Interference Mitigation in STAP Using the Two-Dimensional Wold Decomposition Model

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***Abstract*—We propose a novel parametric approach for modeling, estimation, and detection in space-time adaptive processing (STAP) radar systems. The proposed parametric interference mitigation procedures can be applied even when information in only a single range gate is available, thus achieving high performance gain when the data in the different range gates cannot be assumed stationary. The model is based on the Wold-like decomposition of two-dimensioanl (2-D) random fields. It is first shown that the same parametric model that results from the 2-D Wold-like orthogonal decomposition naturally arises as the physical model in the problem of space-time processing of airborne radar data. We exploit this correspondence to derive computationally efficient fully adaptive and partially adaptive detection algorithms. Having estimated the models of the noise and interference components of the field, the estimated parameters are substituted into the parametric expression of the interference-plus-noise covariance matrix. Hence, an estimate of the fully adaptive weight vector is obtained, and a corresponding test is derived. Moreover, we prove that it is sufficient to estimate only the spectral support parameters of each interference component in order to obtain a projection matrix onto the subspace orthogonal to the interference subspace. The resulting partially adaptive detector is simple to implement, as only a very small number of unknown parameters need to be estimated, rather than the field covariance matrix. The performance of the proposed methods is illustrated using numerical examples.**

***Index Terms*—Airborne radar, clutter, detection, evanescent fields, interference mitigation, jamming, STAP, two-dimensional random fields, Wold decomposition.**

I. Introduction

We PROPOSE a new approach for parametric modeling and estimation of space-time airborne radar data, based on the two-dimensional (2-D) Wold-like decomposition of random fields. Most interestingly, the proposed parametric estimation algorithms of the interference components provide new tools to estimate and mitigate the Doppler ambiguous clutter. The algorithms we develop enable estimation of the interference signals using the observations in *only* a single range gate. This property makes the proposed method particularly suitable for nonstationary clutter and jamming environments. Our modeling approach also provides a new analytical insight into the space-time adaptive processing (STAP) problem.

The goal of STAP is to manipulate the available data to achieve high gain at the target's angle and Doppler and maximal mitigation along both the jamming and clutter lines. Because the interference covariance matrix is unknown *a priori*, it is typically estimated using sample covariances obtained from averaging over a few range gates. Next, a weight vector is computed from the inverse of the sample covariance matrix [1]–[5]. It is shown in [6] that the dominant eigenvectors of the space-time covariance matrix contain all the information required to mitigate the interference. Thus, the weight vector is constrained to be in the subspace orthogonal to the dominant eigenvectors. In [8], a reduced-rank constant false alarm (CFAR) detection test is developed, assuming the dominant eigenvectors of the interference are known, and in [9], a multistage partially adaptive CFAR detection algorithm is introduced. In [17], an approach that bypasses the need to estimate the covariance matrix is presented: The data collected in a single range gate is employed to obtain a least-squares estimate of the signal power at each hypothesized direction of arrival, through evaluation of a weight vector constrained to null the unknown interference and noise. In [18], a simple *ad hoc* model of the clutter signal and covariance matrix is proposed. The model represents the spectral density of the clutter as a sum of Gaussian-shaped humps along the support of the clutter ridge. In [19], this model is employed to estimate the clutter covariance matrix from the data observed in a single range gate.

In this paper, we adopt the 2-D Wold-like decomposition of random fields [10] as the parametric model of the observed data. Employing this model, we derive computationally efficient algorithms useful for parametrically estimating both the jamming and clutter fields. The estimation procedure we propose is capable of estimating the interference parameters from the information in a single range gate. Hence, no averaging over a few range gates is required. This property provides significant advantage in the practical case where data in the different range gates is nonstationary. Having estimated the interference terms parametric models, their covariance matrix can be evaluated based on the estimated parameters. Moreover, the problem of evaluating the rank of the low-rank covariance matrix of the interference is solved as a byproduct of obtaining the parametric estimates of the interference components. Once the parametric models of the interference components have been estimated, several alternative detection procedures

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are available. In this paper, we present two such methods: the parametric fully-adaptive processing and the parametric partially-adaptive processing.

The paper is organized as follows. In Section II, we briefly summarize the main results of the 2-D Wold-like decomposition and the resulting random field model. Next, in Section III, the correspondence between this model and the physical model of the STAP data is identified. In Section IV, we elaborate on the parametric representation of the covariances of the different components of the random field. The estimation algorithm of the random field parametric model is presented and analyzed in Section V. After the method for estimating the parametric models of the different components of the data field has been established, we present the parametric fully adaptive processing method and the computationally more efficient parametric partially adaptive processing method in Sections VI and VII, respectively. The performance of both methods is illustrated using synthetic data examples. We summarize our conclusions in Section VIII.

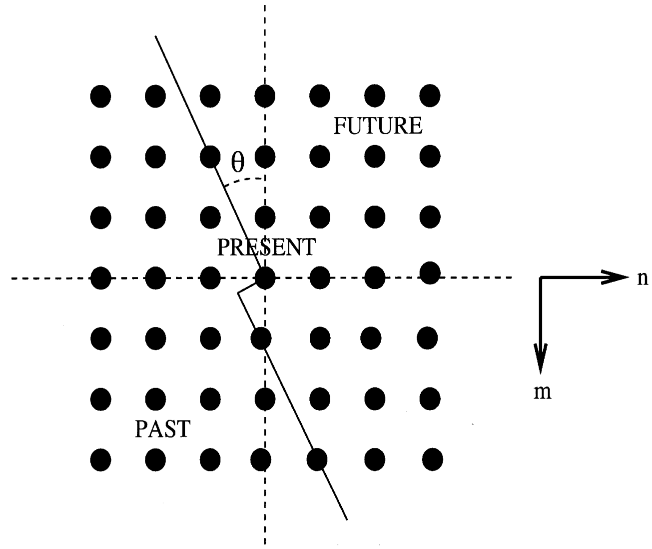

Fig. 1. Rational nonsymmetrical half-plane support; example with $a = 2$ and $b = -1$.

II. Random Field Model

In this section, we briefly review the 2-D Wold-like decomposition of random fields and the resulting random field model. In the next section, the applicability of this model to STAP data will be explained. It is shown in [10] that any 2-D regular and homogeneous discrete random field can be represented as a sum of two mutually orthogonal components: a *purely indeterministic* (unpredictable in the mean-square sense) field and a *deterministic* (predictable in the mean-square sense) one. The purely indeterministic component has a unique white innovations driven nonsymmetrical half-plane (NSHP) moving average representation. The deterministic component is further orthogonally decomposed into a *harmonic* field and a countable number of mutually orthogonal *evanescent* fields. This decomposition results in a corresponding decomposition of the spectral measure of the regular random field into a countable sum of mutually singular spectral measures. The purely indeterministic component has an absolutely continuous spectral distribution function. The spectral measure of the deterministic component is singular with respect to the Lebesgue measure, and therefore, it is concentrated on a set of Lebesgue measure zero in the frequency plane. It is shown in [12] that under some mild assumptions (that always hold in practice), the spectral supports of the different evanescent components have the form of lines whose slope is a rational number.

