

Fourier-Motzkin Elimination software for Information Theory User Guide

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July 22, 2015

<http://www.ee.bgu.ac.il/~fmeit/>

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1 Introduction

Fourier-Motzkin elimination (FME) is often used on information theoretic applications in order to eliminate meaningless variables, such as auxiliary rates. This procedure can be done by this software, which provides a basic Graphic User Interface (GUI) for that cause. This *user-guide* provides basic knowledge about the usage of the software. Focusing on information theoretic applications, the output system is reduced by removing in-active constraints, considering those that are implied by *basic information inequalities* as explained in [1, Chapter 14]. Some inequalities are implied to be in-active neither by basic information inequalities nor by other constraints in the system, therefore, those inequalities won't be removed.

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3 Installation

1. Download the latest release from <http://www.ee.bgu.ac.il/~fmeit/>.
2. Unpack the .zip file to you MATLAB working directory.
3. Set default setting by executing `sti_setup` in your MATLAB console.

3.1 Graphic user interface

A graphic user interface (GUI) is provided together with the FME-IT. Type `fme_GUI` in MATLAB console to open the GUI.

3.2 Optimization solver

The FME-IT package supports the following solvers:

- **MATLAB** - MATLAB's `linprog` optimization tool is included with MATLAB student suite.
- **GUROBI** - Gurobi Optimizer includes a fast and very accurate linear programming solver. Gurobi offers a free academic license: <http://www.gurobi.com/academia/for-universities>.
- **CVX** - CVX optimization tool is offered with a free or professional license: <http://cvxr.com/>. The CVX supports several solvers. By setting FME-IT to use CVX, the CVX preferred solver will be selected.

To change your preferred solver, execute `sti_setup -solver`.

4 Basic Usage

4.1 Graphic User Interface

First, run the FME-IT software with GUI, by executing `fme_GUI`.

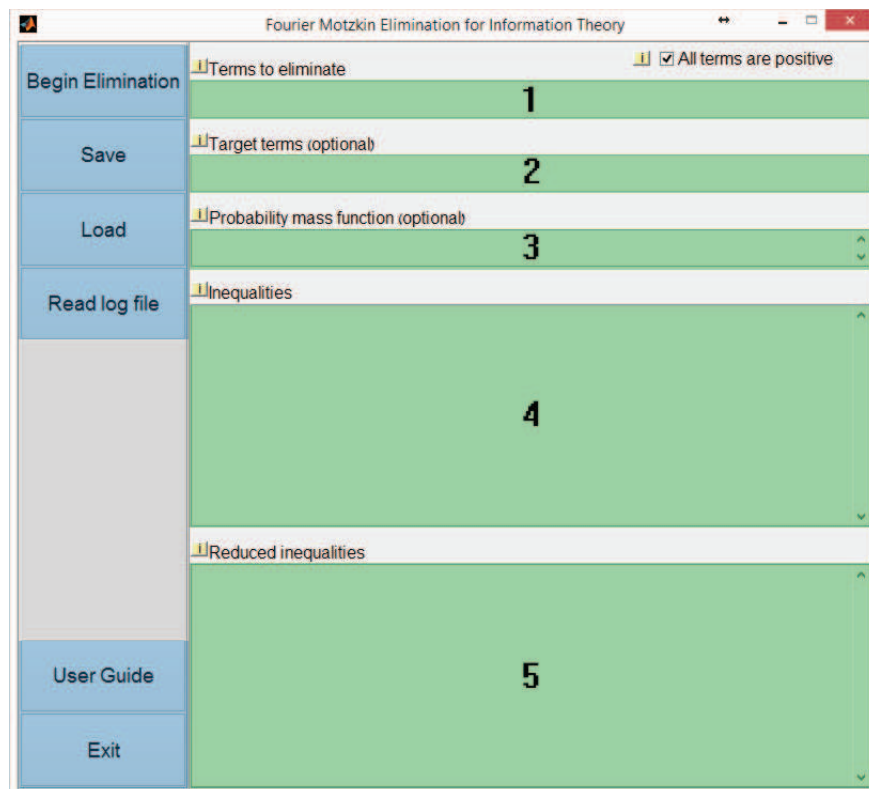


Figure 1: FME-IT User interface

Once done, a graphics user interface (GUI) will open as in Figure 1.
For instant help about syntax for the various input fields, press the  button that appears before each field.
The inputs are divided as follows:

