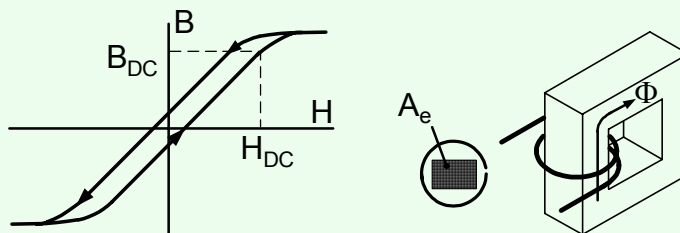


Magnetics Design

- 3.1 Important magnetic equations
- 3.2 Magnetic losses
- 3.3 Transformer
 - 3.3.1 Ideal transformer (voltages and currents)
 - 3.3.2 Equivalent circuit of transformer (coupling, magnetization current)
 - 3.3.3 Design of transformer
- 3.4 Inductor design

Faraday's law



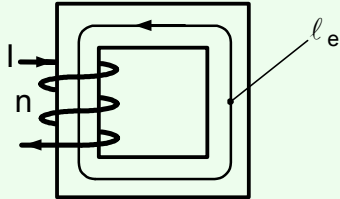
$$V = n \frac{d\Phi}{dt} = n A_e \frac{dB}{dt}; \quad \mu = \frac{\Delta B}{\Delta H}$$

Φ - magnetic flux Weber [Wb] V - voltage [V]

B - flux density $\frac{\text{Wb}}{\text{m}^2} = \text{Tesla [T]}$

Also : Gauss [G] 1T = 10,000 G

Ampere's law



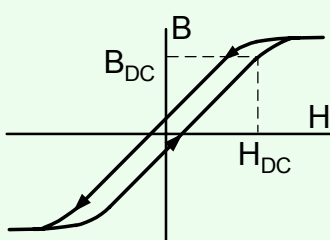
H - magnetic field [A/m]

$$\oint H dl = n \cdot I$$

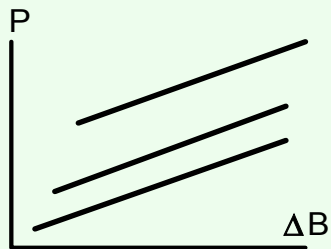
$$n \cdot I = H \cdot l_e$$

$$H = \frac{n \cdot I}{l_e} \text{ [A/m]}$$

Magnetic losses



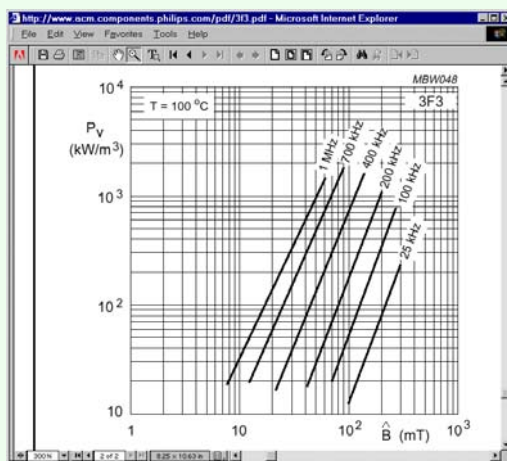
$\frac{\text{mW}}{\text{cm}^3}$



Magnetic losses $\sim \Delta B$

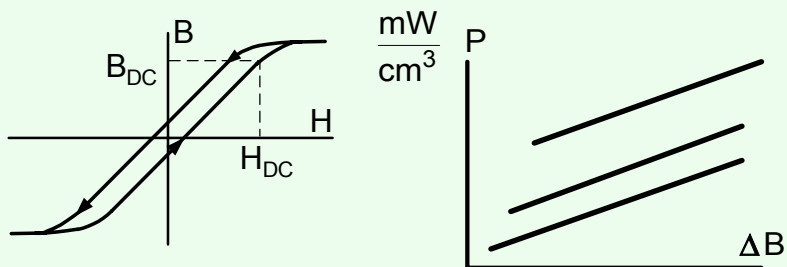
“Good number” = $100 \text{mW/cm}^3 = 100 \text{KW/m}^3$

