Random Delay in Network Coding for Bidirectional Relaying

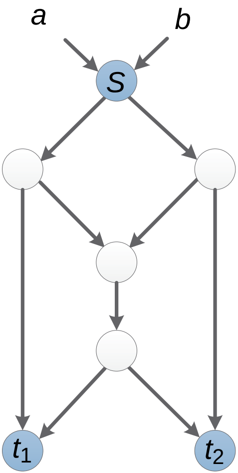
Niv Voskoboynik, Haim Permuter and Asaf Cohen

Ben Gurion University

June, 2014

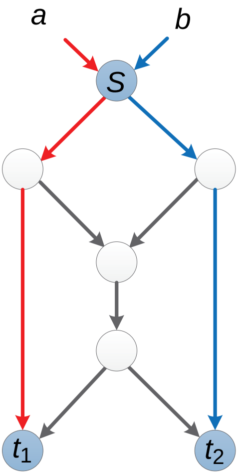
What is Network Coding?

Butterfly network



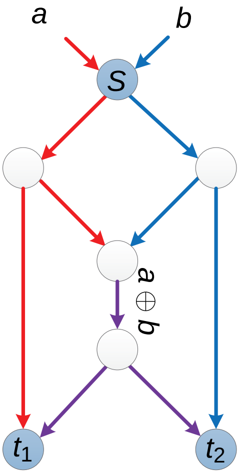
What is Network Coding?

Using simple routing:



What is Network Coding?

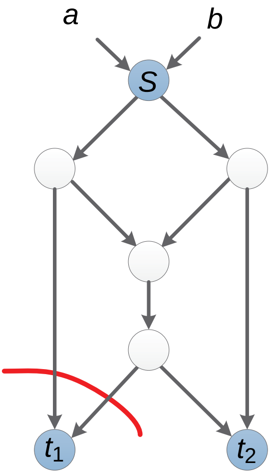
Using Network Coding:



Max-Flow Min-Cut Theorem

The maximum value of a flow is equal to the minimum cut

(R.Ahlswede *et al.* 2000)

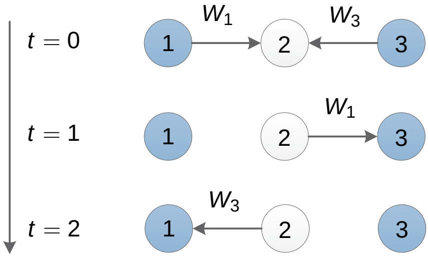


Motivation

How to minimize the number of transmissions?

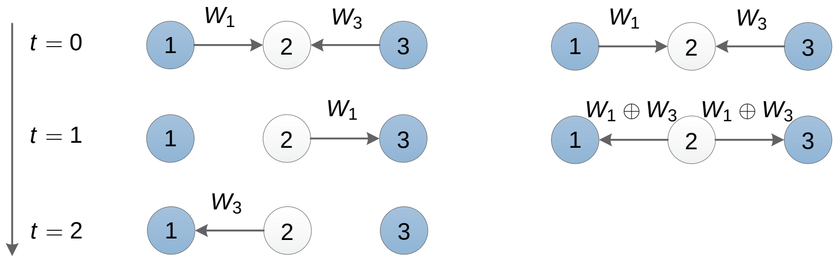
Motivation

How to minimize the number of transmissions?



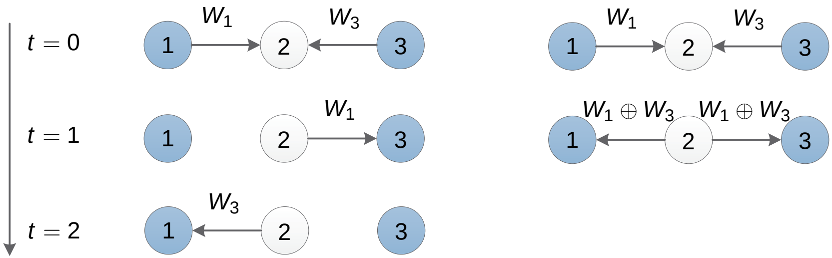
Motivation

How to minimize the number of transmissions?



Motivation

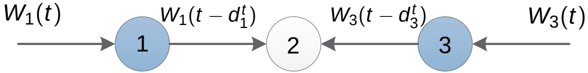
How to minimize the number of transmissions?



