

Homework Set #4
Gaussian MAC and Compound Channel

1. **Converse for the Gaussian multiple access channel.** Prove the converse for the Gaussian multiple access channel by extending the converse in the discrete case to take into account the power constraint on the codewords.

2. **A multiple access identity.**

Let $C(x) = \frac{1}{2} \log(1 + x)$ denote the channel capacity of a Gaussian channel with signal to noise ratio x . Show

$$C\left(\frac{P_1}{N}\right) + C\left(\frac{P_2}{P_1 + N}\right) = C\left(\frac{P_1 + P_2}{N}\right).$$

This suggests that 2 independent users can send information as well as if they had pooled their power.

3. **Gaussian multiple access.**

A group of m users, each with power P , is using a Gaussian multiple access channel at capacity, so that

$$\sum_{i=1}^m R_i = C\left(\frac{mP}{N}\right), \quad (1)$$

where $C(x) = \frac{1}{2} \log(1 + x)$ and N is the receiver noise power.

A new user of power P_0 wishes to join in.

- (a) At what rate can he send without disturbing the other users?
 - (b) What should his power P_0 be so that the new users rate is equal to the combined communication rate $C(mP/N)$ of all the other users?
4. **Frequency Division Multiple Access (FDMA).** Maximize the throughput $R_1 + R_2 = W_1 \log(1 + \frac{P_1}{NW_1}) + (W - W_1) \log(1 + \frac{P_2}{N(W - W_1)})$ over W_1 to show that bandwidth should be proportional to transmitted power for FDMA.

5. **Compound channel with feedback.** In the class we introduced the memoryless compound channel $(\mathcal{X}; p(y|x, s); \mathcal{Y})$ where $s \in \mathcal{S}$ is the state of the channel. Throughout this question we assume that the alphabets $\mathcal{X}, \mathcal{Y}, \mathcal{S}$ are all finite. A $(2^{nR}, n)$ code for the compound channel is defined in the same way as for the DMC (see lecture notes). The average probability of error is defined as

$$P_e^{(n)} = \sup_s P \left\{ \hat{W} \neq W, s \text{ is the actual channel} \right\}$$

A rate R is achievable if there exists a sequence of $(2^{nR}, n)$ codes with $P_e^{(n)} \rightarrow 0$.

- (a) What is the capacity of the discrete compound channel with feedback? Prove converse and achievability.
- (b) Compute the capacity of the compound binary eraser channel with feedback where the probability of an eraser is one of the four values $(0, 0.1, 0.2, 0.25)$.
- (c) Write an expression and then sort from lower to higher the
 - (i) Capacity of compound channel with feedback when the state is not known.
 - (ii) Capacity of compound channel with no feedback when the state is not known.
 - (iii) Capacity of compound channel with feedback when the state known only at the encoder.
 - (iv) Capacity of compound channel with no feedback when the state known at the encoder.
- (d) If the probability distribution that achieves the capacity of each channel is the same, does it imply that the capacity with feedback and without feedback are equal? If it does, prove it and if it does not give a counter example.
- (e) If the capacity of the compound channel without feedback is zero, does it imply that the capacity with feedback is also zero? If it does, prove it and if it does not give a counter example.
- (f) Under what conditions the capacity of the compound channel with feedback and without feedback has the same capacity.