Broadcast Channels with Cooperation: Capacity and Duality for the Semi-Deterministic Case

Ziv Goldfeld, Haim H. Permuter and Gerhard Kramer

Ben Gurion University and Technische Universität München

IEEE Information Theory Wrokshop

April-May, 2015

Outline

- Channel-source duality for BCs
- Semi-deterministic BC with decoder cooperation
- Source coding dual
- Capacity results
- Summary

Duality - Preface

"There is a curious and provocative duality between the properties of a source with a distortion measure and those of a channel..."
(C. E. Shannon, 1959)

Duality - Preface

"There is a curious and provocative duality between the properties of a source with a distortion measure and those of a channel..."
(C. E. Shannon, 1959)

PTP Duality: [Shannon, 1959], [Cover and Chiang, 2002], [Pradhan *et al.*, 2003], [Gupta and Verdú, 2011].

Duality - Preface

"There is a curious and provocative duality between the properties of a source with a distortion measure and those of a channel..."
(C. E. Shannon, 1959)

PTP Duality: [Shannon, 1959], [Cover and Chiang, 2002], [Pradhan *et al.*, 2003], [Gupta and Verdú, 2011].

- The solutions are dual - Information measures coincide.

Duality - Preface

"There is a curious and provocative duality between the properties of a source with a distortion measure and those of a channel..."
(C. E. Shannon, 1959)

PTP Duality: [Shannon, 1959], [Cover and Chiang, 2002], [Pradhan *et al.*, 2003], [Gupta and Verdú, 2011].

- The solutions are dual - Information measures coincide.
- A formal proof of duality is still absent.

Duality - Preface

"There is a curious and provocative duality between the properties of a source with a distortion measure and those of a channel..."
(C. E. Shannon, 1959)

PTP Duality: [Shannon, 1959], [Cover and Chiang, 2002], [Pradhan *et al.*, 2003], [Gupta and Verdú, 2011].

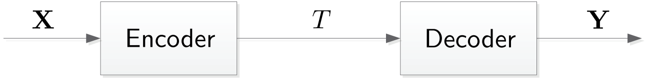
- The solutions are dual - Information measures coincide.
- A formal proof of duality is still absent.
- Solving one problem $\Longrightarrow$ Valuable insight into solving dual.

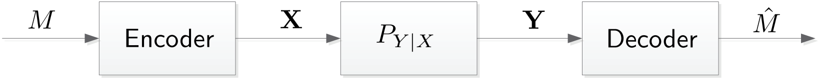
Duality - Preface

Point-to-Point Case:

Duality - Preface

Point-to-Point Case:

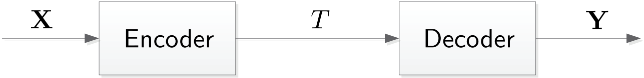


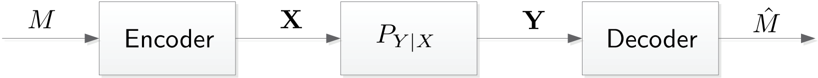


Duality - Preface

Point-to-Point Case:

Empirical Coordination: $(\mathbf{X}, \mathbf{Y}) \in \mathcal{T}_\epsilon^{(n)}(P_X P^\star_{Y|X})$

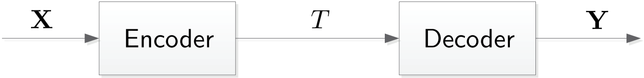


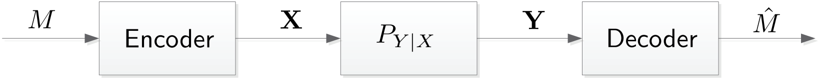


Duality - Preface

Point-to-Point Case:

Empirical Coordination: $(\mathbf{X}, \mathbf{Y}) \in \mathcal{T}_{\epsilon}^{(n)}(P_X P^{\star}_{Y|X})$

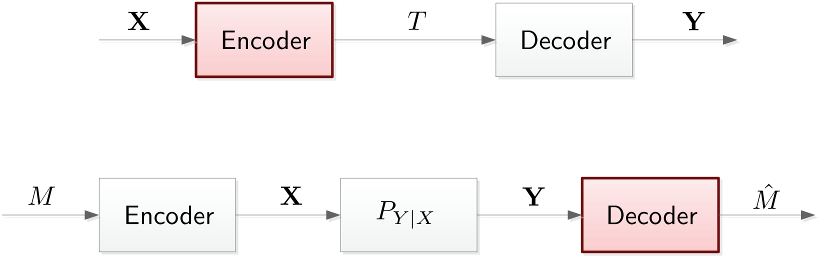




Fixed-Type Code: $(\mathbf{X}, \mathbf{Y}) \in \mathcal{T}_{\epsilon}^{(n)}(P^{\star}_X P_{Y|X})$

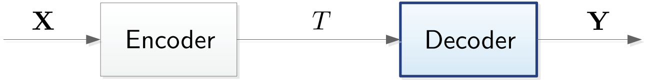
Duality - Preface

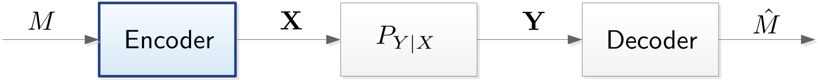
Point-to-Point Case:



Duality - Preface

Point-to-Point Case:

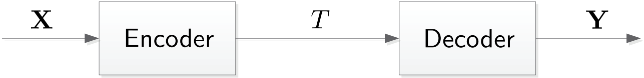


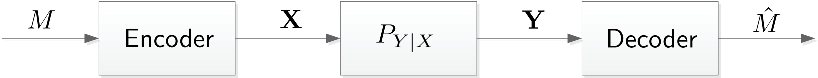


Duality - Preface

Point-to-Point Case:

$$R^\star = I(X;Y)$$



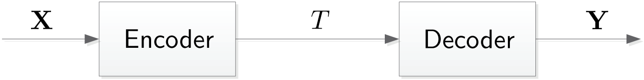


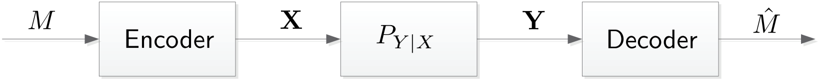
$$C = I(X;Y)$$

Duality - Preface

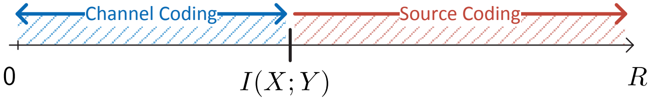
Point-to-Point Case:

$R^\star = I(X;Y)$

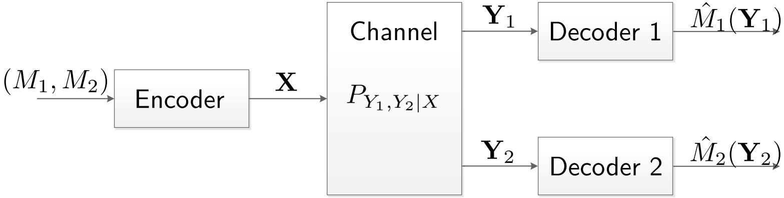


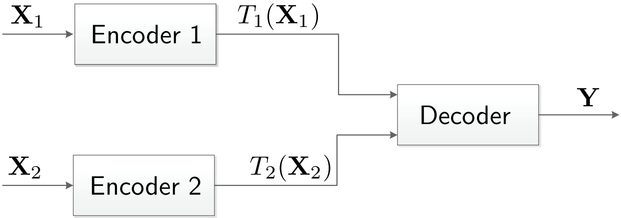


$C = I(X;Y)$

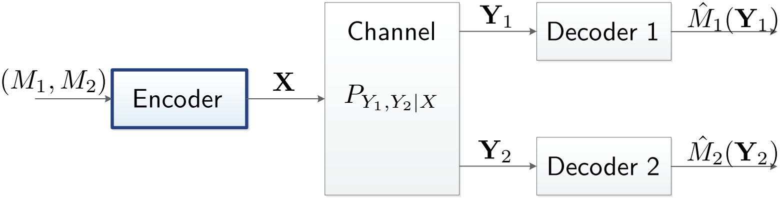


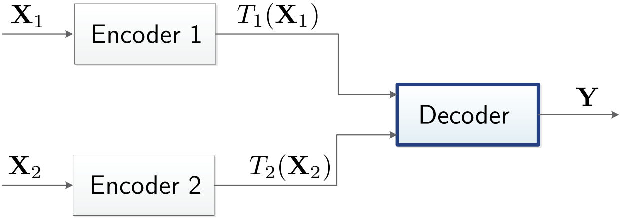
Multi-User Duality - Broadcast Channels



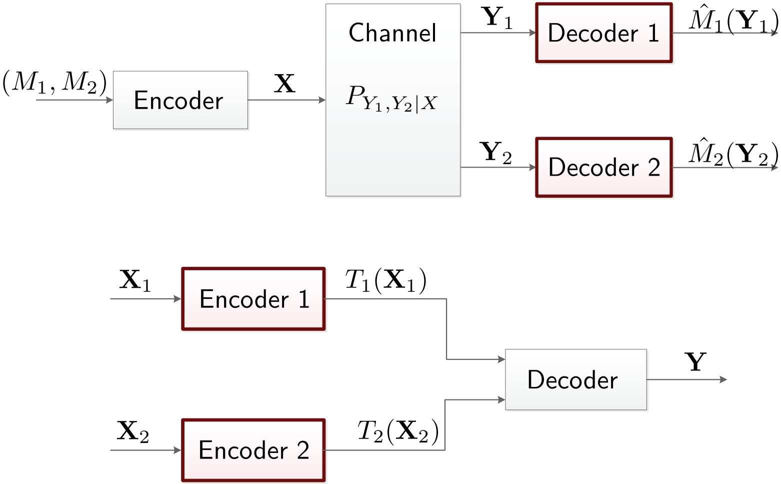


Multi-User Duality - Broadcast Channels





Multi-User Duality - Broadcast Channels



Multi-User Duality - Broadcast Channels

Probabilistic relations are preserved:

Multi-User Duality - Broadcast Channels

Probabilistic relations are preserved:

Broadcast Channel

$$(\mathbf{X}, \mathbf{Y}_1, \mathbf{Y}_2) \in \mathcal{T}_\epsilon^{(n)}\Big(P_X^\star P_{Y_1,Y_2|X}\Big)$$

⟷

Dual Source Coding Setting

$$(\mathbf{X}_1, \mathbf{X}_2, \mathbf{Y}) \in \mathcal{T}_\epsilon^{(n)}\Big(P_{X_1,X_2} P^\star_{Y|X_1,X_2}\Big)$$

Multi-User Duality - Broadcast Channels

Probabilistic relations are preserved:

Broadcast Channel		**Dual Source Coding Setting**
$(\mathbf{X}, \mathbf{Y}_1, \mathbf{Y}_2) \in \mathcal{T}_\epsilon^{(n)}\left(P_X^\star P_{Y_1,Y_2\|X}\right)$	⟷	$(\mathbf{X}_1, \mathbf{X}_2, \mathbf{Y}) \in \mathcal{T}_\epsilon^{(n)}\left(P_{X_1,X_2} P_{Y\|X_1,X_2}^\star\right)$

e.g., Markov relations, deterministic functions, etc.