Magnetic Losses



- “Good number” = 100mW/cm³ = 100 kW/m³

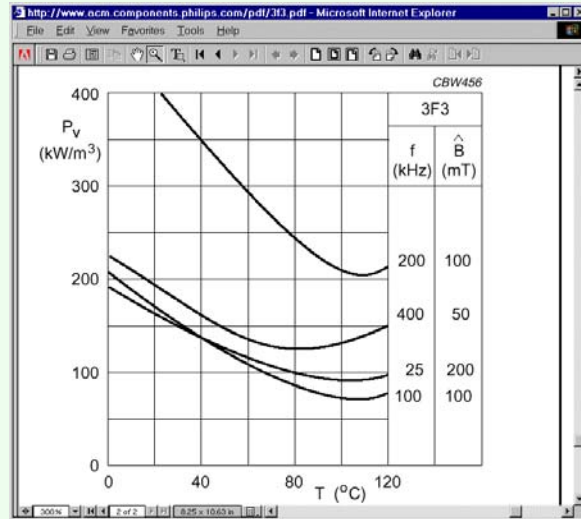
Magnetic losses



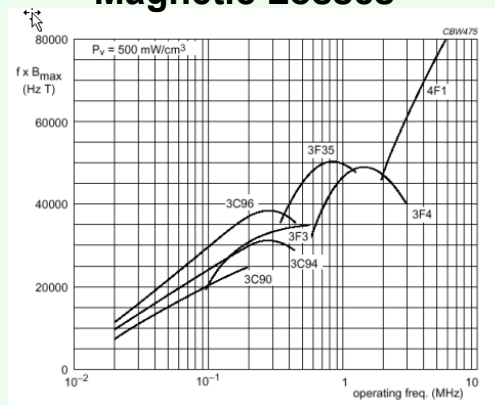
Magnetic losses $\sim \Delta B$

“Good number” = 100mW/cm³ = 100KW/m³

Magnetic Losses

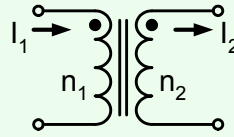
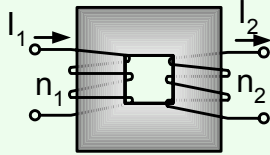


Magnetic Losses



- Curves for constant loss: 500 mW/cm^3
- Figure of merit $B \cdot f$
- Each material has optimum operating temperature (minimum loss)

Transformer currents



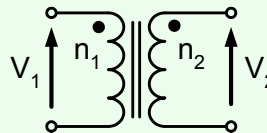
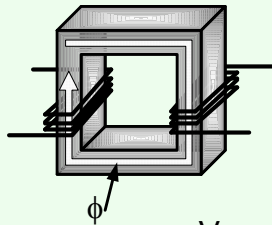
For ideal transformer $n_1 I_1 = n_2 I_2$ $\frac{I_1}{I_2} = \frac{n_2}{n_1}$

At any given moment $n_1 I_1 = n_2 I_2$

I_1, I_2 opposite direction.

No magnetic energy stored due to useful currents I_1, I_2 (they cancel each other)

Transformer voltages



$$V_1 = n_1 \frac{d\phi_1}{dt} \quad V_2 = n_2 \frac{d\phi_2}{dt}$$

Assuming $\phi_1 = \phi_2$

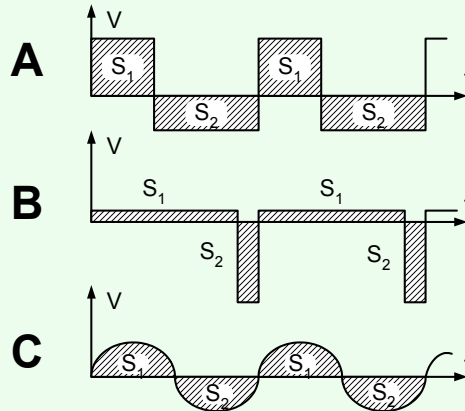
$$\frac{d\phi_1}{dt} = \frac{d\phi_2}{dt}$$