- Decoding: $(W_1 \oplus W_3) \oplus W_3 = W_1$

Motivation - Random Delay

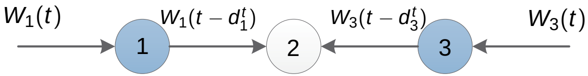
Challenge: Random delay



- At time t: Node 2 has $W_1(t-d_1^t)$ and $W_3(t-d_3^t)$, where $d_1^t, d_3^t \sim unif(1,2,...,D)$

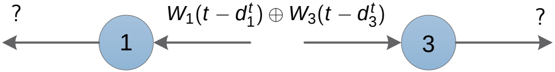
Motivation - Random Delay

Challenge: Random delay



- At time t: Node 2 has $W_1(t-d_1^t)$ and $W_3(t-d_3^t)$, where $d_1^t, d_3^t \sim unif(1, 2, ..., D)$

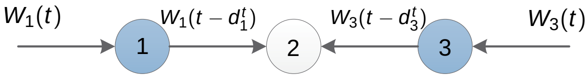
Problem: How to decode?



- With which message to XOR with?

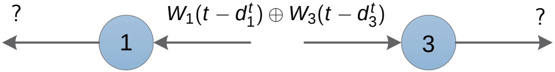
Motivation - Random Delay

Challenge: Random delay



- At time t: Node 2 has $W_1(t-d_1^t)$ and $W_3(t-d_3^t)$, where $d_1^t, d_3^t \sim unif(1, 2, ..., D)$

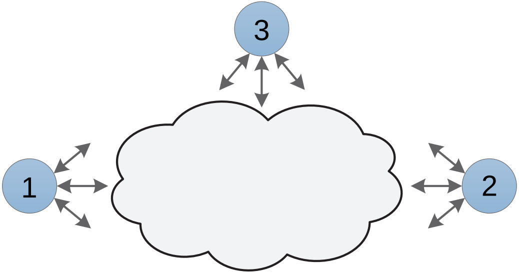
Problem: How to decode?



- With which message to XOR with?
- Which message is decoded?

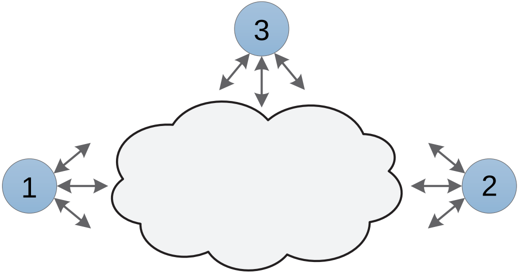
Goal

Find a coding scheme for a bidirectional graph with random delay



Goal

Find a coding scheme for a bidirectional graph with random delay

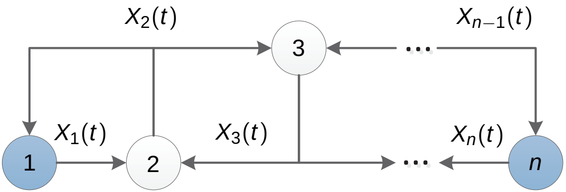


- Approach: Use building blocks

Building Block 1 - Line Topology

One generalization of the first example is a line topology

(Y.Wu *et al.* 2004)



Building Block 1 - Line Topology

Coding scheme

- Assuming $d_1^t, d_n^t = 1, \forall t$

Building Block 1 - Line Topology

Coding scheme

- Assuming $d_1^t, d_n^t = 1, \forall t$
- Node r transmits:

$$X_r(t) = W_1(t-(r-1)) + W_n(t-(n-r))$$

Building Block 1 - Line Topology

Coding scheme

- Assuming $d_1^t, d_n^t = 1, \forall t$
- Node r transmits:

$$X_r(t) = W_1(t-(r-1)) + W_n(t-(n-r))$$

$$X_1(t) = W_1(t) + W_n(t-(n-1))$$

Building Block 1 - Line Topology

Coding scheme

- Assuming $d_1^t, d_n^t = 1, \forall t$
- Node r transmits:

$$X_r(t) = W_1(t-(r-1)) + W_n(t-(n-r))$$

$$X_1(t) = W_1(t) + W_n(t-(n-1))$$

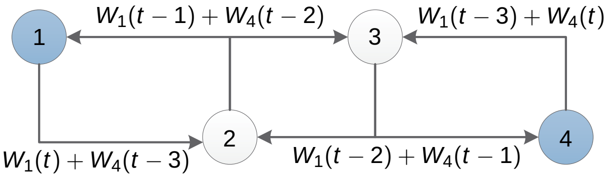
- Generated by

$$X_r(t) = X_{r+1}(t-1) + X_{r-1}(t-1) + X_r(t-2)$$

Building Block 1 - Line Topology

Example

- A line topology with four nodes



Building Block 1 - Line Topology

A generalization to random delay

$$X_r(t) = W_1(p) + W_n(q)$$

Building Block 1 - Line Topology

Summary

- A coding scheme for a line topology with rate of 1

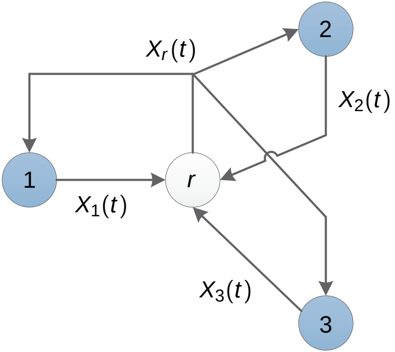
Building Block 1 - Line Topology

Summary

- A coding scheme for a line topology with rate of 1
- A first building block for a general topology

Building Block 2 - Star Topology

A star topology with three source nodes



Building Block 2 - Star Topology

Coding Scheme

- Node r transmits:

$$X_r(t) = a_1 W_1(t-1) + a_2 W_2(t-1) + a_3 W_3(t-1)$$

Building Block 2 - Star Topology

Coding Scheme

- Node r transmits:

$$X_r(t) = a_1W_1(t-1) + a_2W_2(t-1) + a_3W_3(t-1)$$
$$X_r(t+1) = b_1W_1(t-1) + b_2W_2(t-1) + b_3W_3(t-1)$$

Building Block 2 - Star Topology

Coding Scheme

- Node r transmits:

$$X_r(t) = a_1W_1(t-1) + a_2W_2(t-1) + a_3W_3(t-1)$$
$$X_r(t+1) = b_1W_1(t-1) + b_2W_2(t-1) + b_3W_3(t-1)$$

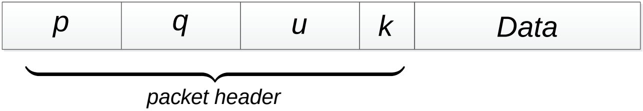
- Generated by

$$X_r(t+2) = X_1(t) + X_2(t) + X_3(t)$$

Building Block 2 - Star Topology

A generalization to random delay

$$X_r(t) = k_1 W_1(p) + k_2 W_2(q) + k_3 W_3(u).$$



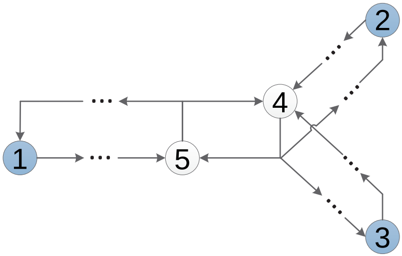
Building Block 2 - Star Topology

Summary

- A coding scheme for a star topology with rate of 0.5

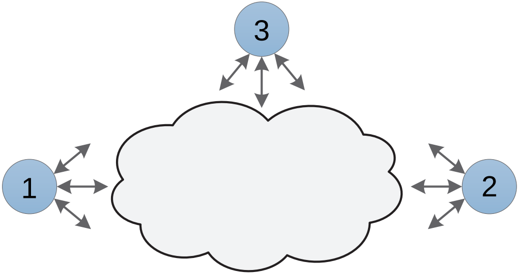
Summary

- A coding scheme for a star topology with rate of 0.5
- A combination is called a line-star topology