Multi-User Duality - Broadcast Channels

Probabilistic relations are preserved:

Broadcast Channel		**Dual Source Coding Setting**
$(\mathbf{X}, \mathbf{Y}_1, \mathbf{Y}_2) \in \mathcal{T}_\epsilon^{(n)}\left(P_X^\star P_{Y_1,Y_2\vert X}\right)$	$\longleftrightarrow$	$(\mathbf{X}_1, \mathbf{X}_2, \mathbf{Y}) \in \mathcal{T}_\epsilon^{(n)}\left(P_{X_1,X_2} P_{Y\vert X_1,X_2}^\star\right)$

e.g., Markov relations, deterministic functions, etc.

Additional Principles:

Multi-User Duality - Broadcast Channels

Probabilistic relations are preserved:

Broadcast Channel		**Dual Source Coding Setting**
$(\mathbf{X}, \mathbf{Y}_1, \mathbf{Y}_2) \in \mathcal{T}_\epsilon^{(n)}\left(P_X^\star P_{Y_1,Y_2|X}\right)$	⟷	$(\mathbf{X}_1, \mathbf{X}_2, \mathbf{Y}) \in \mathcal{T}_\epsilon^{(n)}\left(P_{X_1,X_2} P^\star_{Y|X_1,X_2}\right)$

e.g., Markov relations, deterministic functions, etc.

Additional Principles:

- Causal/non-causal **encoder** CSI $\longleftrightarrow$ Causal/non-causal **decoder** SI

Multi-User Duality - Broadcast Channels

Probabilistic relations are preserved:

Broadcast Channel		**Dual Source Coding Setting**
$(\mathbf{X},\mathbf{Y}_1,\mathbf{Y}_2)\in\mathcal{T}_\epsilon^{(n)}\Big(P_X^\star P_{Y_1,Y_2\|X}\Big)$	⟷	$(\mathbf{X}_1,\mathbf{X}_2,\mathbf{Y})\in\mathcal{T}_\epsilon^{(n)}\Big(P_{X_1,X_2}P^\star_{Y\|X_1,X_2}\Big)$

e.g., Markov relations, deterministic functions, etc.

Additional Principles:

- Causal/non-causal **encoder** CSI $\longleftrightarrow$ Causal/non-causal **decoder** SI
- **Decoder** cooperation $\longleftrightarrow$ **Encoder** cooperation

Multi-User Duality - Broadcast Channels

Probabilistic relations are preserved:

Broadcast Channel		**Dual Source Coding Setting**
$(\mathbf{X},\mathbf{Y}_1,\mathbf{Y}_2)\in\mathcal{T}_\epsilon^{(n)}\Big(P^\star_X P_{Y_1,Y_2\|X}\Big)$	$\longleftrightarrow$	$(\mathbf{X}_1,\mathbf{X}_2,\mathbf{Y})\in\mathcal{T}_\epsilon^{(n)}\Big(P_{X_1,X_2}P^\star_{Y\|X_1,X_2}\Big)$

e.g., Markov relations, deterministic functions, etc.

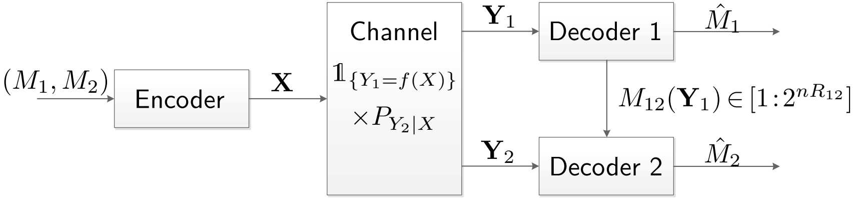
Additional Principles:

- Causal/non-causal **encoder** CSI $\longleftrightarrow$ Causal/non-causal **decoder** SI
- **Decoder** cooperation $\longleftrightarrow$ **Encoder** cooperation

⋆ **Result Duality:** Information measures at the corner points coincide! ⋆

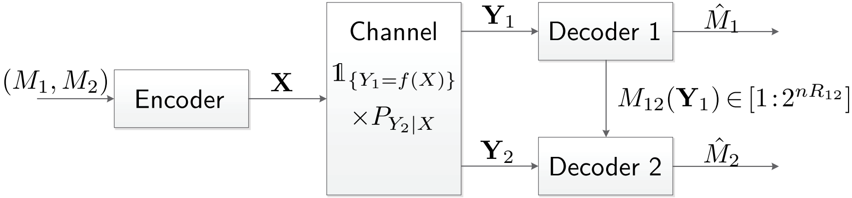
Cooperative SD-BC vs. Cooperative WAK Problem

Without cooperation: [Gelfand vs. Pinsker, 1980] and [Wyner, 1975]&[Ahlswede-Körner, 1975]



Cooperative SD-BC vs. Cooperative WAK Problem

Without cooperation: [Gelfand vs. Pinsker, 1980] and [Wyner, 1975]&[Ahlswede-Körner, 1975]

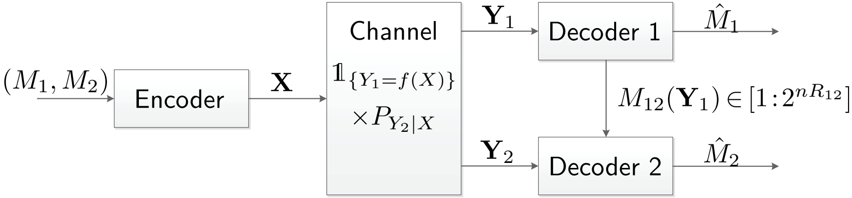


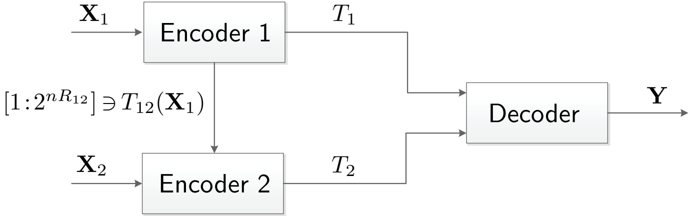
BCs with Cooperation:

- Physicaly degraded (PD) [Dabora and Servetto, 2006].
- Relay-BC [Liang and Kramer, 2007].
- State-dependent PD [Dikstein, Permuter and Steinberg, 2014].
- Degraded message sets / PD with parallel conf. [Steinberg, 2015].

Cooperative SD-BC vs. Cooperative WAK Problem

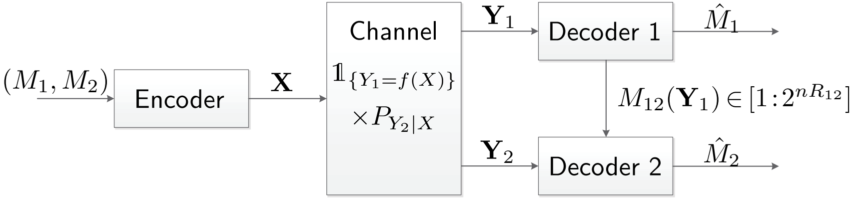
Without cooperation: [Gelfand vs. Pinsker, 1980] and [Wyner, 1975]&[Ahlswede-Körner, 1975]

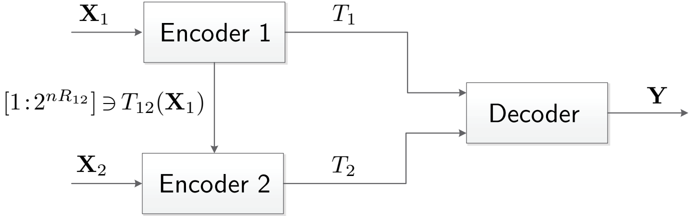




Cooperative SD-BC vs. Cooperative WAK Problem

Without cooperation: [Gelfand vs. Pinsker, 1980] and [Wyner, 1975]&[Ahlswede-Körner, 1975]



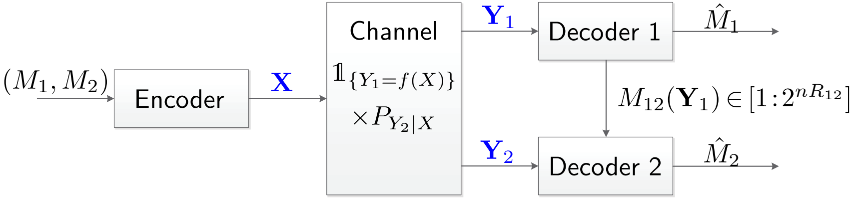


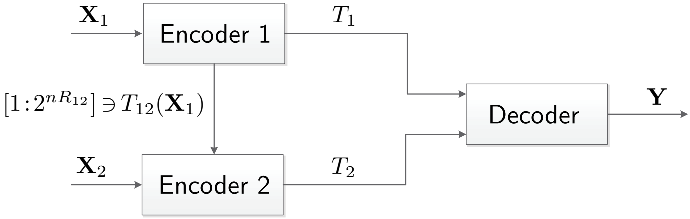
Semi-Deterministic BC **WAK Problem**

$(\mathbf{X},\mathbf{Y}_1,\mathbf{Y}_2)\in\mathcal{T}_\epsilon^{(n)}(P_X^\star\mathbb{1}_{\{Y_1=f(X)\}}P_{Y_2|X})$ ⟷ $(\mathbf{Y},\mathbf{X}_1,\mathbf{X}_2)\in\mathcal{T}_\epsilon^{(n)}(P_Y\mathbb{1}_{\{X_1=f(Y)\}}P_{X_2|Y}^\star)$

Cooperative SD-BC vs. Cooperative WAK Problem

Without cooperation: [Gelfand vs. Pinsker, 1980] and [Wyner, 1975]&[Ahlswede-Körner, 1975]



