voltage step applied to the H4 lamp

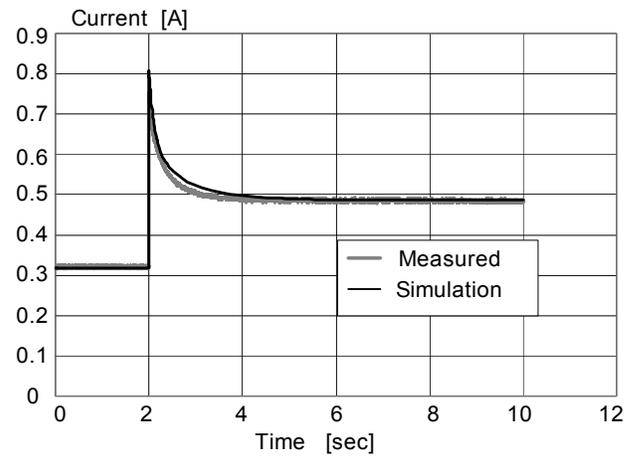


Figure 9. Measured and simulated current responses of a fluorescent lamp filament (OSRAM 36W) to a 2V to 5V voltage step. Model was fitted by a 0V to 6V voltage step.

#### APPENDIX A ESTIMATING THE INITIAL VALUES

To help the optimizer converge to the physical values of  $C_f$ - $C_s$ - $R_s$ , the initial values and the upper and lower limits of these values should be close to the final values. This could be accomplished by a sequence of trial and error runs. A better approach is to base the initial guess on an estimate of  $C_f$ - $C_s$ - $R_s$  assuming that the initial heating process is adiabatic. Integrating the input power for the first 0.1 s of the waveforms in Figs. 5 and 6, for example, gives an estimate of the amount of heat energy transferred to the filament during that time interval. The sum of  $C_f$  and  $C_s$  can be assumed to be equal to that energy divided by the rise in temperature above the initial value,  $T_a$ . The heated temperature is calculated by solving (5) for  $T_h$ , and then using the voltage divided by the current at the end of the heating interval as the hot resistance. The initial guess values of  $C_f$  and  $C_s$  can each be set equal to half of the estimated thermal capacitance. An initial value for  $R_s$  can be

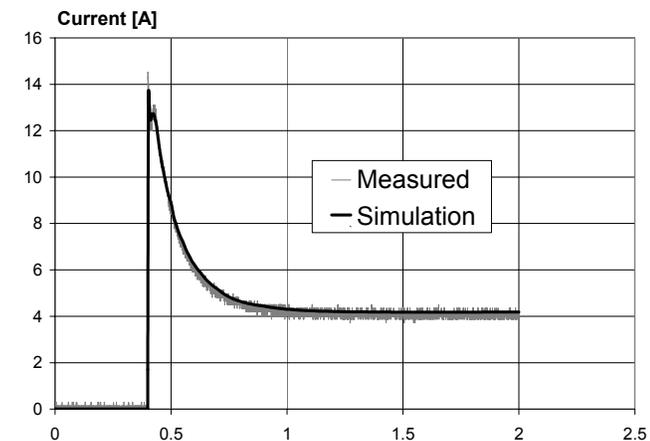


Figure 7. Measured and simulation current responses of the H4 lamp to a 0V to 7V voltage step.

obtained by dividing the heating time interval by the value of the two thermal capacitances connected in series, which is  $\frac{1}{4}$  of the estimated thermal capacitance. For the experiment corresponding to Figs. 5 and 6, the estimated heating energy was for the first 0.1 s was 10.6 J, and the temperature rise was 1248 K. The estimated values of  $C_f$  and  $C_s$  were 4.1 mF (corresponding to 4.1 mJ/K). The estimated value of  $R_s$  was 49 ohms, (corresponding to 49 K/W).

#### APPENDIX B POSSIBLE MODEL EXTENSION

A potential problem with the PSpice implementation is that the dissipated power ( $P_c + P_r$ ) calculated by GTABLE G3 from the filament resistance is limited to the value corresponding to highest resistance value in the table. Thus, it is important to ensure that the driving voltage used in the simulations never exceeds the highest voltage used in the static fitting unless some extrapolated values are added to the table.

An appropriate equation curve-fitted to the static measurements could extend the input voltage range of the model. The radiated power is approximately proportional to the fourth power of the filament temperature, while the conducted power is approximately proportional to the difference between the filament temperature and the ambient temperature [13]. This suggests a possible form for an equation to compute the static power dissipation from the filament temperature calculated with (5) and the ambient temperature:

$$P_{cr} = P_c + P_r = a(T_f - T_a) + bT_f^4 \quad (6)$$

It was found that (6) provided an excellent fit to the static data of the, and it was used to extrapolate two points to enter into GTABLE G3 for the H4 lamp simulations of shown in Figs. 5-7 because the nominal 12 V input voltage used for the dynamic measurement reached 12.8 V, but the maximum voltage used in the static measurement was 12.2 V. The fitted coefficient values for (6) are:  $a = 7.812 \cdot 10^{-4}$  W/K and  $b = 1.947 \cdot 10^{-12}$  W/K<sup>4</sup>.

An alternative PSpice model could be realized in which GTABLE G3 was replaced by a GVALUE source that implements (6), and uses  $V(T_f)$  and  $V(T_a)$  as inputs.

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