$$\frac{V_1}{V_2} = \frac{n_1}{n_2}$$

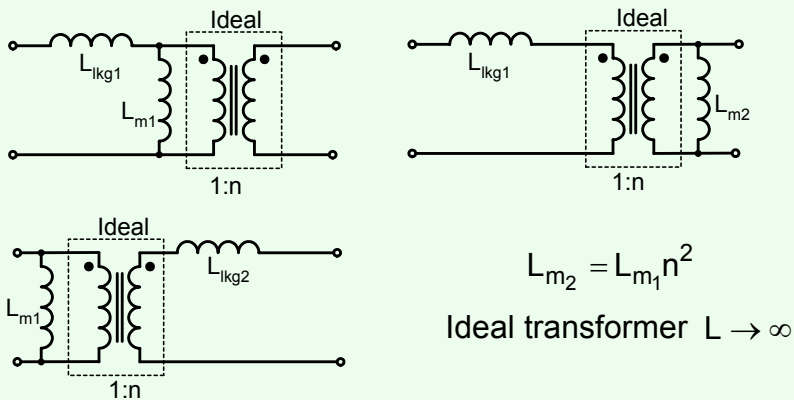
Voltages

Since each winding also represents an inductance, therefore for any winding $\overline{V_n} = 0$

Permissible voltages: AC only on any winding

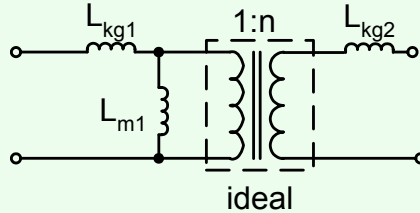
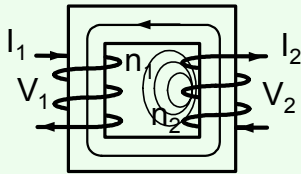


Equivalent circuit (preliminary)



Leakage

- Leakage inductance
- Leakage inductance is the uncoupled magnetic flux



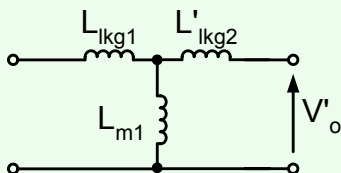
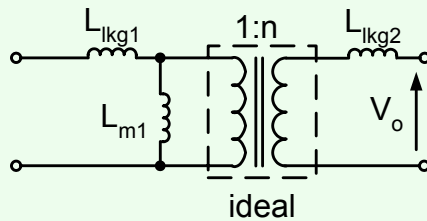
- Relationship between L_{lkg} , M and k (coupling coefficient).