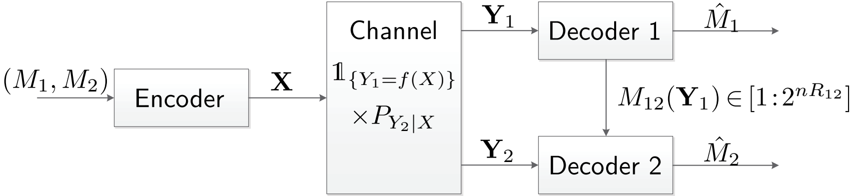


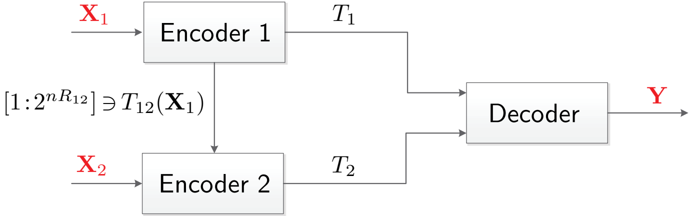
Semi-Deterministic BC **WAK Problem**

$$(\mathbf{X}, \mathbf{Y}_1, \mathbf{Y}_2) \in \mathcal{T}_\epsilon^{(n)}(P_X^\star \mathbb{1}_{\{Y_1=f(X)\}} P_{Y_2|X}) \longleftrightarrow (\mathbf{Y}, \mathbf{X}_1, \mathbf{X}_2) \in \mathcal{T}_\epsilon^{(n)}(P_Y \mathbb{1}_{\{X_1=f(Y)\}} P_{X_2|Y}^\star)$$

Cooperative SD-BC vs. Cooperative WAK Problem

Without cooperation: [Gelfand vs. Pinsker, 1980] and [Wyner, 1975]&[Ahlswede-Körner, 1975]





Semi-Deterministic BC **WAK Problem**

$$(\mathbf{X}, \mathbf{Y}_1, \mathbf{Y}_2) \in \mathcal{T}_\epsilon^{(n)}(P_X^\star \mathbb{1}_{\{Y_1=f(X)\}} P_{Y_2|X}) \longleftrightarrow (\mathbf{Y}, \mathbf{X}_1, \mathbf{X}_2) \in \mathcal{T}_\epsilon^{(n)}(P_Y \mathbb{1}_{\{X_1=f(Y)\}} P_{X_2|Y}^\star)$$

Cooperative WAK Problem - Solution

Theorem (Coordination-Capacity Region)

For a desired coordination PMF $P_{X_2}P_{Y|X_2}\mathbb{1}_{\{X_1=f(Y)\}}$*:*

$$\mathcal{C}_{\mathit{WAK}} = \bigcup \left\{ \begin{array}{c} R_{12} \geq I(V;X_1) - I(V;X_2) \\ R_1 \geq H(X_1|V,U) \\ R_2 \geq I(U;X_2|V) - I(U;X_1|V) \\ R_1 + R_2 \geq H(X_1|V,U) + I(V,U;X_1,X_2) \end{array} \right\}$$

where the union is over all $P_{X_1,X_2}P_{V|X_1}P_{U|X_2,V}P_{Y|X_1,U,V}$ *with* $P_{X_2}P_{Y|X_2}\mathbb{1}_{\{X_1=f(Y)\}}$ *as marginal.*

Cooperative WAK Problem - Solution

Theorem (Coordination-Capacity Region)

For a desired coordination PMF $P_{X_2}P_{Y|X_2}\mathbb{1}_{\{X_1=f(Y)\}}$*:*

$$\mathcal{C}_{WAK}=\bigcup\left\{\begin{array}{c} R_{12}\geq I(V;X_1)-I(V;X_2) \\ R_1\geq H(X_1|V,U) \\ R_2\geq I(U;X_2|V)-I(U;X_1|V) \\ R_1+R_2\geq H(X_1|V,U)+I(V,U;X_1,X_2) \end{array}\right\}$$

where the union is over all $P_{X_1,X_2}P_{V|X_1}P_{U|X_2,V}P_{Y|X_1,U,V}$ *with* $P_{X_2}P_{Y|X_2}\mathbb{1}_{\{X_1=f(Y)\}}$ *as marginal.*

Achievability via Wyner-Ziv coding, superposition coding and Slepian-Wolf binning.

Cooperative WAK Problem - Solution

Theorem (Coordination-Capacity Region)

For a desired coordination PMF $P_{X_2}P_{Y|X_2}\mathbb{1}_{\{X_1=f(Y)\}}$:

$$\mathcal{C}_{WAK}=\bigcup\left\{\begin{array}{r}R_{12}\geq \boldsymbol{I(V;X_1)-I(V;X_2)}\\ R_1\geq H(X_1|V,U)\\ R_2\geq \boldsymbol{I(U;X_2|V)-I(U;X_1|V)}\\ R_1+R_2\geq H(X_1|V,U)+I(V,U;X_1,X_2)\end{array}\right\}$$

where the union is over all $P_{X_1,X_2}P_{V|X_1}P_{U|X_2,V}P_{Y|X_1,U,V}$ *with* $P_{X_2}P_{Y|X_2}\mathbb{1}_{\{X_1=f(Y)\}}$ *as marginal.*

Achievability via **Wyner-Ziv coding**, superposition coding and Slepian-Wolf binning.

Cooperative WAK Problem - Solution

Theorem (Coordination-Capacity Region)

For a desired coordination PMF $P_{X_2}P_{Y|X_2}\mathbb{1}_{\{X_1=f(Y)\}}$*:*

$$\mathcal{C}_{WAK} = \bigcup \left\{ \begin{array}{c} R_{12} \geq I(V;X_1) - I(V;X_2) \\ R_1 \geq H(X_1|V,U) \\ R_2 \geq \boldsymbol{I(U;X_2|V) - I(U;X_1|V)} \\ R_1 + R_2 \geq H(X_1|V,U) + I(V,U;X_1,X_2) \end{array} \right\}$$

where the union is over all $P_{X_1,X_2}P_{V|X_1}P_{U|X_2,V}P_{Y|X_1,U,V}$ *with* $P_{X_2}P_{Y|X_2}\mathbb{1}_{\{X_1=f(Y)\}}$ *as marginal.*

Achievability via Wyner-Ziv coding, **superposition coding** and Slepian-Wolf binning.

Cooperative WAK Problem - Solution

Theorem (Coordination-Capacity Region)

For a desired coordination PMF $P_{X_2}P_{Y|X_2}\mathbb{1}_{\{X_1=f(Y)\}}$*:*

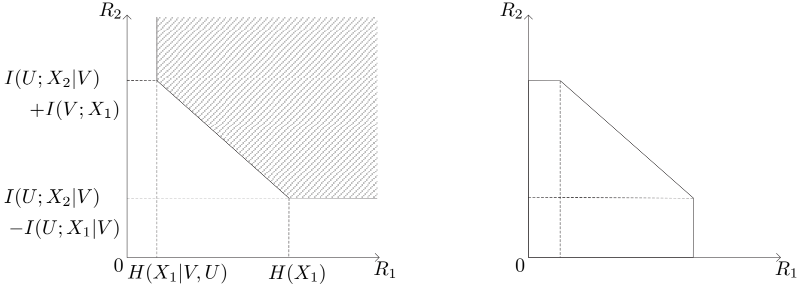
$$\mathcal{C}_{WAK} = \bigcup \left\{ \begin{array}{c} R_{12} \geq I(V;X_1) - I(V;X_2) \\ R_1 \geq \boldsymbol{H(X_1|V,U)} \\ R_2 \geq I(U;X_2|V) - I(U;X_1|V) \\ R_1 + R_2 \geq H(X_1|V,U) + I(V,U;X_1,X_2) \end{array} \right\}$$

where the union is over all $P_{X_1,X_2}P_{V|X_1}P_{U|X_2,V}P_{Y|X_1,U,V}$ *with* $P_{X_2}P_{Y|X_2}\mathbb{1}_{\{X_1=f(Y)\}}$ *as marginal.*

Achievability via Wyner-Ziv coding, superposition coding and **Slepian-Wolf binning**.

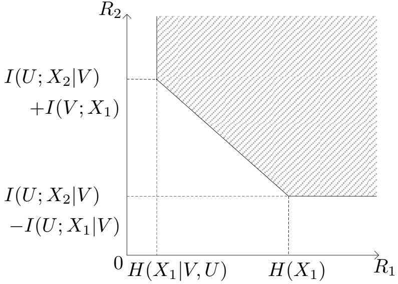
Corner Point Correspondence

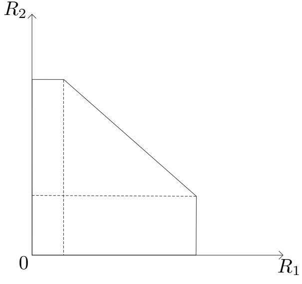
For fixed joint PMFs and R_{12}:



Corner Point Correspondence

For fixed joint PMFs and R_{12}:

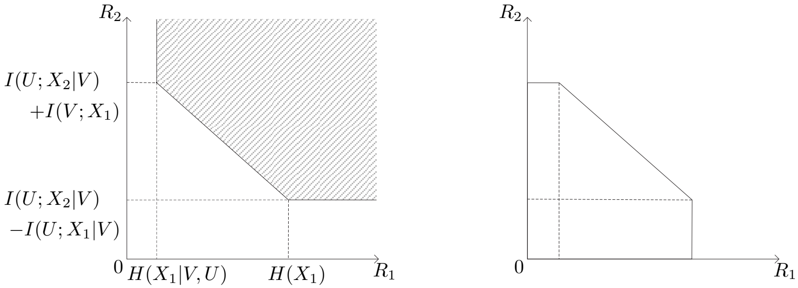




Cooperative WAK Problem	**Cooperative Semi-Deterministic BC**
$R_{12} = I(V;X_1) - I(V;X_2)$	
(R_1, R_2) at Lower Corner Point: $\Big(H(X_1)\ ,\ I(U;X_2\|V) - I(U;X_1\|V) \Big)$	(R_1, R_2) at Lower Corner Point:
(R_1, R_2) at Upper Corner Point: $\Big(H(X_1\|V,U)\ ,\ I(U;X_2\|V) + I(V;X_1) \Big)$	(R_1, R_2) at Upper Corner Point:

Corner Point Correspondence

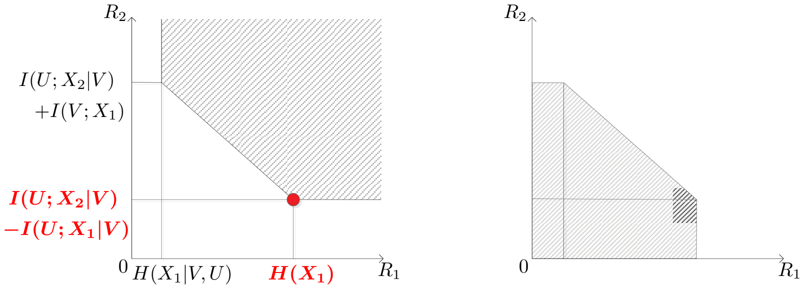
For fixed joint PMFs and R_{12}:



Cooperative WAK Problem	**Cooperative Semi-Deterministic BC**
$\boldsymbol{R_{12} = I(V;X_1) - I(V;X_2)}$	$\boldsymbol{R_{12} = I(V;Y_1) - I(V;Y_2)}$
(R_1, R_2) at Lower Corner Point: $\Big(H(X_1)\ ,\ I(U;X_2|V) - I(U;X_1|V)\Big)$	(R_1, R_2) at Lower Corner Point:
(R_1, R_2) at Upper Corner Point: $\Big(H(X_1|V,U)\ ,\ I(U;X_2|V) + I(V;X_1)\Big)$	(R_1, R_2) at Upper Corner Point:

Corner Point Correspondence

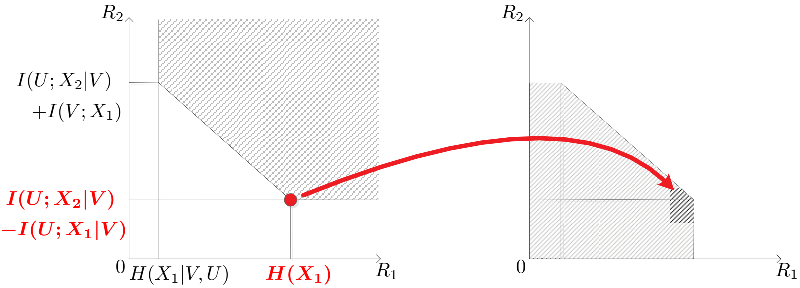
For fixed joint PMFs and R_{12}:



Cooperative WAK Problem	Cooperative Semi-Deterministic BC
$R_{12} = I(V;X_1) - I(V;X_2)$	$R_{12} = I(V;Y_1) - I(V;Y_2)$
(R_1, R_2) at Lower Corner Point: $\big(H(X_1)\ ,\ I(U;X_2\|V) - I(U;X_1\|V) \big)$	(R_1, R_2) at Lower Corner Point:
(R_1, R_2) at Upper Corner Point: $\big(H(X_1\|V,U)\ ,\ I(U;X_2\|V) + I(V;X_1) \big)$	(R_1, R_2) at Upper Corner Point:

Corner Point Correspondence

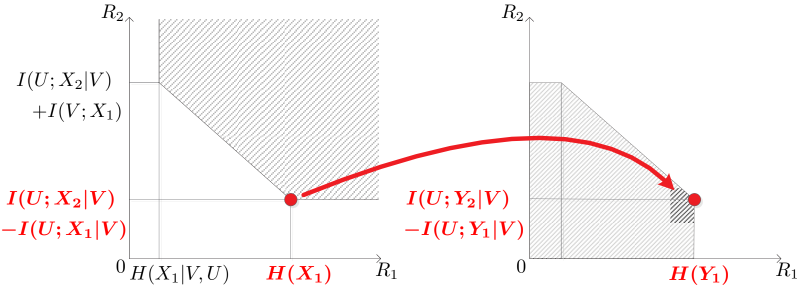
For fixed joint PMFs and R_{12}:



Cooperative WAK Problem	Cooperative Semi-Deterministic BC
$R_{12} = I(V;X_1) - I(V;X_2)$	$R_{12} = I(V;Y_1) - I(V;Y_2)$
(R_1, R_2) at Lower Corner Point: $\big(H(X_1)\ ,\ I(U;X_2\|V) - I(U;X_1\|V)\big)$	(R_1, R_2) at Lower Corner Point:
(R_1, R_2) at Upper Corner Point: $\big(H(X_1\|V,U)\ ,\ I(U;X_2\|V) + I(V;X_1)\big)$	(R_1, R_2) at Upper Corner Point:

Corner Point Correspondence

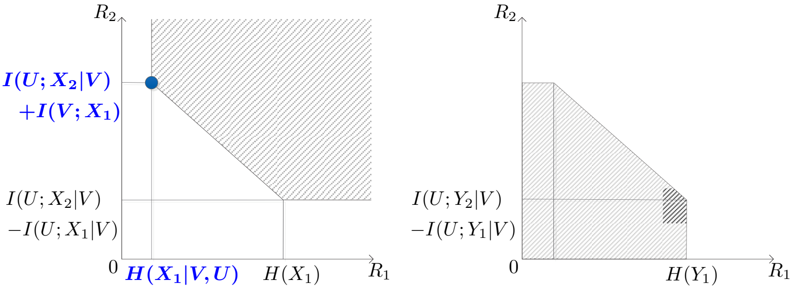
For fixed joint PMFs and R_{12}:



Cooperative WAK Problem	**Cooperative Semi-Deterministic BC**
$R_{12} = I(V;X_1) - I(V;X_2)$	$R_{12} = I(V;Y_1) - I(V;Y_2)$
(R_1, R_2) at Lower Corner Point: $\boldsymbol{\big(H(X_1)\ ,\ I(U;X_2\vert V) - I(U;X_1\vert V)\big)}$	(R_1, R_2) at Lower Corner Point: $\boldsymbol{\big(H(Y_1)\ ,\ I(U;Y_2\vert V) - I(U;Y_1\vert V)\big)}$
(R_1, R_2) at Upper Corner Point: $\big(H(X_1\vert V,U)\ ,\ I(U;X_2\vert V) + I(V;X_1)\big)$	(R_1, R_2) at Upper Corner Point:

Corner Point Correspondence

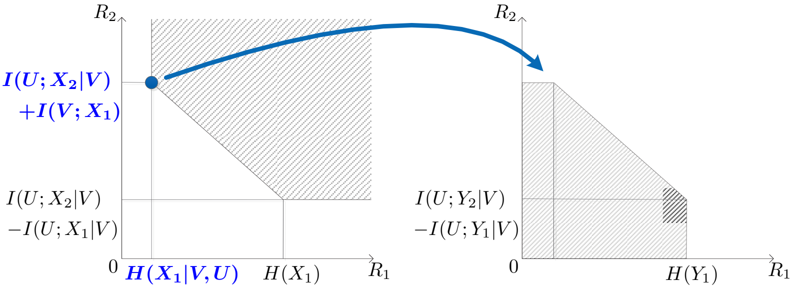
For fixed joint PMFs and R_{12}:



Cooperative WAK Problem	Cooperative Semi-Deterministic BC
$R_{12} = I(V;X_1) - I(V;X_2)$	$R_{12} = I(V;Y_1) - I(V;Y_2)$
(R_1, R_2) at Lower Corner Point: $\big(H(X_1)\ ,\ I(U;X_2\|V) - I(U;X_1\|V)\big)$	(R_1, R_2) at Lower Corner Point: $\big(H(Y_1)\ ,\ I(U;Y_2\|V) - I(U;Y_1\|V)\big)$
(R_1, R_2) at Upper Corner Point: $\big(H(X_1\|V,U)\ ,\ I(U;X_2\|V) + I(V;X_1)\big)$	(R_1, R_2) at Upper Corner Point:

Corner Point Correspondence

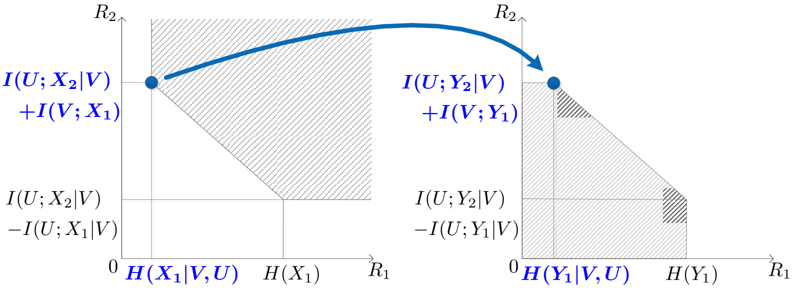
For fixed joint PMFs and R_{12}:



Cooperative WAK Problem	**Cooperative Semi-Deterministic BC**
$R_{12} = I(V;X_1) - I(V;X_2)$	$R_{12} = I(V;Y_1) - I(V;Y_2)$
(R_1, R_2) at Lower Corner Point: $\big(H(X_1)\ ,\ I(U;X_2\|V) - I(U;X_1\|V) \big)$	(R_1, R_2) at Lower Corner Point: $\big(H(Y_1)\ ,\ I(U;Y_2\|V) - I(U;Y_1\|V) \big)$
(R_1, R_2) at Upper Corner Point: $\boldsymbol{\big(H(X_1\|V,U)\ ,\ I(U;X_2\|V) + I(V;X_1) \big)}$	(R_1, R_2) at Upper Corner Point:

Corner Point Correspondence

For fixed joint PMFs and R_{12}:



Cooperative WAK Problem	Cooperative Semi-Deterministic BC
$R_{12} = I(V;X_1) - I(V;X_2)$	$R_{12} = I(V;Y_1) - I(V;Y_2)$
(R_1, R_2) at Lower Corner Point: $\Big(H(X_1)\ ,\ I(U;X_2\|V) - I(U;X_1\|V) \Big)$	(R_1, R_2) at Lower Corner Point: $\Big(H(Y_1)\ ,\ I(U;Y_2\|V) - I(U;Y_1\|V) \Big)$
(R_1, R_2) at Upper Corner Point: $\Big(\boldsymbol{H(X_1\|V,U)\ ,\ I(U;X_2\|V) + I(V;X_1)} \Big)$	(R_1, R_2) at Upper Corner Point: $\Big(\boldsymbol{H(Y_1\|V,U)\ ,\ I(U;Y_2\|V) + I(V;Y_1)} \Big)$

Semi-Deterministic BC with Cooperation - Solution

Theorem (Capacity Region)

The capacity region is:

$$\mathcal{C}_{BC} = \bigcup \left\{ \begin{array}{c} R_{12} \geq I(V;Y_1) - I(V;Y_2) \\ R_1 \leq H(Y_1) \\ R_2 \leq I(V,U;Y_2) + R_{12} \\ R_1 + R_2 \leq H(Y_1|V,U) + I(U;Y_2|V) + I(V;Y_1) \end{array} \right\}$$

where the union is over all $P_{V,U,Y_1,X} P_{Y_2|X} \mathbb{1}_{\{Y_1=f(X)\}}$.