More specifically, let $\{y(n, m), (n, m) \in \mathcal{Z}^2\}$ be a complex valued, regular, homogeneous random field. Then, $y(n, m)$ can be uniquely represented by the orthogonal decomposition

$$y(n, m) = w(n, m) + v(n, m). \tag{1}$$

The field $\{v(n, m)\}$ is a deterministic random field. The field $\{w(n, m)\}$ is purely indeterministic and has a unique white innovations driven moving average representation, which is given by

$$w(n, m) = \sum_{(0,0) \preceq (k, \ell)} b(k, \ell) u(n-k, m-\ell) \tag{2}$$

where $\sum_{(0,0) \preceq (k,\ell)} b^2(k, \ell) < \infty$; $b(0, 0) = 1$, and $\{u(n, m)\}$ is the innovations field of $\{y(n, m)\}$. The notation $\preceq$ implies that the weighted summation includes $u(n, m)$ and all the samples in its "past," where the past is defined with respect to any selected NSHP total-ordering on the 2-D lattice (see, for example, Fig. 1).

We call a 2-D deterministic random field $[e_o(n, m)]$ *evanescent w.r.t. the NSHP total-order* o if it spans a Hilbert space identical to the one spanned by its *column-to-column innovations* at each coordinate (n, m) (w.r.t. the total-order o). The deterministic field column-to-column innovation at each coordinate $(n, m) \in \mathcal{Z}^2$ is defined as the difference between the actual value of the field and its projection on the Hilbert space spanned by the deterministic field samples in all previous columns.

It is possible to define [10] a family of NSHP total-order definitions such that the boundary line of the NSHP has a rational slope. A NSHP of this type is called *rational nonsymmetrical half-plane* (RNSHP), (see, for example, Fig. 1). Let a and b be two coprime integers, such that both $a, b \neq 0$. The slope of the RNSHP is then given by $-a/b$ (and $\cot\theta = -b/a$). For the case where $a = 0$, the RNSHP is uniquely defined by setting $b = 1$. (For the case where $b = 0$, the RNSHP is uniquely defined by setting $a = 1$.) We denote by O the set of all possible RNSHP definitions on the 2-D lattice (i.e., the set of all NSHP definitions in which the boundary line of the NSHP has a rational slope). The introduction of the family of RNSHP total-ordering definitions results in the following countably infinite orthogonal decomposition of the deterministic component of the random field:

$$v(n, m) = p(n, m) + \sum_{(a,b) \in O} e_{(a,b)}(n, m). \tag{3}$$

The random field $\{p(n, m)\}$ is *half-plane deterministic*, i.e., it has no column-to-column innovations w.r.t. any RNSHP total-ordering definition. The field $\{e_{(a,b)}(n, m)\}$ is the evanescent component that generates the column-to-column innovations of the deterministic field w.r.t. the RNSHP total-ordering definition $(a, b) \in O$.

Hence, if $\{y(n, m)\}$ is a 2-D regular and homogeneous random field, then $y(n, m)$ can be uniquely represented by the orthogonal decomposition

$$y(n, m) = w(n, m) + p(n, m) + \sum_{(a, b) \in O} e_{(a, b)}(n, m). \quad (4)$$

In the following, all spectral measures are defined on the square $K = [-1/2, 1/2] \times [-1/2, 1/2]$. It is shown in [10] and [11] that the spectral measures of the decomposition components in (4) are mutually singular. The spectral distribution function of the purely indeterministic component is absolutely continuous, whereas the spectral measures of the half-plane deterministic component and of all the evanescent components are concentrated on a set of Lebesgue measure zero in K. A model for the evanescent field that corresponds to the RNSHP defined by $(a, b) \in O$ is given by

$$\begin{aligned} e_{(a, b)}(n, m) &= \sum_{i=1}^{I^{(a, b)}} e_i^{(a, b)}(n, m) \\ &= \sum_{i=1}^{I^{(a, b)}} s_i^{(a, b)}(na + mb) \\ &\quad \cdot \exp\left(j2\pi\nu_i^{(a, b)}(nc + md)\right) \end{aligned} \quad (5)$$

where c and d are coprime integers satisfying $ad - bc = 1$. For the case where $(a, b) = (0, 1)$, we have $(c, d) = (1, 0)$, and for $(a, b) = (1, 0)$, we have $(c, d) = (0, 1)$. The 1-D purely indeterministic, complex-valued processes $\{s_i^{(a, b)}(na + mb)\}$ and $\{s_j^{(a, b)}(na + mb)\}$ are zero-mean and mutually orthogonal for all $i \neq j$. Hence, the "spectral density function" of each evanescent field has the form of a sum of 1-D delta functions that are supported on lines of rational slope in the 2-D spectral domain. The amplitude of each of these delta functions is determined by the spectral density of the 1-D modulating process. Since the spectral density of the modulating process can rapidly decay to zero, so will the "spectral density" of the evanescent field, hence, the name "evanescent." Since interchanging the roles of past and future in any total-order definition amounts to substituting $\nu_i^{(a, b)}$ by $-\nu_i^{(a, b)}$ in the model (5), we assume without limiting the generality of the derivation that $a \geq 0$, and b can assume any integer value.

One of the half-plane-deterministic field components, which is of prime importance in the STAP problem is the harmonic random field

$$h(n, m) = \sum_{p=1}^{P} C_p \exp\left(j2\pi(n\omega_p + m\nu_p)\right) \quad (6)$$

where the C_ps are mutually orthogonal random variables, and (ω_p, ν_p) are the spatial frequencies of the pth harmonic.

III. STAP Model and the 2-D Wold Decomposition

The random field parametric model that results from the 2-D Wold-like orthogonal decomposition naturally arises as the physical model in the problem of space-time processing of airborne radar data. Let n denote the sensor index, and let m be the time index. In the STAP problem, the target signal is modeled as a random amplitude complex exponential where the exponential is defined by a space-time steering vector that has the target's angle and Doppler. In other words, in the space-time domain the target model is that of a 2-D harmonic component similar to (6). The sum of the white noise field due to the internally generated receiver amplifier noise, and the colored noise field due to the sky noise contribution, is the purely indeterministic component of the space-time field decomposition. The presence of a jammer results in a barrage of noise localized in angle and uniformly distributed over all Doppler frequencies. Hence, in the space-time domain, each jammer is modeled as an evanescent component with $(a, b) = (0, 1)$ such that its 1-D modulating process $s_i^{(0, 1)}(m)$ is the random process of the jammer amplitudes. The jammer samples from different pulses are uncorrelated. In the angle-Doppler domain each jammer contributes a 1-D delta function, parallel to the Doppler axis and located at a specific angle $\nu_i^{(0, 1)}$ [using the notation of (5)]. The ground clutter results in an additional evanescent component of the observed 2-D space-time field. The clutter's echo from a single ground patch has a Doppler frequency that linearly depends on its aspect with respect to the platform. Hence, clutter from all angles lies in a "clutter ridge," which is supported on a diagonal line (that generally wraps around in Doppler) in the angle-Doppler domain. A model of the clutter field is then given by (5) with the slope of the clutter ridge given by b/a and with $s_i^{(a, b)}(na + mb)$ being a 1-D colored noise process. Since the rational numbers are dense in the set of real numbers, an irrational slope of the clutter ridge can be approximated arbitrarily close by a rational one. Hence, any clutter signal can be either exactly modeled or approximated by an evanescent field.