Text boxes

1. **Elimination terms** - terms in the inequalities to be eliminated by the FME procedure.
2. **Target terms** - these terms will be on the left hand side in the output inequalities (optional).
3. **Probability mass function** - joint probability structure of the random variables (optional).
4. **Inequalities** - initial inequalities system.
5. **Reduced inequalities** - output text box for the reduced system of inequalities.

Note: The random variables are NOT case sensitive.

Buttons

- **Begin Elimination** - starts eliminating terms and reduces redundant inequalities.
- **Save** - saves current session in a `.mat` file.
- **Load** - loads a session from a `.mat` file.
- **Read log file** - opens elimination history.
- **User guide** - opens the attached user guide (this document).
- **Exit** - closes current session.

Elimination terms and Target terms are to be separated by commas, e.g., `R1,R2,R3`. Inequalities of the following forms are recognized:

- `EXP1 >= EXP2` (or `=>`)
- `EXP1 > EXP2`
- `EXP1 <= EXP2` (or `=<`)
- `EXP1 < EXP2`

Probability mass function input is of the following form:

- Multiplication is denoted by `*` or not denoted at all.
- Deterministic function is denoted by `1`.

- Probability function is denoted by P/p.
- The probability function imply Markov chains, deterministic function and independencies.
- For instance, the probability function $P(X)P(Y|X) * P(Z|Y)1(W|X, Z)$ is legit, and interpreted as follows:

$$P(X)P(Y|X) \underbrace{P(Z|Y)}_{I(Z;X|Y)=0} \underbrace{1(W|X, Z)}_{H(W|X,Z)=0}. \quad (1)$$

4.2 Function Usage

The FME-IT algorithm supports execution via MATLAB console.

```
result=FME_ALGORITHM(model)
```

FME-IT algorithm which performs Fourier-Motzkin elimination and remove redundant constraints from the final output system, according to the joint probability mass function.

This function returns the reduced system in matrix form with terms dictionary and the reduced system as inequalities in cell array.

Arguments:

model.equations	- Cell array with inequalities.
model.eliminations	- Cell array with terms to eliminate.
model.target	- Cell array with target terms.
model.omega	- Cell array with all random variables (optional).
model.pmf	- Joint probability mass function (optional).
model.allpositive	- (1) all terms are positive (0) otherwise.
model.dispconsole	- (1) display process on console (0) otherwise.
model.ShowWaitBar	- (1) show waitbar (0) otherwise.
model.monitor	- (1) save log.txt (0) otherwise.

Outputs:

result.status	- Status: OK/ERROR/WARNING
result.ineq_str	- Strings cell array of the reduced system.
result.ineq_mat	- Matrix of coefficient of the reduced system.
result.rvs	- List of random variables.
result.pmf	- Probability mass function.
result.dictionary	- List of all terms.
result.allpositive	- (1) all terms are positive (0) otherwise.
result.error	- Error description (if exists).

An example code for calling the algorithm as a function:

```

model.eliminations={'R10'};
model.target={'R0','R1'};
model.equations={'R0+R10<I(U;Y2)', ...
    'R1-R10<I(X;Y1|U)', ...
    'R0+R1<I(X;Y1)',...
    'R10<R1'};
model.allpositive=1;
reduced=fme_algorithm(model);

```

The algorithm will produce the following output:

```

Running FME algorithm for IT...
Processing input...
FME elimination begins...
Eliminating redundant inequalities...
Preparing results...

Reduced system of inequalities:
-----
R0<I(U;Y2)
R0+R1<I(U;Y2)+I(X;Y1|U)
R0+R1<I(X;Y1)

```

The reduced system is stored in the structure `reduced`.

5 Example

5.1 DM-BC with Degraded Message Sets

We begin with a simple example on the capacity region of a discrete memoryless broadcast channel with degraded message set [2, Theorem 8.1]. You can either open the prepared example from your extracted files, by hitting the **Load** button and opening `GUIExamples\BCwDMS.mat`, or typing the inputs given in this example.