Semi-Deterministic BC with Cooperation - Solution

Theorem (Capacity Region)

The capacity region is:

$$\mathcal{C}_{BC} = \bigcup \left\{ \begin{array}{r} R_{12} \geq I(V;Y_1) - I(V;Y_2) \\ R_1 \leq H(Y_1) \\ R_2 \leq I(V,U;Y_2) + R_{12} \\ R_1 + R_2 \leq H(Y_1|V,U) + I(U;Y_2|V) + I(V;Y_1) \end{array} \right\}$$

where the union is over all $P_{V,U,Y_1,X} P_{Y_2|X} \mathbb{1}_{\{Y_1=f(X)\}}$.

- Later: Achievability and converse proofs for an alternative region.

Semi-Deterministic BC with Cooperation - Solution

Theorem (Capacity Region)

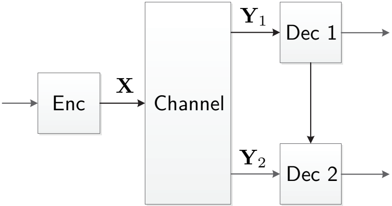
The capacity region is:

$$\mathcal{C}_{BC} = \bigcup \left\{ \begin{array}{c} R_{12} \geq I(V;Y_1) - I(V;Y_2) \\ R_1 \leq H(Y_1) \\ R_2 \leq I(V,U;Y_2) + R_{12} \\ R_1 + R_2 \leq H(Y_1|V,U) + I(U;Y_2|V) + I(V;Y_1) \end{array} \right\}$$

where the union is over all $P_{V,U,Y_1,X} P_{Y_2|X} \mathbb{1}_{\{Y_1=f(X)\}}$.

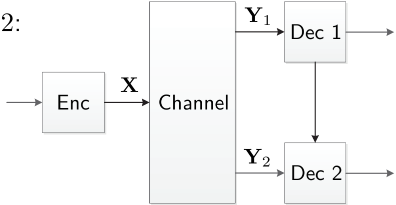
- Later: Achievability and converse proofs for an alternative region.
- $\mathcal{C}_{\text{BC}}$ emphasizes duality.

Cooperative Semi-Deterministic BC - Achievability Outline



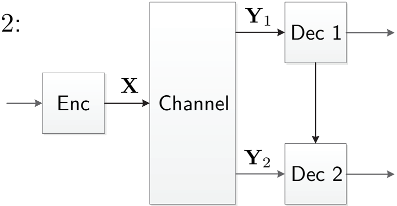
Cooperative Semi-Deterministic BC - Achievability Outline

- **Rate Splitting:** $M_j = (M_{j0}, M_{jj})$, $j = 1, 2$:



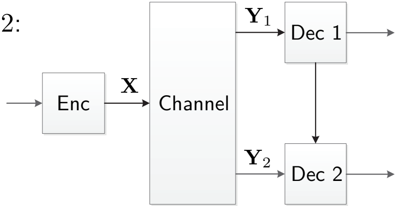
Cooperative Semi-Deterministic BC - Achievability Outline

- **Rate Splitting:** $M_j = (M_{j0}, M_{jj})$, $j = 1, 2$:
  - (M_{10}, M_{20}) - Common message;



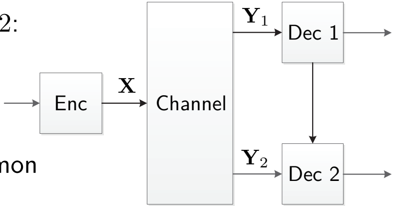
Cooperative Semi-Deterministic BC - Achievability Outline

- **Rate Splitting:** $M_j = (M_{j0}, M_{jj})$, $j = 1, 2$:
  - (M_{10}, M_{20}) - Common message;
  - (M_{11}, M_{22}) - Private messages.



Cooperative Semi-Deterministic BC - Achievability Outline

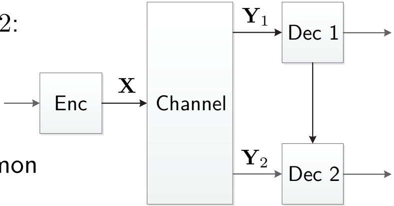
- **Rate Splitting:** $M_j = (M_{j0}, M_{jj})$, $j = 1, 2$:
  - (M_{10}, M_{20}) - Common message;
  - (M_{11}, M_{22}) - Private messages.

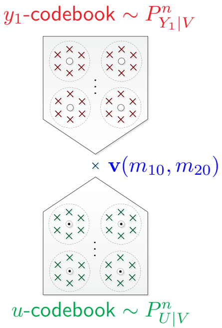


- **Codebook Structure:** Marton (with common message).

Cooperative Semi-Deterministic BC - Achievability Outline

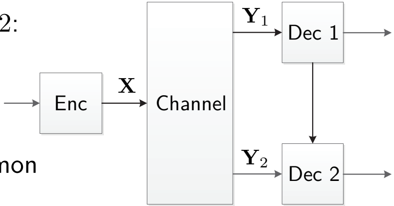
- **Rate Splitting:** $M_j = (M_{j0}, M_{jj})$, $j = 1, 2$:
  - (M_{10}, M_{20}) - Common message;
  - (M_{11}, M_{22}) - Private messages.
- **Codebook Structure:** Marton (with common message).





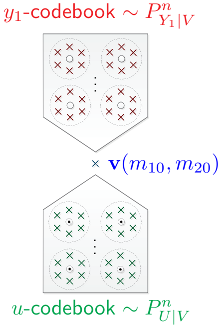
Cooperative Semi-Deterministic BC - Achievability Outline

- **Rate Splitting:** $M_j = (M_{j0}, M_{jj})$, $j = 1, 2$:
  - (M_{10}, M_{20}) - Common message;
  - (M_{11}, M_{22}) - Private messages.



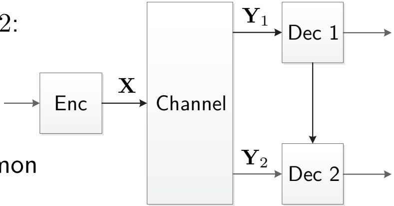
- **Codebook Structure:** Marton (with common message).

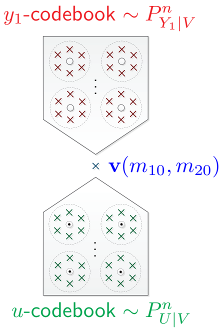
- **Cooperation:**



Cooperative Semi-Deterministic BC - Achievability Outline

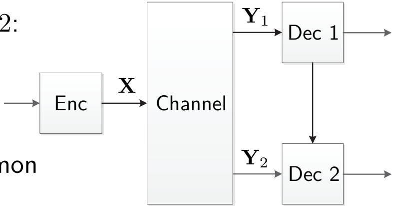
- **Rate Splitting:** $M_j = (M_{j0}, M_{jj})$, $j = 1, 2$:
  - (M_{10}, M_{20}) - Common message;
  - (M_{11}, M_{22}) - Private messages.
- **Codebook Structure:** Marton (with common message).
- **Cooperation:**
  1. Partition common message c.b. into $2^{nR_{12}}$ bins.

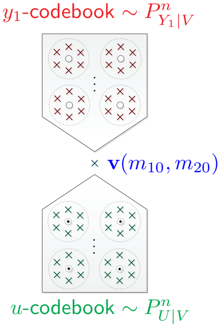




Cooperative Semi-Deterministic BC - Achievability Outline

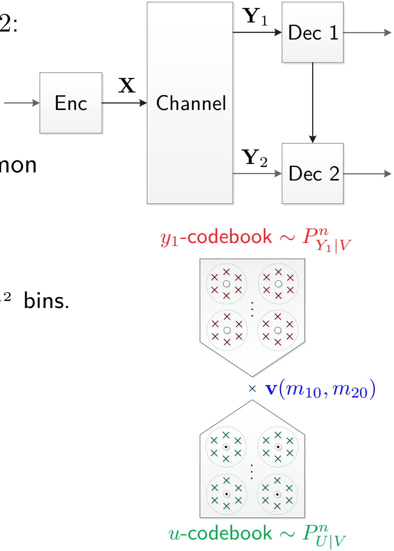
- **Rate Splitting:** $M_j = (M_{j0}, M_{jj})$, $j = 1, 2$:
  - (M_{10}, M_{20}) - Common message;
  - (M_{11}, M_{22}) - Private messages.
- **Codebook Structure:** Marton (with common message).
- **Cooperation:**
  1. Partition common message c.b. into $2^{nR_{12}}$ bins.
  2. Convey bin number via link.





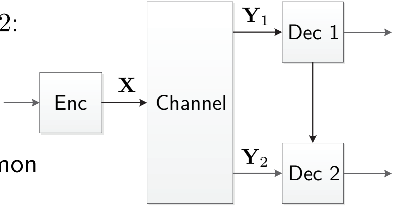
Cooperative Semi-Deterministic BC - Achievability Outline

- **Rate Splitting:** $M_j = (M_{j0}, M_{jj})$, $j = 1, 2$:
  - (M_{10}, M_{20}) - Common message;
  - (M_{11}, M_{22}) - Private messages.
- **Codebook Structure:** Marton (with common message).
- **Cooperation:**
  1. Partition common message c.b. into $2^{nR_{12}}$ bins.
  2. Convey bin number via link.
- **User 2 Gain:**



Cooperative Semi-Deterministic BC - Achievability Outline

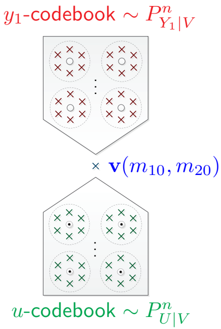
- **Rate Splitting:** $M_j = (M_{j0}, M_{jj})$, $j = 1, 2$:
  - (M_{10}, M_{20}) - Common message;
  - (M_{11}, M_{22}) - Private messages.



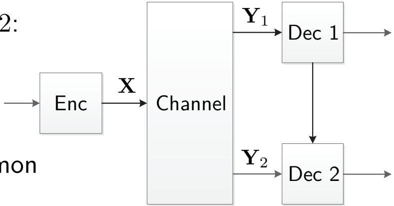
- **Codebook Structure:** Marton (with common message).