Fig. 2 graphically illustrates a typical example of the matching between the 2-D Wold decomposition based parametric random field model and the physical model of STAP data. In this synthetic example, the observed random field is the sum of two evanescent components that correspond to the clutter component with $(a, b) = (1, 2)$, $\nu^{(1, 2)} = 0$ and a jammer with $\nu^{(0, 1)} = 0.2$. Fig. 2 depicts the magnitude of the DFT of the observed field.

We therefore conclude that the foregoing derivation opens the way for new *parametric* solutions that can simplify and improve existing methods of STAP.

IV. Covariance Structure of the Observed Field

Based on the random field model derived in the previous sections, we derive in this section a closed-form parametric expression for the covariance matrix of the observed STAP data field in terms of the model parameters. We begin by stating our assumptions.

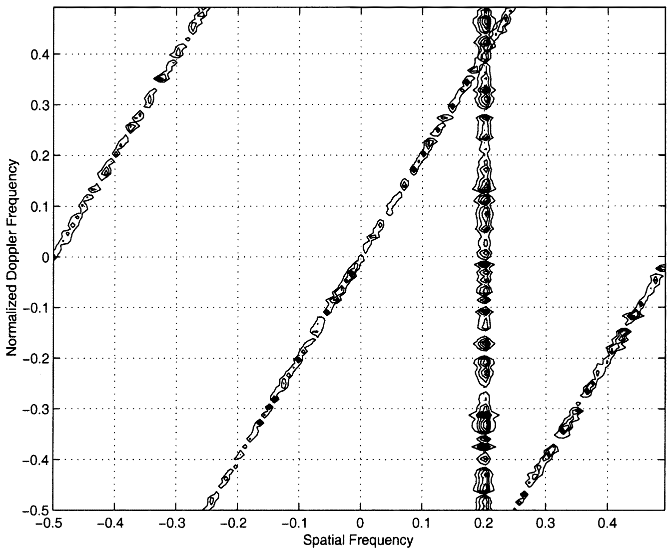

Fig. 2. Magnitude of the DFT of an observed field containing two evanescent components that correspond to a clutter component with $(a, b) = (1, 2)$, $\nu^{(1, 2)} = 0$, and a jammer with $\nu^{(0, 1)} = 0.2$.

Let $\{y(n, m)\}$, $(n, m) \in D$, where $D = \{(i, j) | 0 \le i \le S-1, 0 \le j \le T-1\}$, be the observed random field.

Assumption 1: The purely indeterministic component $\{w(n, m)\}$ is a zero mean circular complex valued random field.

Assumption 2: The number $I = \sum_{(a, b) \in O} I^{(a, b)}$ of evanescent components in the field is *a priori* known. This assumption can be later relaxed.

Assumption 3: For each evanescent field $\{e_i^{(a, b)}\}$, the modulating 1-D purely indeterministic process $\{s_i^{(a, b)}\}$ is a zero-mean circular complex valued process.

Let

$$\begin{aligned}\mathbf{y} = &[y(0,0), \ldots, y(0,T-1), y(1,0), \ldots, y(1,T-1) \\ &\ldots, \ldots, y(S-1,0), \ldots, y(S-1,T-1)]^T \qquad (7)\end{aligned}$$

$$\begin{aligned}\mathbf{w} = &[w(0,0), \ldots, w(0,T-1), w(1,0), \ldots, w(1,T-1) \\ &\ldots, \ldots, w(S-1,0), \ldots, w(S-1,T-1)]^T \qquad (8)\end{aligned}$$

$$\begin{aligned}\mathbf{e}_i^{(a,b)} = &\Big[e_i^{(a,b)}(0,0), \ldots, e_i^{(a,b)}(0,T-1), e_i^{(a,b)}(1,0) \\ &\ldots, e_i^{(a,b)}(1,T-1), \ldots, \ldots, e_i^{(a,b)}(S-1,0) \\ &\ldots, e_i^{(a,b)}(S-1,T-1)\Big]^T. \qquad (9)\end{aligned}$$

Let

$$\begin{aligned}\boldsymbol{\xi}_i^{(a, b)} = &\Big[s_i^{(a, b)}(0), s_i^{(a, b)}(b), \ldots, s_i^{(a, b)}((T-1)b) \\ &s_i^{(a, b)}(a), s_i^{(a, b)}(a+b), \ldots, s_i^{(a, b)}(a+(T-1)b) \\ &\ldots, s_i^{(a, b)}((S-1)a), s_i^{(a, b)}((S-1)a+b) \\ &\ldots, s_i^{(a, b)}((S-1)a+(T-1)b)\Big]^T \qquad (10)\end{aligned}$$

be the vector whose elements are the observed samples from the 1-D modulating process $\{s_i^{(a, b)}\}$. Define

$$\begin{aligned}\mathbf{v}^{(a, b)} = &[0, d, \ldots, (T-1)d, c, c+d, \ldots \\ &c+(T-1)d, \ldots, \ldots, (S-1)c, (S-1)c+d \\ &\ldots, (S-1)c+(T-1)d]^T. \qquad (11)\end{aligned}$$

Given a scalar function $f(v)$, we will denote the matrix, or column vector, consisting of the values of $f(v)$ evaluated for all the elements of $\mathbf{v}$, where $\mathbf{v}$ is a matrix, or a column vector, by $f(\mathbf{v})$. Using this notation, we define

$$\mathbf{d}_i^{(a, b)} = \exp\left(j2\pi\nu_i^{(a, b)}\mathbf{v}^{(a, b)}\right). \qquad (12)$$

Thus, using (5), we have that

$$\mathbf{e}_i^{(a, b)} = \boldsymbol{\xi}_i^{(a, b)} \odot \mathbf{d}_i^{(a, b)} \qquad (13)$$

where $\odot$ denotes an element-by-element product of the vectors.

Note that whenever $na + mb = ka + \ell b$ for some integers n, m, k, ℓ such that $0 \le n, k \le S-1$ and $0 \le m, \ell \le T-1$, the same sample from the modulating process $\{s_i^{(a, b)}\}$ is duplicated in the elements of $\boldsymbol{\xi}_i^{(a, b)}$. It is shown in [15] that for a rectangular observed field of dimensions $S \times T$, the number of *distinct* samples from the random process $\{s_i^{(a, b)}\}$ that are found in the observed field is

$$N_c = (S-1)|a| + (T-1)|b| + 1 - (|a|-1)(|b|-1). \qquad (14)$$

This is because N_c is the number of different "columns" one can define on such a rectangular lattice for a RNSHP defined by (a, b). We note here that in the special case where $a = 1$, (14) provides the well-known Brennan rule [3] on the rank of the clutter covariance matrix.