Consider the following region:

$$R_0 + R_{10} < I(U; Y_2) \quad (2a)$$

$$R_1 - R_{10} < I(X; Y_1|U) \quad (2b)$$

$$R_0 + R_1 < I(X; Y_1) \quad (2c)$$

$$R_{10} < R_1 \quad (2d)$$

$$(2e)$$

together with the non-negativity of the rates R_0, R_1, R_{10} .

We enter the inequalities system into the **Inequalities** text-box.

We select R_{10} to be eliminated by typing it in **Terms to eliminate** text-box, and choose R_0, R_1 to be

isolated in the output system by entering them into the **Target terms** text-box.

Whenever no *target terms* are provided to the software, the terms that are not Shannon's information measures will be isolated by the software. The non-negativity of the rates is considered by checking the box **All terms are positive**.

Begin Elimination Save Load Read log file User Guide Exit	Terms to eliminate ☒ All terms are positive R10
	Target terms (optional) R0,R1
	Probability mass function (optional)
	Inequalities R0+R10<I(U,Y2) R1-R10<I(X,Y1 U) R0+R1<I(X,Y1) R10<R1
	Reduced inequalities

Figure 2: Example - DM-BC with DMS input region.

After all inputs are given, as depicted in Figure 2, we hit the **Begin Elimination** button.

The software will eliminate the variables R_{10} and reduce the output region by removing in-active constraints.

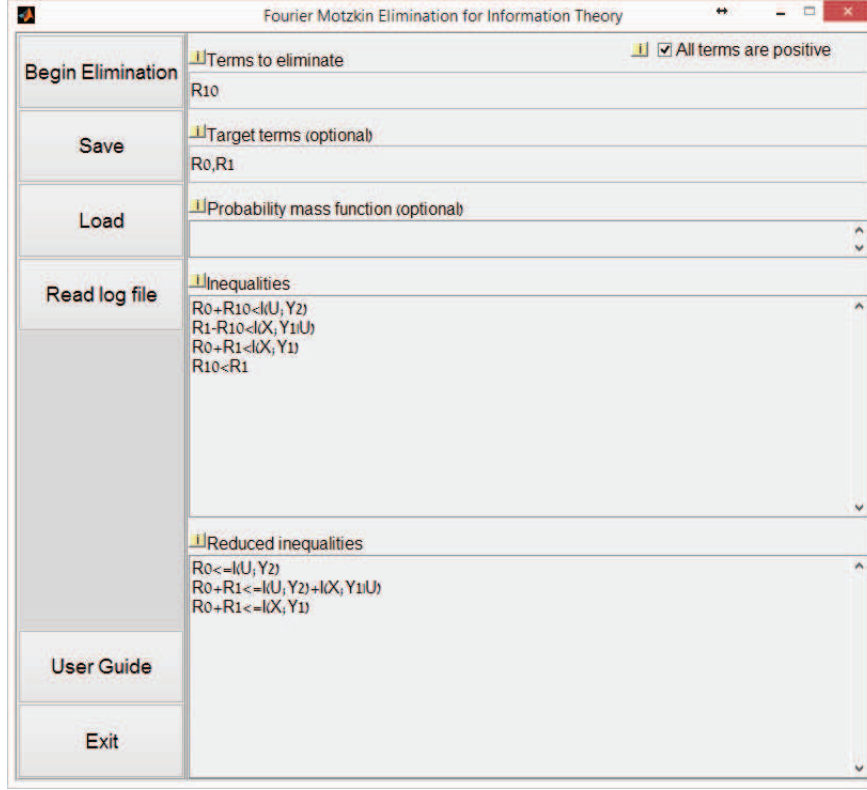


Figure 3: Example - DM-BC with DMS output.

When procedure is done, the GUI should be the same as in Figure 3.
The output system after elimination is

$$R_0 < I(U; Y_2) \quad (3a)$$

$$R_0 + R_1 < I(U; Y_2) + I(X; Y_1|U) \quad (3b)$$

$$R_0 + R_1 < I(X; Y_1) \quad (3c)$$

that is equivalent to the alternative representation of the capacity region given in [2, Theorem 8.1].