- **Cooperation:**
  1. Partition common message c.b. into $2^{nR_{12}}$ bins.
  2. Convey bin number via link.

- **User 2 Gain:** Reduced search space of common message c.w. by R_{12}.

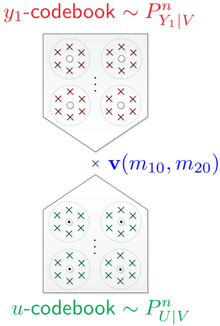


- **Rate Splitting:** $M_j = (M_{j0}, M_{jj})$, $j = 1, 2$:
  - (M_{10}, M_{20}) - Common message;
  - (M_{11}, M_{22}) - Private messages.



- **Codebook Structure:** Marton (with common message).

- **Cooperation:**
  1. Partition common message c.b. into $2^{nR_{12}}$ bins.
  2. Convey bin number via link.



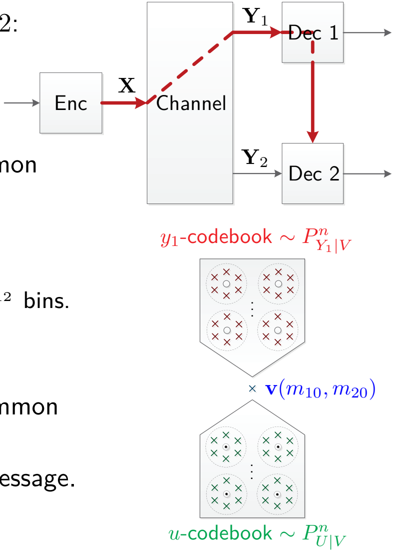
- **User 2 Gain:** Reduced search space of common message c.w. by R_{12}.

  $\Longrightarrow$ More channel resources for private message.

Cooperative Semi-Deterministic BC - Achievability Outline

- **Rate Splitting:** $M_j = (M_{j0}, M_{jj})$, $j = 1, 2$:
  - (M_{10}, M_{20}) - Common message;
  - (M_{11}, M_{22}) - Private messages.
- **Codebook Structure:** Marton (with common message).
- **Cooperation:**
  1. Partition common message c.b. into $2^{nR_{12}}$ bins.
  2. Convey bin number via link.
- **User 2 Gain:** Reduced search space of common message c.w. by R_{12}.

  $\Longrightarrow$ More channel resources for private message.



Summary

- Channel-source duality for BCs.

Summary

- Channel-source duality for BCs.
- Cooperative semi-deterministic BCs:

Summary

- Channel-source duality for BCs.
- Cooperative semi-deterministic BCs:
  - Source coding dual - Cooperative WAK problem.

Summary

- Channel-source duality for BCs.
- Cooperative semi-deterministic BCs:
  - Source coding dual - Cooperative WAK problem.
  - Corner point correspondence.

Summary

- Channel-source duality for BCs.
- Cooperative semi-deterministic BCs:
  - Source coding dual - Cooperative WAK problem.
  - Corner point correspondence.
- Achievability via Marton coding with a common message.

Summary

- Channel-source duality for BCs.
- Cooperative semi-deterministic BCs:
  - Source coding dual - Cooperative WAK problem.
  - Corner point correspondence.
- Achievability via Marton coding with a common message.
- Full version undergoing review for IEEE Trans. Inf. Theory; available on ArXiV at http://arxiv.org/abs/1405.7812.

Summary

- Channel-source duality for BCs.
- Cooperative semi-deterministic BCs:
  - Source coding dual - Cooperative WAK problem.
  - Corner point correspondence.
- Achievability via Marton coding with a common message.
- Full version undergoing review for IEEE Trans. Inf. Theory; available on ArXiV at http://arxiv.org/abs/1405.7812.

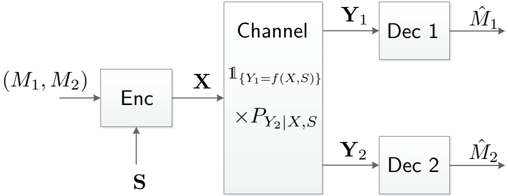
Thank you!

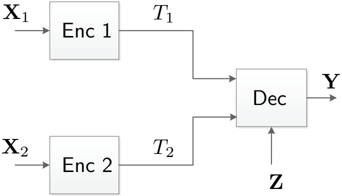
Multi-User Duality - Additional Examples

State-Dependant Semi-Deterministic BC vs. Dual:

Multi-User Duality - Additional Examples

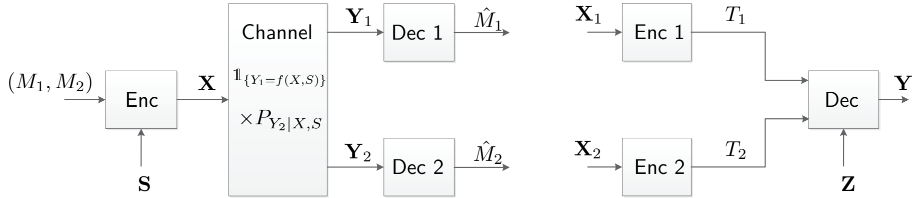
State-Dependant Semi-Deterministic BC vs. Dual:

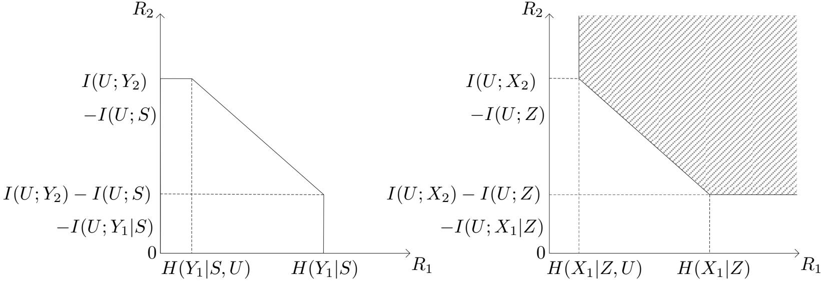




Multi-User Duality - Additional Examples

State-Dependant Semi-Deterministic BC vs. Dual:



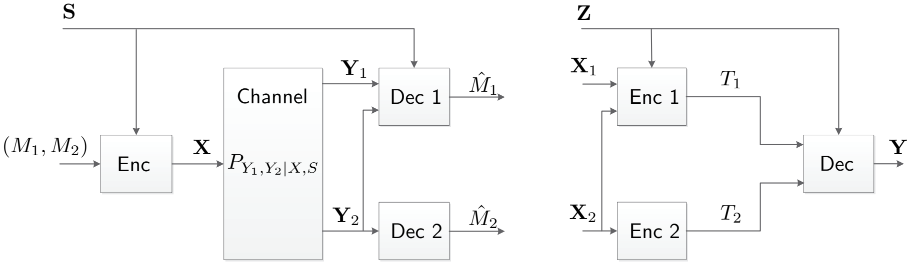


Multi-User Duality - Additional Examples

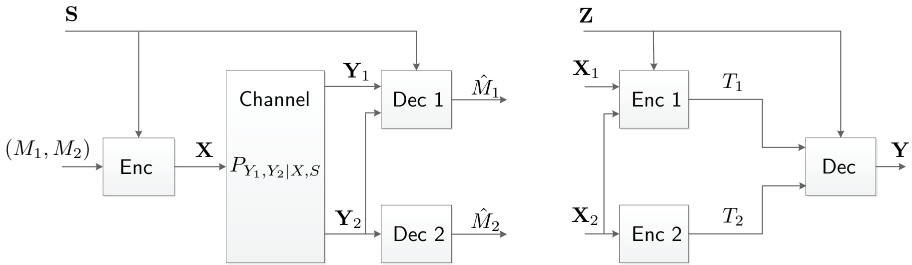
State-Dependant Output-Degraded BC vs. Dual:

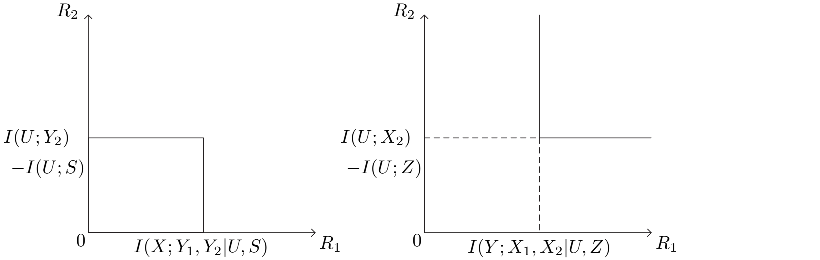
Multi-User Duality - Additional Examples

State-Dependant Output-Degraded BC vs. Dual:



State-Dependant Output-Degraded BC vs. Dual:

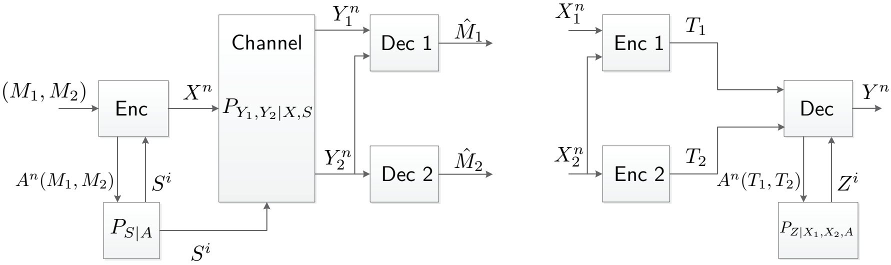




Multi-User Duality - Additional Examples

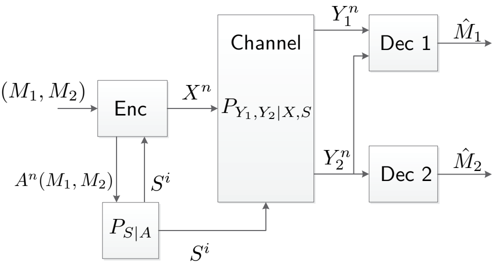
Action-Dependant Output-Degraded BC vs. Dual:

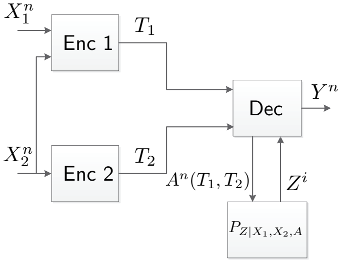
Action-Dependant Output-Degraded BC vs. Dual:

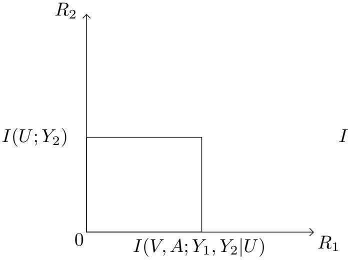


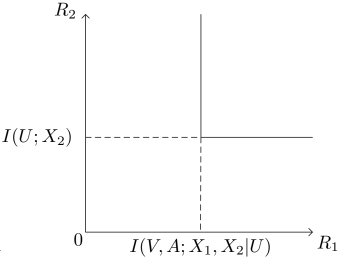
Multi-User Duality - Additional Examples

Action-Dependant Output-Degraded BC vs. Dual:

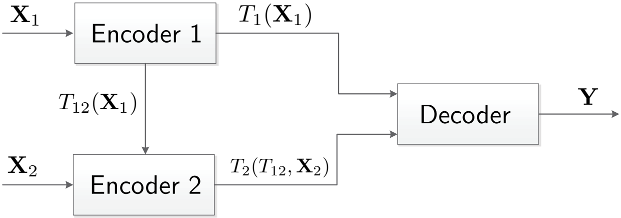




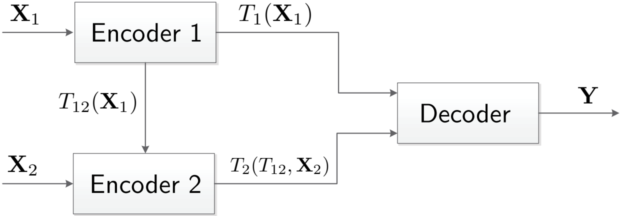




AK Problem with Cooperation - Achievability Outline

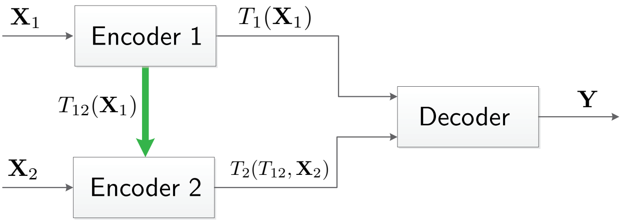


AK Problem with Cooperation - Achievability Outline



Rate	Corner Point 1	Corner Point 2
R_{12}	$I(V;X_1) - I(V;X_2)$	$I(V;X_1) - I(V;X_2)$
R_1	$H(X_1)$	$H(X_1\|V,U)$
R_2	$I(U;X_2\|V) - I(U;X_1\|V)$	$I(U;X_2\|V) + I(V;X_1)$

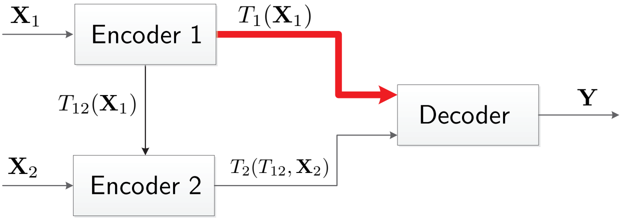
AK Problem with Cooperation - Achievability Outline



Rate	Corner Point 1	Corner Point 2
R_{12}	$\boldsymbol{I(V;X_1) - I(V;X_2)}$	$\boldsymbol{I(V;X_1) - I(V;X_2)}$
R_1	$H(X_1)$	$H(X_1 \mid V, U)$
R_2	$I(U;X_2 \mid V) - I(U;X_1 \mid V)$	$I(U;X_2 \mid V) + I(V;X_1)$

- **Cooperation:** Wyner-Ziv scheme to convey $\mathbf{V}$ via cooperation link.

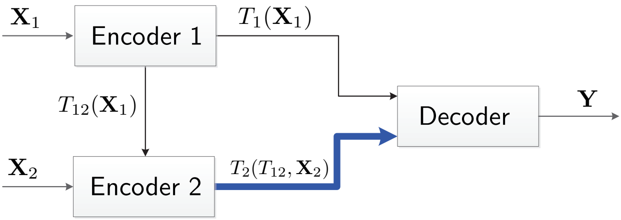
AK Problem with Cooperation - Achievability Outline



Rate	**Corner Point 1**	Corner Point 2
R_{12}	$I(V;X_1) - I(V;X_2)$	$I(V;X_1) - I(V;X_2)$
R_1	$\boldsymbol{H(X_1)}$	$H(X_1 \vert V,U)$
R_2	$\boldsymbol{I(U;X_2 \vert V) - I(U;X_1 \vert V)}$	$I(U;X_2 \vert V) + I(V;X_1)$

- **Cooperation:** Wyner-Ziv scheme to convey $\mathbf{V}$ via cooperation link.
- **Corner Point 1:** $\mathbf{V}$ is transmitted to dec. by Enc. 1 within $\mathbf{X}_1$.

AK Problem with Cooperation - Achievability Outline



Rate	Corner Point 1	**Corner Point 2**
R_{12}	$I(V;X_1) - I(V;X_2)$	$I(V;X_1) - I(V;X_2)$
R_1	$H(X_1)$	$\boldsymbol{H(X_1\vert V,U)}$
R_2	$I(U;X_2\vert V) - I(U;X_1\vert V)$	$\boldsymbol{I(U;X_2\vert V) + I(V;X_1)}$

- **Cooperation:** Wyner-Ziv scheme to convey $\mathbf{V}$ via cooperation link.
- **Corner Point 1:** $\mathbf{V}$ is transmitted to dec. by Enc. 1 within $\mathbf{X}_1$.
- **Corner Point 2:** $\mathbf{V}$ is explicitly transmitted to dec. by Enc. 2.

AK Problem with Cooperation - Proof Outline

Converse:

AK Problem with Cooperation - Proof Outline

Converse:

- Standard techniques while defining

$$V_i = (T_{12}, X_1^{n\setminus i}, X_{2,i+1}^n),$$
$$U_i = T_2,$$

for every $1 \le i \le n$.

AK Problem with Cooperation - Proof Outline

Converse:

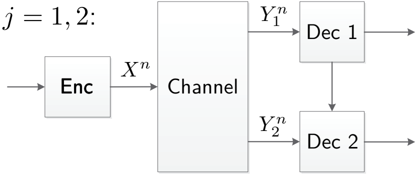
- Standard techniques while defining

$$V_i = (T_{12}, X_1^{n\backslash i}, X_{2,i+1}^n),$$
$$U_i = T_2,$$

  for every $1 \leq i \leq n$.
- Time mixing properties.

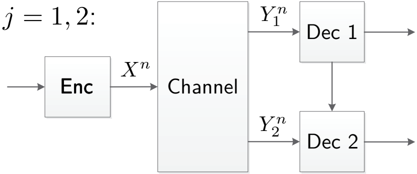
Semi-Deterministic BC with Cooperation - Achievability Outline

- **Rate Splitting:** $M_j = (M_{j0}, M_{jj})$, $j = 1, 2$:



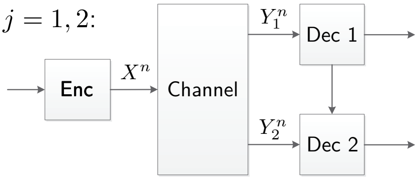
Semi-Deterministic BC with Cooperation - Achievability Outline

- **Rate Splitting:** $M_j = (M_{j0}, M_{jj})$, $j = 1, 2$:
  - (M_{10}, M_{20}) - Public message;



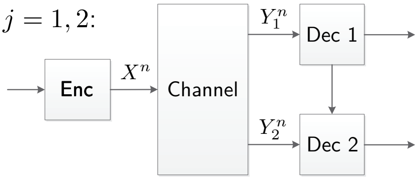
Semi-Deterministic BC with Cooperation - Achievability Outline

- **Rate Splitting:** $M_j = (M_{j0}, M_{jj})$, $j = 1, 2$:
  - (M_{10}, M_{20}) - Public message;
  - (M_{11}, M_{22}) - Private messages.



Semi-Deterministic BC with Cooperation - Achievability Outline

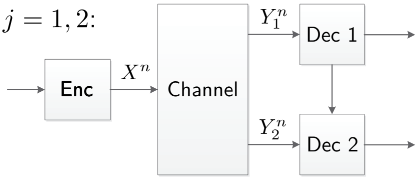
- **Rate Splitting:** $M_j = (M_{j0}, M_{jj})$, $j = 1, 2$:
  - (M_{10}, M_{20}) - Public message;
  - (M_{11}, M_{22}) - Private messages.



- **Codebook Structure:** Marton:

Semi-Deterministic BC with Cooperation - Achievability Outline

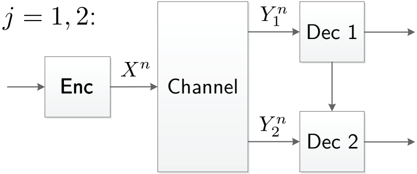
- **Rate Splitting:** $M_j = (M_{j0}, M_{jj})$, $j = 1, 2$:
  - (M_{10}, M_{20}) - Public message;
  - (M_{11}, M_{22}) - Private messages.



- **Codebook Structure:** Marton:
  - Public Message: $(M_{10}, M_{20}) \longrightarrow \mathbf{V}$.

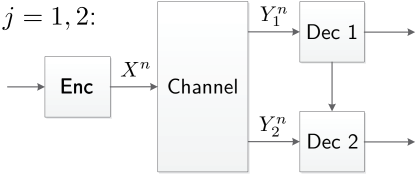
Semi-Deterministic BC with Cooperation - Achievability Outline

- **Rate Splitting:** $M_j = (M_{j0}, M_{jj})$, $j = 1, 2$:
  - (M_{10}, M_{20}) - Public message;
  - (M_{11}, M_{22}) - Private messages.



- **Codebook Structure:** Marton:
  - Public Message: $(M_{10}, M_{20}) \longrightarrow \mathbf{V}$.
  - Private Messages - Superposed on $\mathbf{V}$:

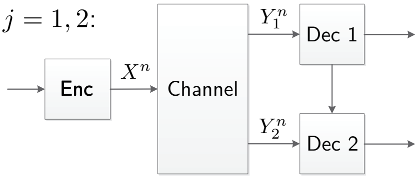
Semi-Deterministic BC with Cooperation - Achievability Outline



- **Rate Splitting:** $M_j = (M_{j0}, M_{jj})$, $j = 1, 2$:
  - (M_{10}, M_{20}) - Public message;
  - (M_{11}, M_{22}) - Private messages.
- **Codebook Structure:** Marton:
  - Public Message: $(M_{10}, M_{20}) \longrightarrow \mathbf{V}$.
  - Private Messages - Superposed on $\mathbf{V}$:
    1. $M_{11} \longrightarrow \mathbf{Y}_1$;

Semi-Deterministic BC with Cooperation - Achievability Outline

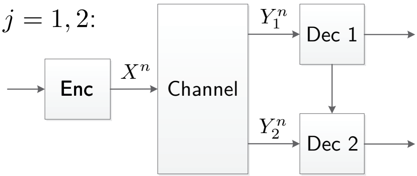
- **Rate Splitting:** $M_j = (M_{j0}, M_{jj})$, $j = 1, 2$:
  - (M_{10}, M_{20}) - Public message;
  - (M_{11}, M_{22}) - Private messages.