We therefore define the *concentrated version* $\mathbf{s}_i^{(a, b)}$ of $\boldsymbol{\xi}_i^{(a, b)}$ to be an N_c-dimensional column vector of nonrepeating samples of the process $\{s_i^{(a, b)}\}$. More specifically, for the case in which $a > 0$ and $b < 0$, $\mathbf{s}_i^{(a, b)}$ is given by

$$\mathbf{s}_i^{(a, b)} = \left[s_i^{(a, b)}((T-1)b), \ldots, \ldots, s_i^{(a, b)}((S-1)a)\right]^T \qquad (15)$$

whereas for the case in which $a \ge 0$ and $b \ge 0$, $\mathbf{s}_i^{(a, b)}$ is given by

$$\begin{aligned}\mathbf{s}_i^{(a, b)} = &\Big[s_i^{(a, b)}(0), \ldots \\ &\ldots, s_i^{(a, b)}((S-1)a + b(T-1))\Big]^T. \qquad (16)\end{aligned}$$

Thus, for any (a, b), we have that

$$\boldsymbol{\xi}_i^{(a, b)} = \mathbf{A}_i^{(a, b)}\mathbf{s}_i^{(a, b)} \qquad (17)$$

where $\mathbf{A}_i^{(a, b)}$ is rectangular matrix of zeros and ones that replicates rows of $\mathbf{s}_i^{(a, b)}$.

Note, however, that due to boundary effects, the vector $\mathbf{s}_i^{(a, b)}$ is not composed of consecutive samples from the process $\{s_i^{(a, b)}\}$ unless $|a| \le 1$ or $|b| \le 1$. In other words, for some arbitrary a and b, there are missing samples in $\mathbf{s}_i^{(a, b)}$.

We note that the covariance matrix $\mathbf{R}_{\mathbf{i}}^{(\mathbf{a},\,\mathbf{b})}$, which characterizes the second-order properties of the process $\{s_i^{(a,\,b)}\}$, is defined in terms of the concentrated version vector $\mathbf{s}_i^{(a,\,b)}$, i.e.,

$$\mathbf{R}_i^{(a,\,b)} = E\left[\mathbf{s}_i^{(a,\,b)}\left(\mathbf{s}_i^{(a,\,b)}\right)^H\right] \tag{18}$$

and not in terms of the covariance matrix

$$\tilde{\mathbf{R}}_i^{(a,\,b)} = E\left[\boldsymbol{\xi}_i^{(a,\,b)}\left(\boldsymbol{\xi}_i^{(a,\,b)}\right)^H\right] \tag{19}$$

of the vector $\boldsymbol{\xi}_i^{(a,\,b)}$. The matrix $\tilde{\mathbf{R}}_i^{(a,\,b)}$ is a singular matrix, where $\tilde{\mathbf{R}}_i^{(a,\,b)} = \mathbf{A}_i^{(a,\,b)}\mathbf{R}_i^{(a,\,b)}(\mathbf{A}_i^{(a,\,b)})^H$.

Since the evanescent components $\{e_i^{(a,\,b)}\}$ are mutually orthogonal and since all the evanescent components are orthogonal to the purely indeterministic component, we conclude that $\boldsymbol{\Gamma}$, which is the covariance matrix of $\mathbf{y}$, has the form

$$\boldsymbol{\Gamma} = \boldsymbol{\Gamma}_{\mathrm{PI}} + \sum_{(a,\,b)\in O}\sum_{i=1}^{I^{(a,\,b)}}\boldsymbol{\Gamma}_i^{(a,\,b)} \tag{20}$$

where $\boldsymbol{\Gamma}_i^{(a,\,b)}$ is the covariance matrix of $\mathbf{e}_i^{(a,\,b)}$.

Using (5) and (13), we find that

$$\boldsymbol{\Gamma}_i^{(a,\,b)} = \left(\mathbf{A}_i^{(a,\,b)}\mathbf{R}_i^{(a,\,b)}\left(\mathbf{A}_i^{(a,\,b)}\right)^T\right) \odot \left(\mathbf{d}_i^{(a,\,b)}\left(\mathbf{d}_i^{(a,\,b)}\right)^H\right). \tag{21}$$

A compact matrix representation of $\boldsymbol{\Gamma}_i^{(a,\,b)}$ for any $(a,\,b)$ cannot be derived due to the dependence of the matrix structure on $(a,\,b)$. However, for the case in which $(a,\,b) = (0,\,1)$ (and similarly for $(a,\,b) = (1,\,0)$), a somewhat more compact representation is possible, using Kronecker products instead of the Hadamard products.

More specifically, for this special case, (13) can be expressed in the form

$$\mathbf{e}_i^{(0,\,1)} = \mathbf{d}_i^{(0,\,1)} \otimes \mathbf{s}_i^{(0,\,1)} \tag{22}$$

where $\otimes$ is the Kronecker product. Hence

$$\begin{aligned}\boldsymbol{\Gamma}_i^{(0,\,1)} &= \left(\mathbf{d}_i^{(0,\,1)}\left(\mathbf{d}_i^{(0,\,1)}\right)^H\right) \otimes \mathbf{R}_i^{(0,\,1)} \\ &= \mathbf{E}_i^{(0,\,1)} \otimes \mathbf{R}_i^{(0,\,1)}\end{aligned} \tag{23}$$

where $\mathbf{R}_i^{(0,\,1)}$ and $\mathbf{E}_i^{(0,\,1)}$ are Toeplitz matrices, given by (24) and (25), as shown at the bottom of the page.

V. Parametric Estimation of the Interference Components

In this section, we derive a computationally efficient algorithm for estimating both the jamming and clutter fields, based on the above results. More specifically, for each interference component of the observed field, we estimate its spectral support parameters a, b, $\nu^{(a,\,b)}$ as well as c, d and the parametric model of the modulating 1-D purely indeterministic process $\{s_i^{(a,\,b)}\}$. In the setting of the radar problem considered here, partial information on the different components of the field is *a priori* known: The jamming signals are localized in angle and distributed over all Doppler frequencies. Thus, each jammer contributes an evanescent component with spectral support parameters $(a,\,b) = (0,\,1)$ and an unknown frequency $\nu_i^{(0,\,1)}$. The clutter signal is also modeled as an evanescent component with $\nu^{(a,\,b)} = 0$ and an unknown $(a,\,b)$ pair, which is uniquely determined by the platform motion parameters.

The proposed estimation algorithm of the spectral support parameters of the evanescent field a, b and $\nu_i^{(a,\,b)}$ is based on the following lemma.