$$M = k\sqrt{L_1 \cdot L_2}$$

$$L_{lkg1} \cong L_{m1}(1-k)$$

$$L_{lkg2} \cong L_{lkg1} \cdot n^2$$

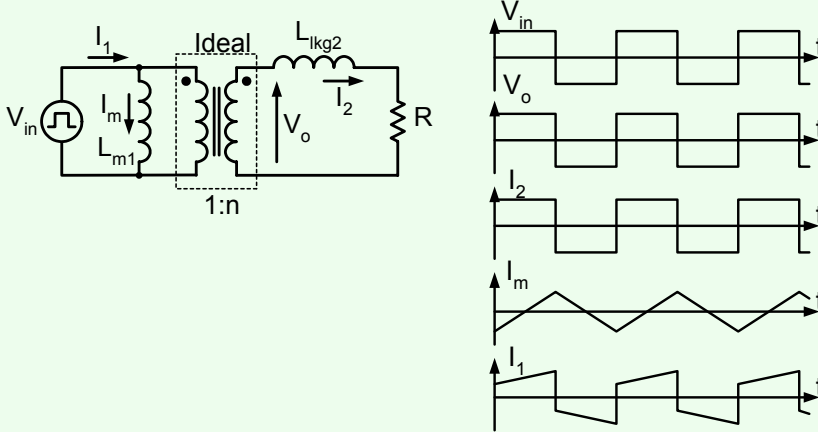
Leakage



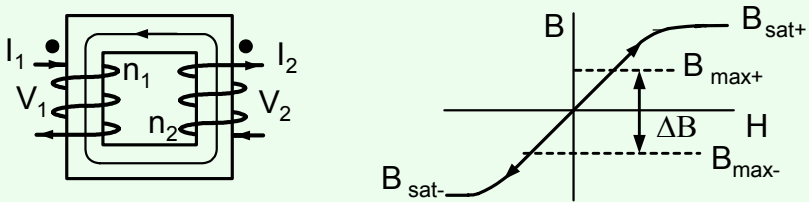
$$V'_o = \frac{V_o}{n^2}$$

$$L'_{lkg2} = \frac{L_{lkg2}}{n^2}$$

Magnetization Current



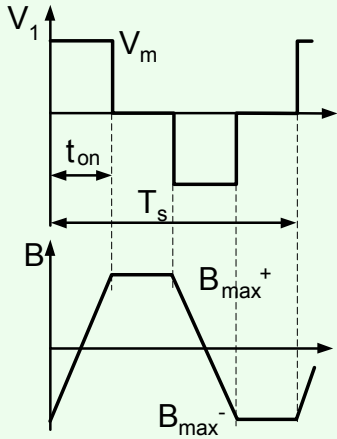
Transformer



1. B_{max} (could be symmetrical or asymmetrical)
2. $B_{max} < B_{sat}$
3. In most case (high frequency) B_{max} limit by magnetic losses.

$$V_1 = n_1 \frac{d\Phi}{dt} = n_1 A_e \frac{dB}{dt}$$

Symmetrical operation



$$B = \frac{1}{n_1 A_e} \int V dt$$

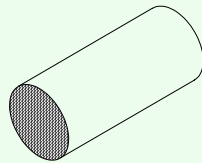
$$|B_{max}^+| = |B_{max}^-|$$

$$\Delta B = 2B_{max} = \frac{V_m t_{on}}{n_1 A_e}$$

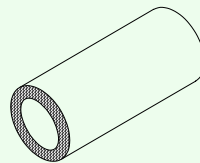
$$n_1 = \frac{\{V_m, t_{on}\}}{2B_{max} A_e}$$

$$n_1 A_e \sim t_{on} \Big|_{t_{on} \rightarrow \frac{T_s}{2}} \sim \frac{1}{f_s}$$

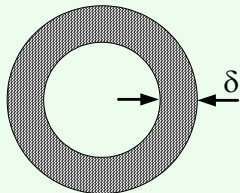
Skin effect



DC



High Frequency



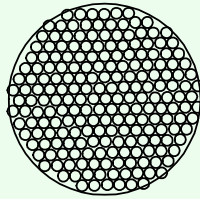
$$\frac{R_{AC}}{R_{DC}} > 1$$

δ – skin depth

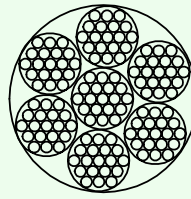
$$\delta \text{ (mm)} = \frac{72}{\sqrt{f}}$$