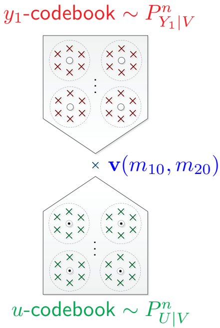
- **Codebook Structure:** Marton:
  - Public Message: $(M_{10}, M_{20}) \longrightarrow \mathbf{V}$.
  - Private Messages - Superposed on $\mathbf{V}$:
    1. $M_{11} \longrightarrow \mathbf{Y}_1$;
    2. $M_{22} \longrightarrow \mathbf{U}$.

Semi-Deterministic BC with Cooperation - Achievability Outline

- **Rate Splitting:** $M_j = (M_{j0}, M_{jj})$, $j = 1, 2$:
  - (M_{10}, M_{20}) - Public message;
  - (M_{11}, M_{22}) - Private messages.

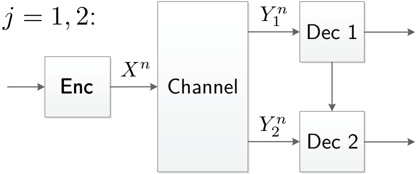


- **Codebook Structure:** Marton:
  - Public Message: $(M_{10}, M_{20}) \longrightarrow \mathbf{V}$.
  - Private Messages - Superposed on $\mathbf{V}$:
    1. $M_{11} \longrightarrow \mathbf{Y}_1$;
    2. $M_{22} \longrightarrow \mathbf{U}$.

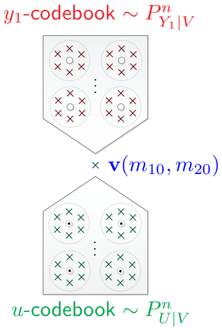


Semi-Deterministic BC with Cooperation - Achievability Outline

- **Rate Splitting:** $M_j = (M_{j0}, M_{jj})$, $j = 1, 2$:
  - (M_{10}, M_{20}) - Public message;
  - (M_{11}, M_{22}) - Private messages.



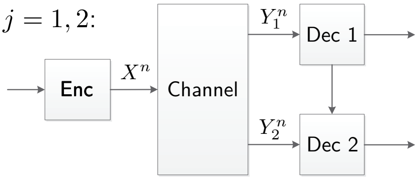
- **Codebook Structure:** Marton:
  - Public Message: $(M_{10}, M_{20}) \longrightarrow \mathbf{V}$.
  - Private Messages - Superposed on $\mathbf{V}$:
    1. $M_{11} \longrightarrow \mathbf{Y}_1$;
    2. $M_{22} \longrightarrow \mathbf{U}$.



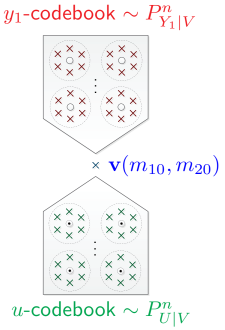
- **Decoding:** Joint typicality decoding.

Semi-Deterministic BC with Cooperation - Achievability Outline

- **Rate Splitting:** $M_j = (M_{j0}, M_{jj})$, $j = 1, 2$:
  - (M_{10}, M_{20}) - Public message;
  - (M_{11}, M_{22}) - Private messages.



- **Codebook Structure:** Marton:
  - Public Message: $(M_{10}, M_{20}) \longrightarrow \mathbf{V}$.
  - Private Messages - Superposed on $\mathbf{V}$:
    1. $M_{11} \longrightarrow \mathbf{Y}_1$;
    2. $M_{22} \longrightarrow \mathbf{U}$.

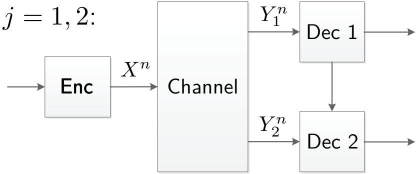


- **Decoding:** Joint typicality decoding.

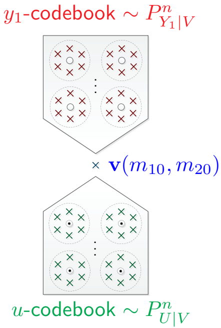
- **Cooperation:** Bin number of V^n - $2^{nR_{12}}$ bins.

Semi-Deterministic BC with Cooperation - Achievability Outline

- **Rate Splitting:** $M_j = (M_{j0}, M_{jj})$, $j = 1, 2$:
  - (M_{10}, M_{20}) - Public message;
  - (M_{11}, M_{22}) - Private messages.



- **Codebook Structure:** Marton:
  - Public Message: $(M_{10}, M_{20}) \longrightarrow \mathbf{V}$.
  - Private Messages - Superposed on $\mathbf{V}$:
    1. $M_{11} \longrightarrow \mathbf{Y}_1$;
    2. $M_{22} \longrightarrow \mathbf{U}$.



- **Decoding:** Joint typicality decoding.

- **Cooperation:** Bin number of V^n - $2^{nR_{12}}$ bins.

- **Gain:** Dec. 2 reduces search space of $\mathbf{V}$ by R_{12}.

Semi-Deterministic BC with Cooperation - Converse Outline

Via telescoping identities:

Semi-Deterministic BC with Cooperation - Converse Outline

Via telescoping identities:

1. Auxiliaries: $V_i = (M_{12}, Y_1^{i-1}, Y_{2,i+1}^n)$ and $U_i = M_2$.

Semi-Deterministic BC with Cooperation - Converse Outline

Via telescoping identities:

1. Auxiliaries: $V_i = (M_{12}, Y_1^{i-1}, Y_{2,i+1}^n)$ and $U_i = M_2$.

2. Telescoping identities [Kramer, 2011], e.g.,

Semi-Deterministic BC with Cooperation - Converse Outline

Via telescoping identities:

1. Auxiliaries: $V_i = (M_{12}, Y_1^{i-1}, Y_{2,i+1}^n)$ and $U_i = M_2$.

2. Telescoping identities [Kramer, 2011], e.g.,

$$H(M_2) - n\epsilon_n$$

Semi-Deterministic BC with Cooperation - Converse Outline

Via telescoping identities:

1. Auxiliaries: $V_i = (M_{12}, Y_1^{i-1}, Y_{2,i+1}^n)$ and $U_i = M_2$.

2. Telescoping identities [Kramer, 2011], e.g.,

$$H(M_2) - n\epsilon_n \leq I(M_2; Y_2^n | M_{12}) + I(M_2; M_{12})$$

Semi-Deterministic BC with Cooperation - Converse Outline

Via telescoping identities:

1. Auxiliaries: $V_i = (M_{12}, Y_1^{i-1}, Y_{2,i+1}^n)$ and $U_i = M_2$.

2. Telescoping identities [Kramer, 2011], e.g.,

$$H(M_2) - n\epsilon_n \leq \boldsymbol{I(M_2; Y_2^n | M_{12})} + I(M_2; M_{12})$$

Semi-Deterministic BC with Cooperation - Converse Outline

Via telescoping identities:

1. Auxiliaries: $V_i = (M_{12}, Y_1^{i-1}, Y_{2,i+1}^n)$ and $U_i = M_2$.

2. Telescoping identities [Kramer, 2011], e.g.,

$$H(M_2) - n\epsilon_n \leq \boldsymbol{I(M_2; Y_2^n | M_{12})} + I(M_2; M_{12})$$
$$= \sum_{i=1}^{n} \Big[\boldsymbol{I(M_2; Y_{2,i}^n | M_{12}, Y_1^{i-1}) - I(M_2; Y_{2,i+1}^n | M_{12}, Y_1^i)} \Big] + I(M_2; M_{12})$$

Semi-Deterministic BC with Cooperation - Converse Outline

Via telescoping identities:

1. Auxiliaries: $V_i = (M_{12}, Y_1^{i-1}, Y_{2,i+1}^n)$ and $U_i = M_2$.
2. Telescoping identities [Kramer, 2011], e.g.,

$$H(M_2) - n\epsilon_n \leq \boldsymbol{I(M_2; Y_2^n | M_{12})} + I(M_2; M_{12})$$

$$= \sum_{i=1}^{n} \left[I(M_2; Y_{2,i}^n | M_{12}, Y_1^{i-1}) - I(M_2; Y_{2,i+1}^n | M_{12}, Y_1^i) \right] + I(M_2; M_{12})$$

$$= \sum_{i=1}^{n} \left[I(M_2; Y_{2,i} | M_{12}, Y_1^{i-1}, Y_{2,i+1}^n) - I(M_2; Y_{1,i} | M_{12}, Y_1^{i-1}, Y_{2,i+1}^n) \right] + I(M_2; M_{12})$$

Semi-Deterministic BC with Cooperation - Converse Outline

Via telescoping identities:

1. Auxiliaries: $V_i = (M_{12}, Y_1^{i-1}, Y_{2,i+1}^n)$ and $U_i = M_2$.

2. Telescoping identities [Kramer, 2011], e.g.,

$$
\begin{aligned}
H(M_2) - n\epsilon_n &\leq \boldsymbol{I(M_2; Y_2^n | M_{12})} + I(M_2; M_{12}) \\
&= \sum_{i=1}^{n} \Big[\boldsymbol{I(M_2; Y_{2,i}^n | M_{12}, Y_1^{i-1}) - I(M_2; Y_{2,i+1}^n | M_{12}, Y_1^{i})} \Big] + I(M_2; M_{12}) \\
&= \sum_{i=1}^{n} \Big[\boldsymbol{I(M_2; Y_{2,i} | M_{12}, Y_1^{i-1}, Y_{2,i+1}^n) - I(M_2; Y_{1,i} | M_{12}, Y_1^{i-1}, Y_{2,i+1}^n)} \Big] \\
&\quad + I(M_2; M_{12})
\end{aligned}
$$

⋆ Replaces 2 uses of Csiszár Sum Identity.