Lemma 1: Let $\{e_i^{(a,\,b)}(n,\,m)\}$ be an evanescent field and let k be an integer. The samples of the evanescent field along a line

$$\mathbf{R}_i^{(0,\,1)} = \begin{bmatrix} r_i^{(0,1)}(0) & r_i^{(0,1)}(-1) & \cdots & r_i^{(0,1)}(-(T-1)) \\ r_i^{(0,1)}(1) & r_i^{(0,1)}(0) & \cdots & r_i^{(0,1)}(-(T-2)) \\ \vdots & \ddots & \ddots & \vdots \\ \vdots & & & r_i^{(0,1)}(-1) \\ r_i^{(0,1)}(T-1) & r_i^{(0,1)}(T-2) & \cdots & r_i^{(0,1)}(0) \end{bmatrix} \tag{24}$$

$$\mathbf{E}_i^{(0,\,1)} = \begin{bmatrix} 1 & \exp\left(-j2\pi\nu_i^{(0,1)}\right) & \cdots & \exp\left(-j2\pi(S-1)\nu_i^{(0,1)}\right) \\ \exp\left(j2\pi\nu_i^{(0,1)}\right) & 1 & \cdots & \exp\left(-j2\pi(S-2)\nu_i^{(0,1)}\right) \\ \vdots & \ddots & \ddots & \vdots \\ \vdots & & & \exp\left(-j2\pi\nu_i^{(0,1)}\right) \\ \exp\left(j2\pi(S-1)\nu_i^{(0,1)}\right) & \exp\left(j2\pi(S-2)\nu_i^{(0,1)}\right) & \cdots & 1 \end{bmatrix} \tag{25}$$

on the sampling grid defined by $k = na + mb$ are the samples of a 1-D constant amplitude harmonic signal, whose frequency is $\nu_i^{(a,b)}$.

Proof: Since for fixed a, b, k, $k = na + mb$ is the linear Diophantine equation (see the Appendix), its solutions are given by

$$n = n_k + tb \tag{26}$$

$$m = m_k - ta \tag{27}$$

where (n_k, m_k) is a solution of the equation, and t is an integer such that the sequence of consecutive values of t corresponds to the different lattice point on the line $k = na + mb$. From (5), we have, for the evanescent field samples along the line $k = na + mb$

$$\begin{aligned} e_i^{(a,b)}(n, m) &= s_i^{(a,b)}(na + mb) \exp\left(j2\pi\nu_i^{(a,b)}(nc + md)\right) \\ &= s_i^{(a,b)}(k) \exp\left(j2\pi\nu_i^{(a,b)}(n_k c + m_k d - t(ad - bc))\right) \\ &= \left[s_i^{(a,b)}(k) \exp\left(j2\pi\nu_i^{(a,b)}(n_k c + m_k d)\right)\right] \\ &\quad \cdot \exp\left(-j2\pi\nu_i^{(a,b)} t\right) \end{aligned} \tag{28}$$

where the last equality is because c, d are coprime integers such that $ad - bc = 1$. Hence, in each realization and for a fixed k, $s_i^{(a,b)}(k) \exp(j2\pi\nu_i^{(a,b)}(n_k c + m_k d))$ is a (random) constant. Hence, the proof follows. ■

The algorithm is implemented by the following four-step procedure:

Initial estimation of a and b: In the presence of an evanescent component, the peaks of the observed field periodogram are concentrated along a straight line such that its slope is defined by the two coprime integers a and b. Hence, several alternative approaches for obtaining an initial estimate of the spectral support parameters of the evanescent component can be derived by taking the Radon or Hough transforms [20] of the observed field periodogram. (The current implementation employs the Hough transform for detecting straight lines in 2-D arrays). However, due to noise presence, this estimate may perturb. Since, on a finite-dimension observed field, only a finite number of possible (a, b) pairs may be defined, the output of the initial stage is a set of possible (a, b) pairs such that the ratio b/a is close to the ratio obtained for the (a, b) pair estimated by the Hough transform.

Estimation of the frequency parameter of the evanescent component: For each possible (a, b) pair, we next evaluate the frequency parameter of the evanescent component $\nu_i^{(a,b)}$. Assuming the considered (a, b) pair is the correct one, we have, from Lemma 1, that in the absence of background noise for a fixed $k = na + mb$ (i.e., along a line on the sampling grid), the samples of the evanescent component are the samples of a 1-D constant amplitude harmonic signal, whose frequency is $\nu_i^{(a,b)}$. Hence, by considering the samples along such a line, we obtain samples of a 1-D constant amplitude harmonic signal whose frequency $\nu_i^{(a,b)}$ can be easily estimated using any standard frequency estimation algorithm (e.g., the 1-D DFT).

Final estimation of the spectral support parameters of each evanescent component: The test for detecting the correct (a, b) and $\nu_i^{(a,b)}$ is then based on multiplying the observed signal $y(n, m)$ by $\exp(-j2\pi\hat{\nu}_i^{(\hat{a},\hat{b})}(n\hat{c} + m\hat{d}))$ for each of the considered $\hat{a}, \hat{b}$ and $\hat{\nu}_i^{(a,b)}$ triplets and evaluating the variance of this signal along a line on the sampling grid such that $k = n\hat{a} + m\hat{b}$. Clearly, the best estimate of a, b, and $\nu_i^{(a,b)}$ is the one that results in minimal variance for the 1-D sequence because in the absence of noise, the correct a, b, and $\nu_i^{(a,b)}$ result in a zero variance. Note that $\hat{c}, \hat{d}$ are two coprime integers satisfying the linear Diophantine equation $ad - bc = 1$ when a, b are replaced by their estimated values. Clearly, $\hat{c}, \hat{d}$ obtained as solutions to the linear Diophantine equation are not unique (see the Appendix). The correct pair $\hat{c}, \hat{d}$ is then determined by employing the symmetry properties of the field covariance sequence (see [12] for details). Since, in the STAP problem, it is *a priori* known that for the jammers $(a, b) = (0, 1)$, whereas for the clutter $\nu^{(a,b)} = 0$, the parameters c, d do not appear in the model and, hence, need not be estimated. Nevertheless, to maintain the generality of the algorithm description, we proceed for the final step of the algorithm with the general description, assuming $\hat{c}, \hat{d}$ have been estimated (or are *a-priori* known as in the STAP case).

Estimating the model of the 1-D purely indeterministic modulating process of the evanescent field: Having estimated the spectral support parameters of each evanescent component, we take the approach of first estimating a *nonparametric* representation of its 1-D purely indeterministic modulating process $\{s_i^{(a,b)}\}$, and only at a second stage do we estimate the parametric models of these processes. Hence, in the first stage, we estimate the particular values that the vectors $\boldsymbol{\xi}_i^{(a,b)}$ take for the given realization, i.e., we treat these as unknown constants. The estimation procedure is implemented as follows: Multiplying the observed signal $y(n, m)$ by $\exp(-j2\pi\hat{\nu}_i^{(\hat{a},\hat{b})}(n\hat{c} + m\hat{d}))$ and evaluating the arithmetic mean of this signal along a line on the sampling grid such that $k = n\hat{a} + m\hat{b}$, we have

$$\hat{s}_i^{(a,b)}(k) = \frac{1}{N_s} \sum_{n\hat{a}+m\hat{b}=k} y(n, m) \cdot \exp(-j2\pi\hat{\nu}_i^{(a,b)}(n\hat{c} + m\hat{d})) \tag{29}$$

where N_s denotes the number of the observed field samples that satisfy the relation $n\hat{a}+m\hat{b} = k$. Once we obtained the sequence of estimated samples from the 1-D modulating process $\{s_i^{(a,b)}\}$, the problem of estimating its parametric model becomes entirely a 1-D estimation problem. Assuming the modulating process is an autoregressive (AR) process and applying to the sequence an AR estimation algorithm (see, e.g., [21]), we obtain estimates of the modulating process parameters as well.