5.2 Han-Kobayashi Inner Bound

We will demonstrate how to use the software on the *Han-Kobayashi inner bound* [3], on the form given in [2, Theorem 6.4].

You can either open the prepared example from your extracted files, by hitting the Load button and opening GUIExamples\Han-Kobayashi.mat, or typing the inputs given in this example.

To begin with, consider the following region [2, Chapter 6.5.1]:

$$R_{11} < I(X_1; Y_1 | U_1, U_2, Q) \quad (4a)$$

$$R_1 < I(X_1; Y_1 | U_2, Q) \quad (4b)$$

$$R_{11} + R_{20} < I(X_1, U_2; Y_1 | U_1, Q) \quad (4c)$$

$$R_1 + R_{20} < I(X_1, U_2; Y_1 | Q) \quad (4d)$$

$$R_{22} < I(X_2; Y_2 | U_1, U_2, Q) \quad (4e)$$

$$R_2 < I(X_2; Y_2 | U_1, Q) \quad (4f)$$

$$R_{22} + R_{10} < I(X_2, U_1; Y_2 | U_2, Q) \quad (4g)$$

$$R_2 + R_{10} < I(X_2, U_1; Y_2 | Q), \quad (4h)$$

together with the identities

$$R_1 = R_{10} + R_{11} \quad (4i)$$

$$R_2 = R_{20} + R_{22}, \quad (4j)$$

and the non-negativity constraints on the rates:

$$R_1 > 0 \quad (4k)$$

$$R_{11} > 0 \quad (4l)$$

$$R_{10} > 0 \quad (4m)$$

$$R_2 > 0 \quad (4n)$$

$$R_{22} > 0 \quad (4o)$$

$$R_{20} > 0. \quad (4p)$$

We intend to eliminate all rates except R_1, R_2 , and give constraints with respect to those rates. Identities are dealt as follows. We use each identity by isolating one variable (rate) that we wish to eliminate, and replace it in the inequalities, according to its identity. For instance, we isolate R_{11} and R_{22} using the identities:

$$R_{11} = R_1 - R_{10} \quad (5a)$$

$$R_{22} = R_2 - R_{20} \quad (5b)$$

and substitute R_{11} and R_{22} in inequalities (4a) and (4k) according to (5a). Notice that the substitution in the non-negativity inequalities yield two additional inequalities,

$$R_{10} < R_1 \quad (6a)$$

$$R_{20} < R_2. \quad (6b)$$

We derive with the following inequalities system:

$$R_1 - R_{10} < I(X_1; Y_1 | U_1, U_2, Q) \quad (7a)$$

$$R_1 < I(X_1; Y_1 | U_2, Q) \quad (7b)$$

$$R_1 - R_{10} + R_{20} < I(X_1, U_2; Y_1 | U_1, Q) \quad (7c)$$

$$R_1 + R_{20} < I(X_1, U_2; Y_1 | Q) \quad (7d)$$

$$R_2 - R_{20} < I(X_2; Y_2 | U_1, U_2, Q) \quad (7e)$$

$$R_2 < I(X_2; Y_2 | U_1, Q) \quad (7f)$$

$$R_2 - R_{20} + R_{10} < I(X_2, U_1; Y_2 | U_2, Q) \quad (7g)$$

$$R_2 + R_{10} < I(X_2, U_1; Y_2 | Q) \quad (7h)$$

$$R_{10} < R_1 \quad (7i)$$

$$R_{20} < R_2, \quad (7j)$$

together with the non-negativity constraints. We are left with two terms to eliminate - R_{10} and R_{20} , which is done by applying the Fourier-Motzkin elimination. We enter the inequalities by typing them into the **Inequalities** text-box in the FME-IT software:

```
R1-R10<I(X1;Y1|U1,U2,Q)
R1<I(X1;Y1|U2,Q)
R1-R10+R20<I(X1,U2;Y1|U1,Q)
R1+R20<I(X1,U2;Y1|Q)
R2-R20<I(X2;Y2|U1,U2,Q)
R2<I(X2;Y2|U1,Q)
R2-R20+R10<I(X2,U1;Y2|U2,Q)
R2+R10<I(X2,U1;Y2|Q)
R10<R1
R20<R2
```

We choose to eliminate R_{10} and R_{20} by typing them into **Terms to eliminate** text-box, separated by commas, that is, **R10, R20**.

