Homework: Convolutional Codes, Viterbi Algorithm, and BCJR Algorithm

Consider the rate-1/2 convolutional code with generator polynomials

$$g_1(D) = 1, \qquad g_2(D) = 1 + D.$$

Assume that the encoder starts in the all-zero state.

Question 1: Convolutional Encoding

(a) Draw the encoder structure.

(b) Determine the memory of the encoder and the number of states.

(c) Construct the complete state transition table.

(d) Encode the information sequence

$$u = (1, 0, 1, 1).$$

(e) Draw the corresponding path on the trellis.

Question 2: Trellis Representation

(a) Draw the trellis diagram of the code for four time steps.

(b) Label each branch by

input/output.

(c) Explain the meaning of a state in this trellis.

(d) Explain why the trellis contains only two states.

Question 3: Hard-Decision Viterbi Decoding

Suppose the received hard-decision sequence is

$$r = (11, 01, 11, 10).$$

(a) Compute the Hamming-distance branch metrics.

(b) Perform the Viterbi algorithm step by step.

(c) Determine the survivor path at each stage.

(d) Find the decoded information sequence.

(e) Compare the decoded sequence with the transmitted sequence from Question 1.

Question 4: Soft-Decision Viterbi Decoding

Assume BPSK modulation

$$0 \to +1, \qquad 1 \to -1.$$

The received samples are

$$y = (-0.8, -1.1,\ 1.2, -0.9,\ -0.7, -0.6,\ -1.1, 0.8).$$

The corresponding hard-decision sequence is

$$r = (11, 01, 11, 10).$$

(a) Verify that applying hard decisions to the received samples produces

$$r = (11, 01, 11, 10).$$

(b) Compute the Euclidean branch metric for every branch in the first trellis section.

(c) Perform soft-decision Viterbi decoding.

(d) Find the decoded information sequence.

(e) Compare the result with the hard-decision Viterbi decoder of Question 3.

(f) Explain the difference between hard-decision and soft-decision Viterbi decoding.

(g) Why does soft-decision decoding usually provide better performance?

Question 5: BCJR Forward and Backward Metrics

(a) Define the forward metric

$$\alpha_t(s).$$

(b) Define the backward metric

$$\beta_t(s).$$

(c) Explain the probabilistic meaning of each metric.

(d) Write the forward recursion.

(e) Write the backward recursion.

Question 6: APP L-values

The APP L-value is defined as

$$L(u_t) = \log \frac{P(u_t = 1 | y^N)}{P(u_t = 0 | y^N)}.$$

(a) Derive the BCJR expression for $L(u_t)$.

(b) Explain the role of the branch metric

$$\gamma_t(s', s).$$

(c) Explain how a hard decision is obtained from the APP L-value.

(d) What does it mean when

$$L(u_t) = 0?$$

(e) What does it mean when

$$|L(u_t)|$$

is very large?

Question 7: Viterbi versus BCJR

(a) What quantity is optimized by the Viterbi algorithm?

(b) What quantity is computed by the BCJR algorithm?

(c) Explain the difference between sequence decoding and symbol-by-symbol decoding.

(d) Which algorithm produces soft outputs?

(e) Why is BCJR preferred in Turbo decoding?