Finally, it is important to note that we solve the difficult problem of evaluating the rank of the low-rank covariance matrix of the interference as a byproduct of obtaining the parametric estimates of the interference components: Denote the number of evanescent components (interference sources) of the field by Q. It is then shown in [16] that the rank of the interference covariance matrix is given by $S\sum_{k=1}^{Q}|a_k| + T\sum_{k=1}^{Q}|b_k| - \sum_{k=1}^{Q}|a_k|\sum_{k=1}^{Q}|b_k|$. In fact, the special case where $Q = 1$ and $a = 1$ is the well-known Brennan rule [3] on the rank of the clutter covariance matrix. Hence, following the estimation of the spectral support parameters of the different evanescent components, the rank of the interference covariance matrix is also determined.

VI. Parametric Fully Adaptive Processing

Having estimated the parametric models of the purely indeterministic and evanescent components of the field, the estimated parameters can be substituted into (20) and (21) to obtain an estimate of the interference-plus-noise covariance matrix $\mathbf{\Gamma}$. In this section, we show how the estimated interference-plus-noise covariance matrix is employed to obtain a fully adaptive space-time filter.

Let $\mathbf{v}_t$ denote the target steering vector given by

$$\mathbf{v}_t = \mathbf{b}(\varpi_t) \otimes \mathbf{a}(\vartheta_t). \tag{30}$$

Assuming a linear, uniformly spaced, sensor array and a uniform coherent-processing interval (CPI) are employed in our model, the spatial steering vector $\mathbf{a}(\vartheta)$ and the temporal steering vector $\mathbf{b}(\varpi)$ are given by

$$\mathbf{a}(\vartheta) = [1,\, e^{j2\pi\vartheta},\, \ldots,\, e^{j2\pi(T-1)\vartheta}]^T$$
$$\mathbf{b}(\varpi) = [1,\, e^{j2\pi\varpi},\, \ldots,\, e^{j2\pi(S-1)\varpi}]^T$$

respectively. Assume for the moment that only a single target may exist in the observed data and that both the target's steering vector and the interference-plus-noise covariance matrix $\mathbf{\Gamma}$ are known. We next derive a fully adaptive detection algorithm based on the generalized likelihood ratio test (GLRT). Since $\mathbf{v}_t$ and $\mathbf{\Gamma}$ are assumed known, the GLR has to be maximized only with respect to C_t, which is the unknown amplitude parameter of the target. Thus, the GLR has the form

$$\Lambda = \frac{\max_{C_t} p_{\mathbf{y}|\mathcal{H}_1}(\mathbf{y};\, C_t|\mathcal{H}_1)}{p_{\mathbf{y}|\mathcal{H}_0}(\mathbf{y}|\mathcal{H}_0)}. \tag{31}$$

Following a standard procedure (see, e.g., [7] and [9]), the GLR test statistic, which we denote by $|z(\varpi,\, \vartheta)|^2$, can be shown to have the equivalent form

$$|z(\varpi_t,\, \vartheta_t)|^2 = \frac{|\mathbf{v}_t^H \mathbf{\Gamma}^{-1}\mathbf{y}|^2}{\mathbf{v}_t^H \mathbf{\Gamma}^{-1}\mathbf{v}_t}. \tag{32}$$

Let $\mathbf{\Psi} = \mathbf{\Gamma}^{-1}\mathbf{y}$. We thus have

$$|z(\varpi_t,\, \vartheta_t)|^2 = \frac{|\mathbf{v}_t^H \mathbf{\Psi}|^2}{\mathbf{v}_t^H \mathbf{\Gamma}^{-1}\mathbf{v}_t} = \frac{|\mathbf{b}^H(\varpi_t) \otimes \mathbf{a}^H(\vartheta_t)\mathbf{\Psi}|^2}{\mathbf{v}_t^H \mathbf{\Gamma}^{-1}\mathbf{v}_t}. \tag{33}$$

Reorganizing the elements of $\mathbf{\Psi}$ into a $S \times T$ matrix $\mathbf{\Upsilon}$ where the elements of the kth row of $\mathbf{\Upsilon}$ are $\mathbf{\Psi}((k-1)T+1) \cdots \mathbf{\Psi}(kT)$, we conclude that for a linear, uniformly spaced, sensor array and uniform CPI

$$\mathbf{b}^H(\varpi) \otimes \mathbf{a}^H(\vartheta)\mathbf{\Psi} = \sum_{p=1}^{S}\sum_{q=1}^{T} e^{-j2\pi(p-1)\varpi} e^{-j2\pi(q-1)\vartheta}\mathbf{\Upsilon}(p,\, q). \tag{34}$$

Thus, $\mathbf{b}^H(\varpi) \otimes \mathbf{a}^H(\vartheta)\mathbf{\Psi}$ and $\mathbf{\Upsilon}$ are a 2-D DFT pair. However, since in fact the steering vector is unknown, the detector must first estimate the frequency where the magnitude of the 2-D DFT of $\mathbf{\Upsilon}$ is maximal, followed by comparison of the value of the test statistic evaluated at this frequency against the threshold. Thus, the GLRT when $\mathbf{\Gamma}$ is perfectly known is given by

$$\max_{\varpi,\vartheta} |z(\varpi,\, \vartheta)|^2 \gtrless \gamma. \tag{35}$$

In other words, in the case of a known covariance matrix, the test is equivalent to finding the 2-D frequency where the magnitude of the 2-D DFT of $\mathbf{\Upsilon}$ is maximal, followed by comparison of the value of the test statistic at this frequency against the threshold.

Note that under both the null hypothesis (no target) $\mathcal{H}_0$ as well as under the alternative hypothesis $\mathcal{H}_1$, $\mathbf{b}^H(\varpi) \otimes \mathbf{a}^H(\vartheta)\mathbf{\Psi}$ is a Gaussian random variable, being a linear transformation of a Gaussian random vector. Assuming $\mathbf{\Gamma}$ is perfectly known, it is not difficult to show [13] that after prewhitening by $\mathbf{\Gamma}^{-(1/2)}$, the probability density function of the GLRT in (35) is χ^2 distributed with two degrees of freedom under $\mathcal{H}_0$ and noncentral χ^2 with two degrees of freedom under $\mathcal{H}_1$.

Finally, since $\mathbf{\Gamma}$ is also unknown, we adopt an approach similar to that employed in the derivation of the adaptive match filter (AMF) in [7] and substitute the unknown covariance matrix with its estimate, which is obtained as explained in the previous sections.