f in Hz

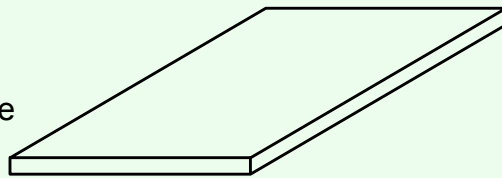
Skin Effect Solutions



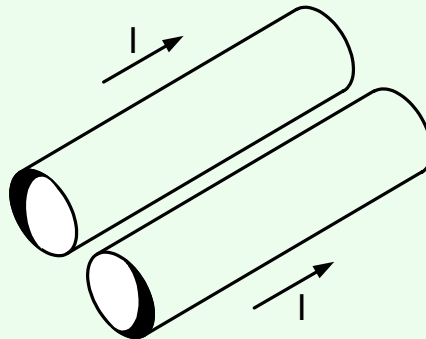
Litz wire



Tape

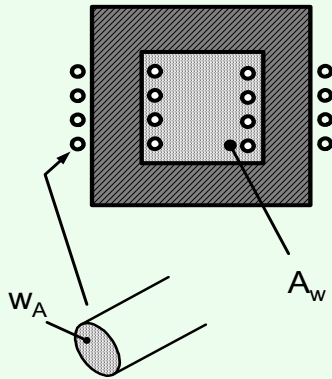


Proximity effect



Current crowding due to magnetic fields

Aw



A_w - winding area

$$A_w = \frac{[w_{A1} \cdot n_1 + w_{A2} \cdot n_2]}{k}$$

k - filling factor $k < 1$

$$w_{A1} = \frac{I_{1rms}}{J}$$

J - current density A/m^2

$$J \cong 4.5 A/mm^2$$

$$I_2 = \frac{n_1}{n_2} I_1$$

Ap

$$A_w = \frac{n_1 I_{1rms}}{Jk} \cdot 2 \quad n_1 = \frac{Jk A_w}{2 \cdot I_{1rms}} \quad \rightarrow \quad \frac{Jk A_w}{I_{1rms} \cdot 2} = \frac{\{V_1, t_{on}\}}{2B_{max} A_e}$$

$$n_1 = \frac{\{V_1, t_{on}\}}{2B_{max} A_e}$$

$$A_p = A_w A_e = \frac{\{V_1, t_{on}\} 2 \cdot I_{1rms}}{\{2B_{max}\} Jk}$$

$$A_p = \frac{\{V_1, t_{on}\} 2 \cdot I_{1rms}}{\Delta B \cdot Jk}$$

$$A_p = \frac{\{V_1, D_{on}\} 2 \cdot I_{1rms}}{f_s \cdot \Delta B \cdot Jk}$$

Transformer design stages

1. Calculate A_p

$$A_p = \frac{\{V_1, D_{on}\} 2 \cdot I_{l_{rms}}}{f_s \cdot \Delta B \cdot Jk}$$

In symmetrical operation

$$\Delta B = B_{max}^+ - B_{max}^-$$

In asymmetrical operation

$$\Delta B = B_{max} - 0$$

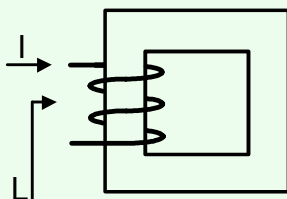
2. Look for core

3. Calculate n_1 by:

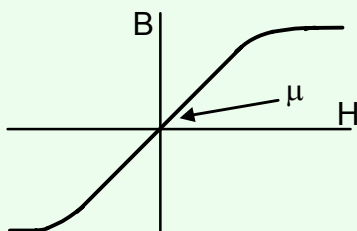
4. Calculate n_2

$$n_1 = \frac{\{V_m, t_{on}\}}{2B_{max} A_e}$$

Inductor design



Need to store energy
(in transformer $n_1 \cdot I_1 = n_2 \cdot I_2$)



$$\mu = \mu_0 \mu_r$$

μ_0 - air (vacuum) permeability

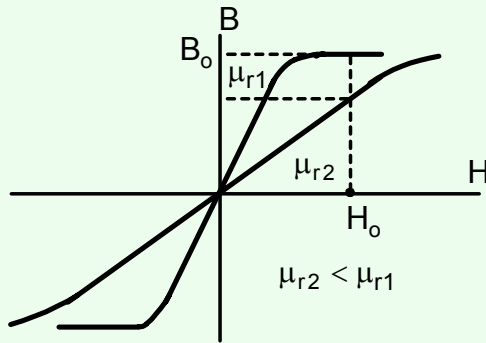
μ_r - relative permeability

Permeability

$$\mu_0 = 1.26 \cdot 10^{-6} \frac{\text{Henry}}{\text{m}}$$

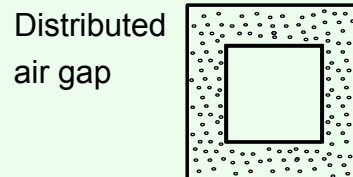
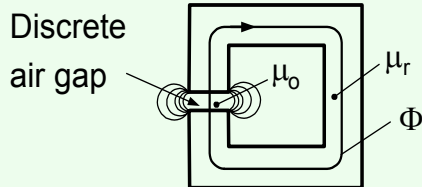
μ_r of ferrites $\sim 2000 - 4000$

$$B = \mu H$$



If μ is high B will reach quickly B_{sat}
Need to slower μ

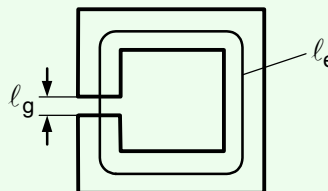
Gaps



Same Φ magnetic lines in ferromagnetic material and in air.