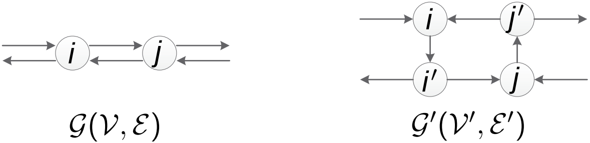
General Topology

- We define a bidirectional graph $\mathcal{G}(\mathcal{V}, \mathcal{E})$



Wired Model

- We construct an equivalent graph $\mathcal{G}'(\mathcal{V}', \mathcal{E}')$

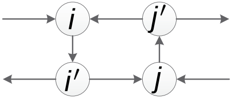


General Topology

Wired Model

- We construct an equivalent graph $\mathcal{G}'(\mathcal{V}',\mathcal{E}')$

$\mathcal{G}(\mathcal{V},\mathcal{E})$



$\mathcal{G}'(\mathcal{V}',\mathcal{E}')$

minimum transmissions

maximum rate

General Topology

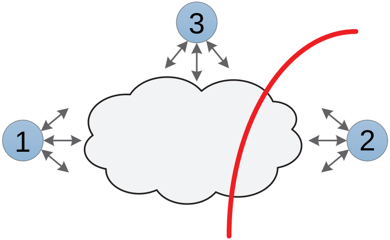
Upper Bound

- $\mathcal{C}_{i,j}$ is the value of the cut-set bound between i and j

General Topology

Upper Bound

- $\mathcal{C}_{i,j}$ is the value of the cut-set bound between i and j
- $h = \min_{i \in \mathcal{S}} \mathcal{C}_{i,\mathcal{S} \setminus \{i\}}$



General Topology

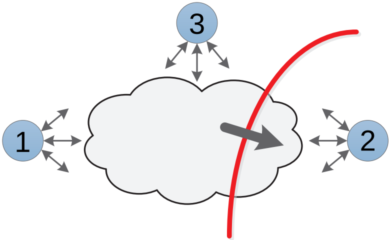
- R_i is the transmission rate of node i

General Topology

- R_i is the transmission rate of node i
- Assumption: all rates are equal ($R_i = R, \forall i \in \mathcal{S}$)

General Topology

- R_i is the transmission rate of node i
- Assumption: all rates are equal ($R_i = R, \forall i \in \mathcal{S}$)
- Upper bound: $R \leq \frac{h}{2}$



General Topology

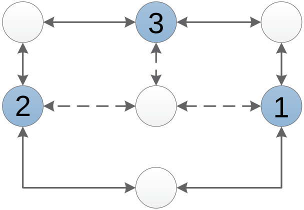
Coding Scheme

- Partition each network $\mathcal{G}'$ into line and star networks

General Topology

Coding Scheme

- Partition each network $\mathcal{G}'$ into line and star networks
- $\mathcal{R}$ represents the line topologies
- $\mathcal{Q}$ represents the star topologies



General Topology

Lemma

There exist $\mathcal{R}$ and $\mathcal{Q}$ such that $|\mathcal{R}| + \frac{|\mathcal{Q}|}{2} \geq \frac{h}{2}$

(N.Voskoboynik *et al.* 2014)

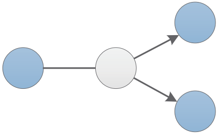
General Topology

Lemma

There exist $\mathcal{R}$ and $\mathcal{Q}$ such that $|\mathcal{R}| + \frac{|\mathcal{Q}|}{2} \geq \frac{h}{2}$

(N.Voskoboynik *et al.* 2014)

- Element in $\mathcal{Q}$:



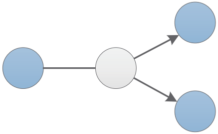
General Topology

Lemma

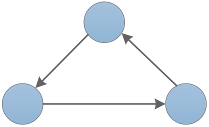
There exist $\mathcal{R}$ and $\mathcal{Q}$ such that $|\mathcal{R}| + \frac{|\mathcal{Q}|}{2} \geq \frac{h}{2}$

(N.Voskoboynik *et al.* 2014)

- Element in $\mathcal{Q}$:



- Element in $\mathcal{R}$:



General Topology

Construct the coding scheme

- Find $\mathcal{R}$

General Topology

Construct the coding scheme

- Find $\mathcal{R}$
- At each line use the coding scheme from building block 1

General Topology

Construct the coding scheme

- Find $\mathcal{R}$
- At each line use the coding scheme from building block 1
- Find $\mathcal{Q}$

General Topology

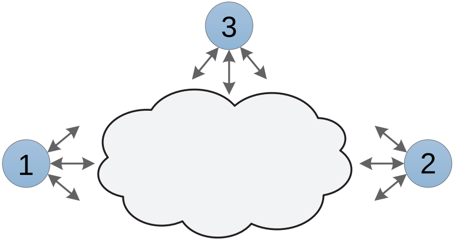
Construct the coding scheme

- Find $\mathcal{R}$
- At each line use the coding scheme from building block 1
- Find $\mathcal{Q}$
- At each star use the coding scheme from building block 2