To illustrate the operation of the proposed solution, we resort to numerical evaluation of some specific examples (see [13] for a detailed performance analysis and additional examples). Consider a 2-D observed random field consisting of a sum of a purely indeterministic component (background noise), a single evanescent (interference) component, and three harmonic components (targets). The purely indeterministic component is a complex valued circular Gaussian white noise field. The evanescent component spectral support parameters are $(a,\, b) = (1,\, 2)$, $\nu^{(1,\,2)} = 0$. The modulating 1-D purely indeterministic process of this evanescent component is a first-order Gaussian AR process, with driving noise variance $(\sigma^{(1,\,2)})^2 = 2$ and $a^{(1,\,2)}(1) = -0.5$. There are three targets that are located at $(0.05,\, 0)$, $(0.15,\, 0.15)$, and $(-0.25,\, 0.15)$, respectively. The observed field dimensions are 48×48.

Let us define the power of each of the field components as $E_w = \mathbf{w}^H\mathbf{w}$ for the purely indeterministic component; $E_e = (\mathbf{e}^{(a,\,b)})^H\mathbf{e}^{(a,\,b)}$ for the evanescent component; and $E_{h_k} = \mathbf{h}_k^H\mathbf{h}_k$, $k = 1,\, 2,\, 3$ for each of the harmonic components, where $\mathbf{h}_k$ is defined in the same way $\mathbf{w}$ and $\mathbf{e}^{(a,\,b)}$ are defined. In this example, we have $E_e/E_w = 6\,\text{dB}$, whereas for the three targets, we have $E_{h_1}/E_w = -12.8\,\text{dB}$, $E_{h_2}/E_w = -14.5\,\text{dB}$, $E_{h_3}/E_w = -15\,\text{dB}$. Due to the strong interference component, the presence of the three targets is hard to detect in the observed data whose power spectral density is depicted in Fig. 3. However, these targets are easily detected by the test statistic $|z(\varpi,\, \vartheta)|$, depicted in Fig. 4. In Fig. 4, $|z(\varpi,\, \vartheta)|$ is depicted as a function of the 2-D frequencies, i.e., angle and Doppler.

VII. Parametric Partially Adaptive Processing

The low rank of the interference covariance matrix is exploited in the partially adaptive STAP to significantly reduce the adaptive problem dimensionality. In this section, we derive a partially adaptive processing algorithm, based on the estimated parametric model of the interference. Moreover, it is proved in

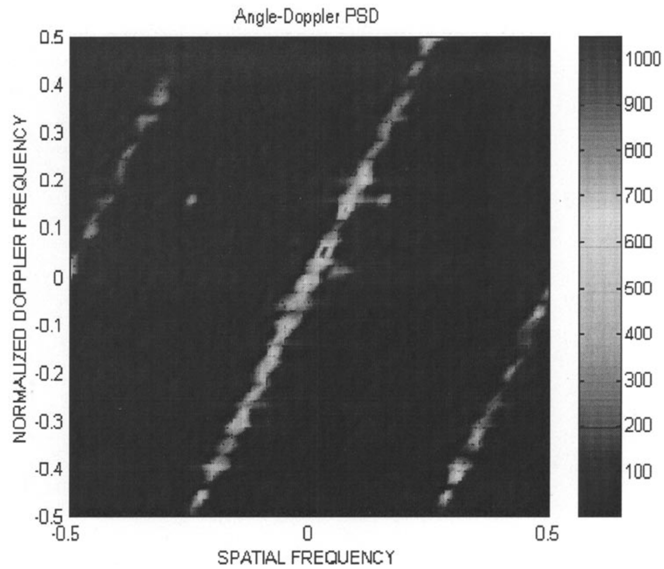

Fig. 3. Power spectral density of the observed field.

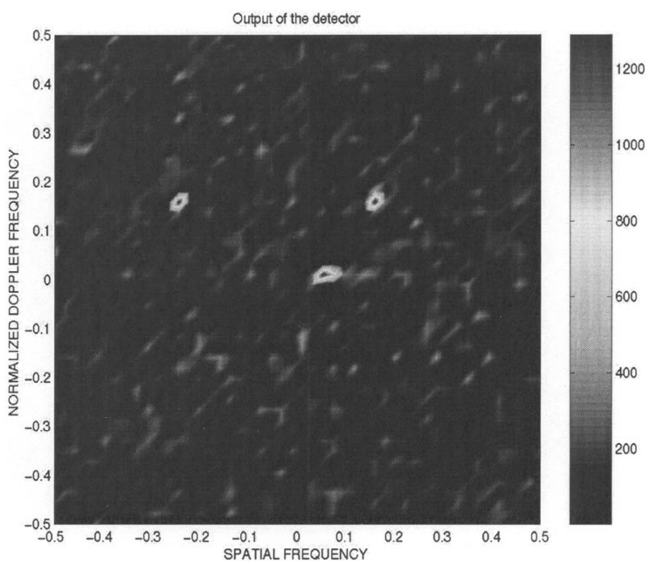

Fig. 4. Test statistic $|z(\varpi, \vartheta)|$.

this section that in order to implement the proposed partially adaptive processing method, *only* the spectral support parameters of the interference need to be estimated, and there is no need whatsoever to estimate the modulating process of the interference model, nor the data covariance matrix.

More specifically, recall that

$$\boldsymbol{\Gamma}_i^{(a,b)} = \left(\mathbf{A}_i^{(a,b)} \mathbf{R}_i^{(a,b)} \left(\mathbf{A}_i^{(a,b)}\right)^T\right) \odot \left(\mathbf{d}_i^{(a,b)} \left(\mathbf{d}_i^{(a,b)}\right)^H\right). \quad (36)$$

Having estimated a, b and $\nu_i^{(a,b)}$ using the algorithm in Section V, the vector $\mathbf{d}_i^{(a,b)}$ is known. Hence, demodulating $\mathbf{e}_i^{(a,b)}$, we conclude using (13) that

$$\boldsymbol{\xi}_i^{(a,b)} = \mathbf{e}_i^{(a,b)} \odot \left((\mathbf{d}_i^{(a,b)})^H\right)^T. \quad (37)$$

However, from (17), we conclude that the covariance matrix of $\boldsymbol{\xi}_i^{(a,b)}$ is given by

$$\tilde{\mathbf{R}}_i^{(a,b)} = \mathbf{A}_i^{(a,b)} \mathbf{R}_i^{(a,b)} \left(\mathbf{A}_i^{(a,b)}\right)^T. \quad (38)$$

In the following, we prove that since a and b are already known, an orthogonal projection matrix onto the low-rank subspace spanned by the evanescent field covariance matrix can be found *without* estimating the parametric model of the evanescent field 1-D modulating process and, hence, without estimating $\mathbf{R}_i^{(a,b)}$. Moreover, this result enables us to avoid the need in both evaluating the field covariance matrix and in employing a computationally intensive eigenanalysis to the estimated covariance matrix. More specifically, let us construct the following orthogonal projection matrix:

$$\mathbf{T}_i^{(a,b)} = \mathbf{A}_i^{(a,b)} \left(\left(\mathbf{A}_i^{(a,b)}\right)^T \mathbf{A}_i^{(a,b)}\right)^{-1} \left(\mathbf{A}_i^{(a,b)}\right)^T. \quad (39)$$

It is easily verified (by substitution) that $\mathbf{T}_i^{(a,b)}$ is an orthogonal projection onto the range space of $\tilde{\mathbf{R}}_i^{(a,b)}$ since for any ST-dimensional vector $\mathbf{v}$

$$\tilde{\mathbf{R}}_i^{(a,b)} \mathbf{v} = \tilde{\mathbf{R}}_i^{(a,b)} \mathbf{T}_i^{(a,b)} \mathbf{v} = \mathbf{T}_i^{(a,b)} \tilde{\mathbf{R}}_i^{(a,b)} \mathbf{v}. \quad (40)$$

In addition, $(\mathbf{T}_i^{(a,b)})^2 = \mathbf{T}_i^{(a,b)}$, and $(\mathbf{T}_i^{(a,b)})^T = \mathbf{T}_i^{(a,b)}$.