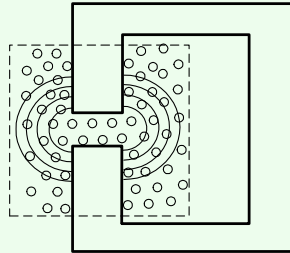
$$l_g \ll l_e$$

$$l_g + l_e \cong l_e$$



Current Crowding

Current crowding due to magnetic fields

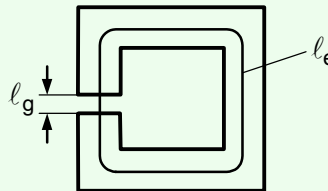


R_{AC} high around gap

Inductance with Gap

$$l_g \ll l_e$$

$$l_g + l_e \cong l_e$$



$$\Phi = \text{constant} \quad B \cong \text{constant}$$

$$H_g = \frac{B}{\mu_0} \quad H_m = \frac{B}{\mu_m}$$

$$nI = l_e H = H_m l_e + H_g l_g$$

$$\longrightarrow H l_e = \frac{B l_e}{\mu_m} + \frac{B l_g}{\mu_0}$$

Inductance with Gap

$$Hl_e = \frac{Bl_e}{\mu_m} + \frac{Bl_g}{\mu_o}$$

•Dividing out l_e and defining $\mu_e = \frac{B}{H}$

$$H = \frac{B}{\mu_e} = \frac{B_m}{\mu_m} + \frac{B_a}{\mu_o} \frac{l_e}{l_g}$$

Gap Calculation

$$\frac{1}{\mu_e} = \frac{1}{\mu_m} + \frac{1}{\mu_o \left(\frac{l_e}{l_g} \right)}$$

$$\frac{1}{\mu_{re}\mu_o} = \frac{1}{\mu_{rm}\mu_o} + \frac{1}{\mu_o \left(\frac{l_e}{l_g} \right)}$$

$$\frac{1}{\mu_{re}} = \frac{1}{\mu_{rm}} + \frac{1}{\left(\frac{l_e}{l_g} \right)}$$

$$\frac{1}{\mu_{re}} = \frac{\left(\frac{l_e}{l_g} \right) + \mu_{rm}}{\mu_{rm} \left(\frac{l_e}{l_g} \right)}$$

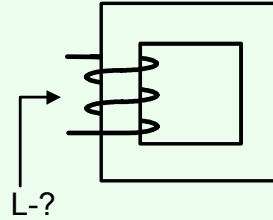
$$\mu_{re} = \frac{\mu_{rm} \left(\frac{l_e}{l_g} \right)}{\left(\frac{l_e}{l_g} \right) + \mu_{rm}}$$

If $\frac{l_e}{l_g} < \mu_{rm}$ $\mu_{re} \approx \left(\frac{l_e}{l_g} \right)$

Inductance

$$V = L \frac{di}{dt} \quad V = n \frac{d\Phi}{dt}$$

$$L \frac{di}{dt} = n \frac{d\Phi}{dt}$$



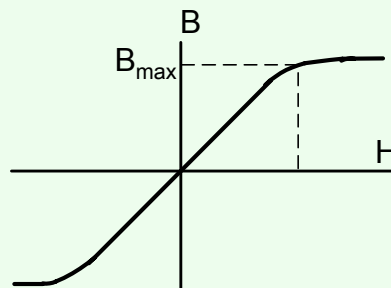
$$n \frac{d\Phi}{dt} = n A_e \frac{dB}{dt} = n A_e \mu \frac{dH}{dt} = n A_e \mu \frac{n di}{\ell_e dt}$$

$$L \frac{di}{dt} = \frac{n^2 A_e \mu di}{\ell_e dt} \quad L = \frac{n^2 A_e \mu}{\ell_e}$$