Blanks and white spaces are discarded in the input.

The **Target terms** text-box is used to select which terms are to be isolated in the left hand side (LHS) of the output equations.

We choose R_1 and R_2 to be the target terms by typing **R1, R2**.

The non-negativity constraints on the rates are considered by checking **All terms positive** box in the upper-right corner. The probability mass function of the random variables is fed into the optional box **Probability mass function**.

Fourier Motzkin Elimination for Information Theory

Begin Elimination

Terms to eliminate ☒ All terms are positive
R10, R20

Save

Target terms (optional)
R1, R2

Load

Probability mass function (optional)
 $p(x_1, x_2 | p(u_1, x_1, p(u_2, x_2 | p(y_1, y_2 | x_1, x_2))$

Read log file

Inequalities

- $R_1 - R_{10} < I(X_1; Y_1 | U_1, U_2, Q)$
- $R_1 < I(X_1; Y_1 | U_2, Q)$
- $R_1 - R_{10} + R_{20} < I(X_1, U_2; Y_1 | U_1, Q)$
- $R_1 + R_{20} < I(X_1, U_2; Y_1 | Q)$
- $R_2 - R_{20} < I(X_2; Y_2 | U_1, U_2, Q)$
- $R_2 < I(X_2; Y_2 | U_1, Q)$
- $R_2 - R_{20} + R_{10} < I(X_2, U_1; Y_2 | U_2, Q)$
- $R_2 + R_{10} < I(X_2, U_1; Y_2 | Q)$
- $R_{10} < R_1$
- $R_{20} < R_2$

Reduced inequalities

User Guide

Exit

Figure 4: Example - Han-Kobayashi inner bound input region.

When done, your GUI should be the same as depicted in Figure 4.

To begin elimination, hit the **Begin Elimination** button.

Fourier Motzkin Elimination for Information Theory

Begin Elimination ☐ Terms to eliminate ☒ All terms are positive

R10,R20

Save ☐ Target terms (optional)

R1,R2

Load ☐ Probability mass function (optional)

p(qp(u1,x1)qp(u2,x2)qp(y1,y2|x1,x2))

Read log file ☐ Inequalities

R1-R10<I(X1,Y1|U1,U2,Q)
R1<I(X1,Y1|U2,Q)
R1-R10+R20<I(X1,U2,Y1|U1,Q)
R1+R20<I(X1,U2,Y1|Q)
R2-R20<I(X2,Y2|U1,U2,Q)
R2<I(X2,Y2|U1,Q)
R2-R20+R10<I(X2,U1,Y2|U2,Q)
R2+R10<I(X2,U1,Y2|Q)
R10<R1
R20<R2

☐ Reduced inequalities

R1<=I(X1,Y1|U1,U2,Q)+I(X2,U1,Y2|U2,Q)
R2<=I(X1,U2,Y1|U1,Q)+I(X2,Y2|U1,U2,Q)
R1<=I(X1,Y1|U2,Q)
R2<=I(X2,Y2|U1,Q)
R1+R2<=I(X1,Y1|U1,U2,Q)+I(X2,U1,Y2|Q)
R1+R2<=I(X1,U2,Y1|U1,Q)+I(X2,U1,Y2|U2,Q)
R1+R2<=I(X1,U2,Y1|Q)+I(X2,Y2|U1,U2,Q)
2R1+R2<=I(X1,Y1|U1,U2,Q)+I(X1,U2,Y1|Q)+I(X2,U1,Y2|U2,Q)
R1+2R2<=I(X1,U2,Y1|U1,Q)+I(X2,Y2|U1,U2,Q)+I(X2,U1,Y2|Q)

User Guide

Exit

Figure 5: Example - Han-Kobayashi inner bound output.