General Topology

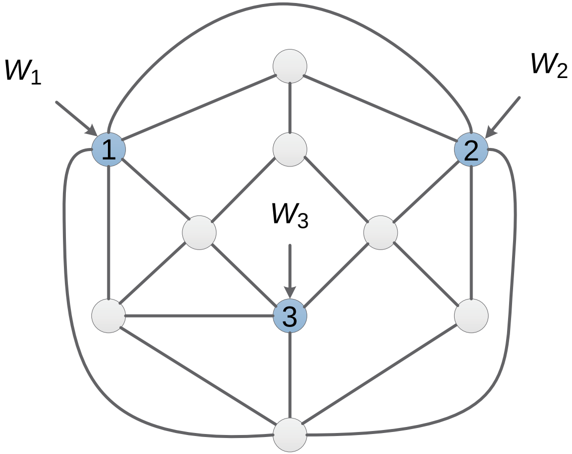
Summary

- Using line and star topologies, we construct a coding scheme for a general topology



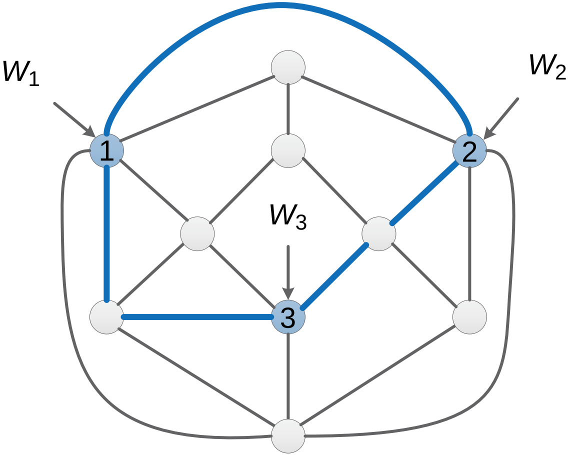
Example 1

$h = 4$ and therefore, the upper bound is 2



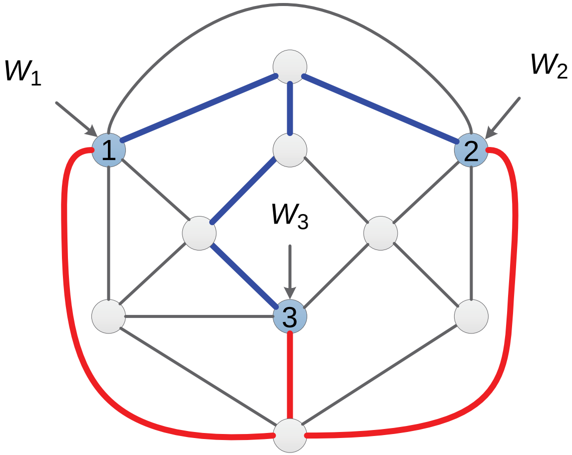
Example 1

The set $\mathcal{R}$ includes one ring, i.e., $|\mathcal{R}| = 1$



Example 1

The set $\mathcal{Q}$ includes two star networks, i.e., $|\mathcal{Q}| = 2$



Example 2

Maximum values of $|\mathcal{R}|$ and $|\mathcal{Q}|$ in a graph $\mathcal{G}(\mathcal{V}, \mathcal{E})$

Maximum value	General graph	Complete graph
$\|\mathcal{R}\|$	$\lfloor \frac{\|\mathcal{V}\|}{3} \rfloor$	1
$\|\mathcal{Q}\|$	$\|\mathcal{V}\| - 2$	$\|\mathcal{V}\| - 3$

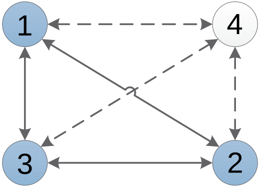


Figure: Complete graph

Summary

- Coding scheme for practical NC is presented

Summary

- Coding scheme for practical NC is presented
- The coding scheme obviates the need for synchronicity

Summary

- Coding scheme for practical NC is presented
- The coding scheme obviates the need for synchronicity
- Equal rate upper bound is achieved

Summary

- Coding scheme for practical NC is presented
- The coding scheme obviates the need for synchronicity
- Equal rate upper bound is achieved

Thank You!