Two windings on same core

$$\frac{L_1}{L_2} = \left(\frac{n_1}{n_2} \right)^2$$

Inductor design



Saturation Limits

$$L \frac{di}{dt} = n \frac{d\Phi}{dt}$$

$$L \int_0^{I_{pk}} \left(\frac{di}{dt} \right) dt = n A_e \int_0^{B_{max}} \left(\frac{dB}{dt} \right) dt$$

$$L I_{pk} = n A_e B_{max}$$

$$n = \frac{L I_{pk}}{A_e B_{max}} \quad \text{quick design and check}$$

$$A_e = \frac{L I_{pk}}{n B_{max}} \quad n = \frac{J k A_w}{I_{rms}}$$

Ap

$$A_p = A_e A_w = \frac{L I_{pk} I_{rms}}{B_{max} J k}$$

$$L I_{pk} I_{rms} \approx L I^2$$

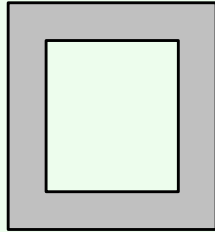
$$\text{Energy stored} = \frac{L I^2}{2}$$

Air gapped core Design

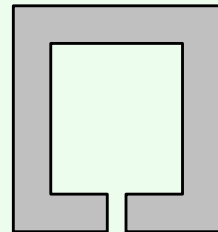
1. Calculate A_p
2. Choose a core
3. Iterate
4. Calculate ℓ_g (or increase gap until L is as required)

Cores

- Transformer core



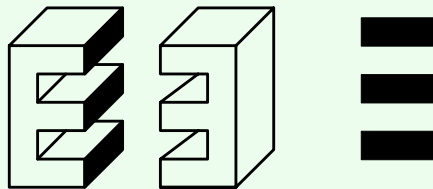
- Inductor core



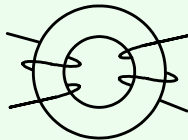
air gap

Cores

1. E - core



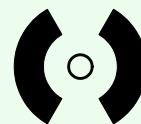
2. TOROID



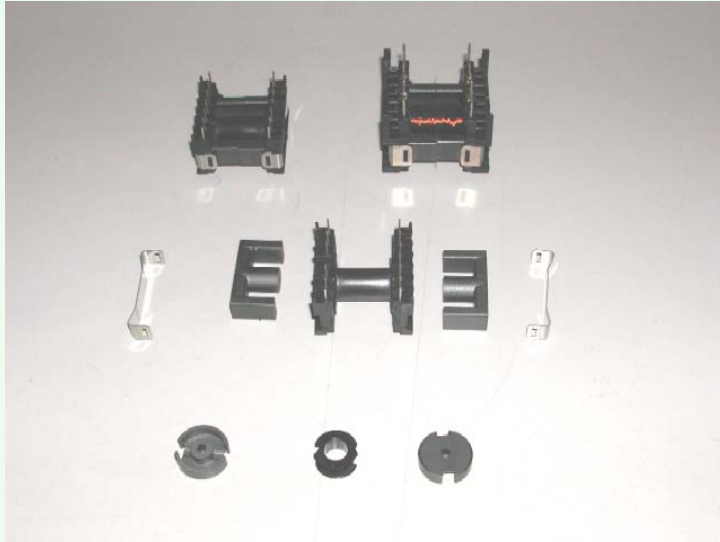
3. ARENCO



4. POT



Commercial cores



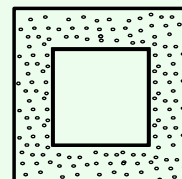
Distributed gap core

- The concept of A_L

$$A_L = \frac{H_y}{\text{turn}} \quad \left(\text{sometime } \frac{H_y}{1000 \text{ turns}} \right)$$

$$L \text{ for } n \text{ turns: } L = n^2 \cdot A_L$$

Distributed
air gap



MAGNETICS Kool Mj Powder Core Catalog - 2.0

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AL and Inductance Considerations

The inductance of a wound core can be calculated from the core geometry by using the following equation:

$$L = \frac{.4\pi\mu N^2 A_e}{l_e \times 10^8}$$

where: L = inductance (Henries)
 μ = core permeability
 N = number of turns
 A_e = core cross-section (cm²)
 l_e = core magnetic path length (cm)

The inductance for a given number of turns is related to the nominal inductance (as listed in the catalog as mH/1000 turns) by the following:

$$L_n = \frac{L_{1000} N^2}{10^6}$$

where: L_n = inductance for N turns (mH)
 L₁₀₀₀ = nominal inductance (mH/1000 turns),
 or the A_L listed on the core pages.