The reduced system of inequalities will appear in the **Reduced system** text-box, as depicted in Figure 5, which is:

$$R_2 \leq I(X_1, U_2; Y_1 | U_1, Q) + I(X_2; Y_2 | U_1, U_2, Q) \quad (8a)$$

$$R_2 \leq I(X_2; Y_2 | U_1, Q) \quad (8b)$$

$$R_1 \leq I(X_1; Y_1 | U_1, U_2, Q) + I(X_2, U_1; Y_2 | U_2, Q) \quad (8c)$$

$$R_1 \leq I(X_1; Y_1 | U_2, Q) \quad (8d)$$

$$2R_1 + R_2 \leq I(X_1; Y_1 | U_1, U_2, Q) + I(X_1, U_2; Y_1 | Q) + I(X_2, U_1; Y_2 | U_2, Q) \quad (8e)$$

$$R_1 + R_2 \leq I(X_1; Y_1 | U_1, U_2, Q) + I(X_2, U_1; Y_2 | Q) \quad (8f)$$

$$R_1 + R_2 \leq I(X_1, U_2; Y_1 | U_1, Q) + I(X_2, U_1; Y_2 | U_2, Q) \quad (8g)$$

$$R_1 + R_2 \leq I(X_1, U_2; Y_1 | Q) + I(X_2; Y_2 | U_1, U_2, Q) \quad (8h)$$

$$R_1 + 2R_2 \leq I(X_1, U_2; Y_1 | U_1, Q) + I(X_2; Y_2 | U_1, U_2, Q) + I(X_2, U_1; Y_2 | Q). \quad (8i)$$

Note that the output system given in (8) is the outcome of the FME procedure together with removing redundant inequalities, as given in [1, Appedix D]. It can be shown that inequalities (8a) and (8c) are

redundant, but not due to basic information inequalities. Therefore, those inequalities can't be removed by this software.

5.3 More Examples

There are more examples given in `GUIExamples` library that is attached to the program. The library includes the following files:

- `BCTwoReverslyDegraded.mat` - *The Capacity Region of the product of two reversely degraded DM-BC* [2, Proposition 9.1].
- `ElGamalCoverInnerBound.mat` - *El Gamal-Cover Inner Bound* [2, Theroem 13.1].
- `BCwDMS.mat` - *Broadcast Channels with Degraded Message set* [2, Theorem 8.1].
- `BCw3RecieversMultilevel.mat` - *Three-Reciever Multilevel DM-BC* [2, Theorem 8.2].
- `HanKobayashi.mat` - *Han-Kobayashi inner bound* [3], on the form given in [2, Theorem 6.4].
- `LiangBCrelay.mat` - *Rate Regions for Relay Broadcast Channels* [4].
- `MartonInnerCommon.mat` - *Marton's Inner Bound with Common Message* [2, Theorem 8.4].
- `MulticodingIC.mat` - *Multicoding schemes for interference channels* [5].
- `SDBCwithCOOP.mat` - *Duality of a Source Coding Problem and the Semi-Deterministic Broadcast Channel with Rate-Limited Cooperation* [6].
- `StateDependentPartiallyCribbing.mat` - *Multiple Access Channels with Combined Cooperation and Partial Cribbing* [7].
- `BCwLeakageConstraints.mat` - *Broadcast Channels with Privacy Leakage Constraints* [8].
- `ZhengBergerInnerBound` - *Zhang-Berger Inner Bound* [2, Theorem 13.4]

These files can be open using the software, to demonstrate the algorithm's abilities.

6 Mathematical Background - Overview

The mathematical approach that was established in order to implement the FME-IT algorithm, combines Shannon-type inequalities, a type of inequalities that was studied by Raymond W. Yeung in [1], and identification of inactive constraints in a linear matrix constraints system [9]. The characterization of the Shannon-type inequalities led to a piece of software called ITIP [10] which is able to prove certain information inequalities in information theory. More work on the ITIP was done in [11]. Using the same characterization, we applied [9, Theorem 2.1] while restricting the region where information inequalities hold. In consequence, inequalities that are redundant due to *basic information inequalities* and constraints.

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