## Homework set on Tree Distribution

1. Tree Distribution: In the following items you should write whether the statement is true or false, and explain your answer.
(a) True/False: In our Tree Distribution lecture, we concluded that the criteria for an optimal tree is

$$
\max _{\text {All Trees }} \sum_{i=1}^{n} I\left(x_{i}, x_{j(i)}\right)
$$

where $x_{i}$ is the $i_{t h}$ feature, $x_{j(i)}$ is the parent of the $i_{t h}$ feature, and $I$ is the mutual information between the two features. This criteria is equivalent to the Maximum-Likelihood criteria.
(b) True/False: There can be more than two 'sons' per node in a Tree Distribution.
2. Tree Distribution: You wish to generate a model to predict if a mushroom is poisonous or not. You have some empirical data:

| Example | Is heavy | Is smelly | Is spotted | Is smooth | Is poisonous |
| :--- | :--- | :--- | :--- | :--- | :--- |
| A | 0 | 0 | 0 | 0 | 0 |
| B | 0 | 0 | 1 | 0 | 0 |
| C | 1 | 1 | 0 | 1 | 0 |
| D | 1 | 0 | 0 | 1 | 1 |
| E | 0 | 1 | 1 | 0 | 1 |
| F | 0 | 0 | 1 | 1 | 1 |
| G | 0 | 0 | 0 | 1 | 1 |
| H | 1 | 1 | 0 | 0 | 1 |

(a) Calculate the empirical mutual information between all couples of features (including Is poisonous).
(b) Build tree distribution for the data according to the maximum-likelihood criteria. You have a constraint that the node of 'Is poisonous' must be the main root (head) of the tree.
(c) Use the tree you built to determine by the maximum-likelihood criteria whether $\mathrm{U}, \mathrm{V}, \mathrm{W}$ are poisonous or not. If it happens to be that there is a tie, you define it as poisonous.

| Example | Is heavy | Is smelly | Is spotted | Is smooth | Is poisonous |
| :--- | :--- | :--- | :--- | :--- | :--- |
| U | 0 | 1 | 1 | 1 | $?$ |
| V | 0 | 1 | 0 | 1 | $?$ |
| W | 1 | 1 | 0 | 0 | $?$ |