Note that since $\mathbf{A}_i^{(a,b)}$ is a sparse matrix of zeros and ones *only*, the computation of $\mathbf{T}_i^{(a,b)}$ is very simple. The projection matrix onto the subspace orthogonal to the interference space is therefore given by $(\mathbf{T}_i^{(a,b)})^\perp = \mathbf{I} - \mathbf{T}_\mathbf{i}^{(\mathbf{a},\mathbf{b})}$. Hence, by projecting the demodulated observed data vector $\bar{\mathbf{y}} = \mathbf{y} \odot ((\mathbf{d}_i^{(a,b)})^H)^T$ onto the subspace orthogonal to the interference subspace, a reduced-dimension data vector given by $\tilde{\mathbf{y}} = (\mathbf{T}_i^{(a,b)})^\perp \bar{\mathbf{y}}$ is obtained, such that the interference contribution to the observed signal is mitigated. Remodulating $\tilde{\mathbf{y}}$ by evaluating $\tilde{\mathbf{y}} \odot \mathbf{d}_i^{(a,b)}$, followed by sequentially applying this procedure to mitigate each of the interference sources, the detection problem is reduced to that of detecting a target in the presence of background noise only. Following a similar derivation to the one in (31)–(35), we conclude that in the special case where the background noise is known to be a white noise field, the statistical test is obtained by finding the 2-D frequency where the magnitude of the 2-D DFT of the processed data vector (organized back into a 2-D array) is maximal, followed by comparison of the value of the test statistic at this frequency against the threshold. In the more general case, where the purely indeterministic component of the field is not a white noise field, the observed data vector is first prewhitened by the estimated $\boldsymbol{\Gamma}_{PI}^{-(1/2)}$. It is shown in [13] that the probability density function of the GLR test that upper bounds the performance of the actual detector is χ^2 with two degrees of freedom under $\mathcal{H}_0$ and noncentral χ^2 with two degrees of freedom under $\mathcal{H}_1$.

As an example, consider the same field as in the previous section. Due to the strong interference component, the presence of the three targets is difficult to detect in the observed data, whose power spectral density is depicted in Fig. 3. However, these targets are easily detected in the processed data, as illustrated in Fig. 5. This result is obtained without estimating the parametric model of the evanescent field 1-D modulating process

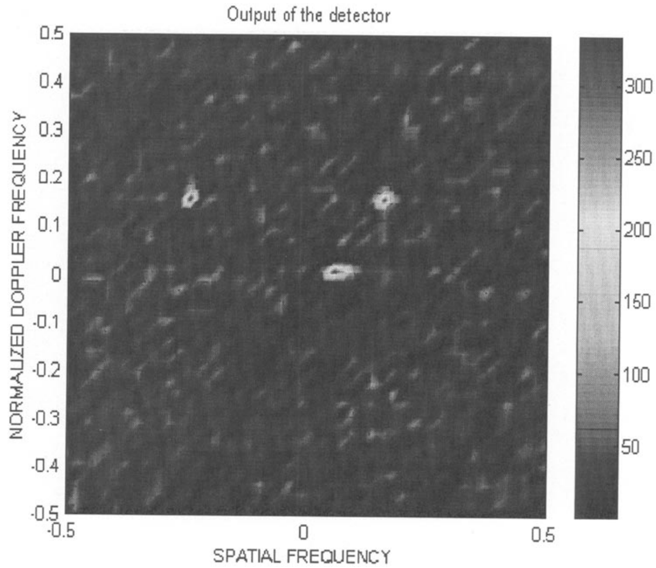

Fig. 5. Test statistic of the parametric partially adaptive processor. The power spectral density of the field after being projected onto the subspace orthogonal to the interference subspace.

and, hence, without estimating the interference-plus-noise covariance matrix. Since both the estimation of the interference-plus-noise covariance matrix, as well as its analysis, are saved, the proposed parametric partially adaptive processing method is robust and computationally attractive (see [13] for a detailed performance analysis and additional examples).

VIII. Conclusions

In this paper, a novel parametric approach for modeling, estimation, and target detection for STAP data has been derived. The proposed parametric interference mitigation procedures employ the information in only a single range gate, thus achieving high performance gain when the data in the different range gates cannot be assumed stationary. The model is based on the results of the 2-D Wold-like decomposition. We showed that the same parametric model that results from the 2-D Wold-like orthogonal decomposition naturally arises as the physical model in the problem of space-time processing of airborne radar data. We exploited this correspondence to derive computationally efficient fully adaptive and partially adaptive detection algorithms. Having estimated the models of the noise and interference components of the field, the estimated parameters are substituted into the parametric expression of the covariance matrix to obtain an estimate of the interference-plus-noise covariance matrix. Hence, the fully adaptive weight vector is obtained, and a corresponding test is derived. Moreover, we proved that it is sufficient to estimate only the spectral support parameters of each interference component in order to obtain a projection matrix onto the subspace orthogonal to the interference subspace. Thus, the resulting detector is statistically superior to the fully adaptive detector as considerably fewer parameters need to be estimated. Since a much smaller number of parameters need to be estimated the proposed partially adaptive detector is also computationally much simpler. Statistical analysis of the performance of the proposed detectors is considered in [13].

Appendix
Linear Diophantine Equation

Let k and ℓ be two nonzero integers and p some other integer. The equation

$$kx - \ell y = p$$

is called the *linear Diophantine equation*. A *solution* of this equation is a pair (x, y) of integers (a *lattice point* in the plane) that satisfies the equation. We use the following well known theorem (e.g., see [23])

Theorem 1: The linear Diophantine equation

$$kx - \ell y = p$$

has a solution if and only if $q \mid p$, (i.e., q divides p), where $q = g.c.d.(k, \ell)$. Furthermore, if (x_0, y_0) is a solution of this equation, then the set of solutions of the equation consists of all integer pairs (x, y) of the form

$$x = x_0 + t\frac{\ell}{q} \quad \text{and} \quad y = y_0 + t\frac{k}{q}, \quad t \in \mathcal{Z}. \tag{41}$$

Note that if k and ℓ are coprime, then there will always be solutions, given by (41).

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