Toroid Data

MAGNETICS Kool Mj Powder Core Catalog - 2.0

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0.400"OD 0.200"ID 0.156"HT (10.2mm 5.08 mm 3.96 mm)

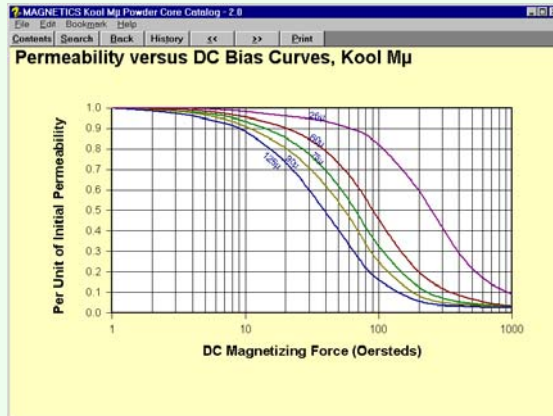
Part Number	Perm. (μ)	A _L ± 12%
77041-A7	60	32
77845-A7	75	40
77844-A7	90	48
77040-A7	125	66

Physical Characteristics		
Window Area	32.400 c.mils	0.164 cm ²
Cross Section	0.01550 in ²	0.1000 cm ²
Path Length	0.906 in.	2.38 cm.
Volume	0.014 in ³	0.238 cm ³
Weight	0.0032 lb.	1.46 gm.
Area Product	0.000395 in ⁴	0.0164 cm ⁴

Core Dimensions (after finish)		
O.D. (max.)	0.425 in.	10.80 mm.
I.D. (min.)	0.180 in.	4.57 mm.
HT. (max.)	0.181 in.	4.60 mm.
Surface Area	0.574 in ²	3.703 cm ²

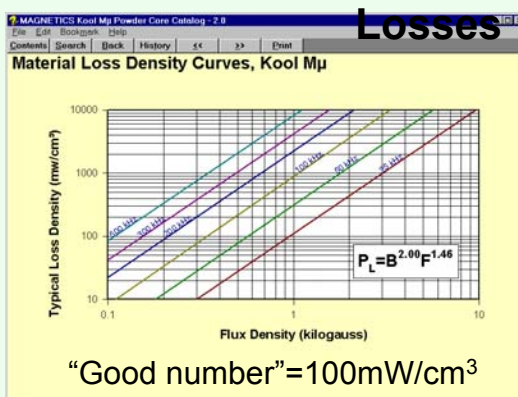
Winding Turn Length AWG Rdc Single Single

Permeability change



1 Amp/m = 79.5 Oe

L decreases with DC current !



“Good number”=100mW/cm³

Misleading notations !
 ΔB NOT B

These curves are measured by feeding ac signals.

If the current is composed of DC + ripple, core loss is due only to ripple component !

DC bias tend to increase loss

MAGNETICS Kool Mj Powder Core Catalog - 2.0

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Contents Search Back History Temp Rise Print

Temp Rise

Temperature Rise Calculations

Temperature rise in a wound core depends on (1) wire resistance and current through the coil (P_{cu}, copper losses), and (2) core excitation (P_{fe}, core losses). Total power loss, defined as P_{fe} + P_{cu} (milliwatts), is in the form of heat, and is dissipated from exposed surfaces of a wound core. The heat dissipated depends on the total exposed surface of the wound unit. Temperature rise cannot be predicted precisely, but can be approximated by the following formula:

$$\text{Temperature Rise (}^\circ\text{C)} = \left[\frac{\text{Total Power Loss (milliwatts)}}{\text{Surface Area (cm}^2\text{)}} \right]^{0.833}$$

In this catalog, surface area is presented in two ways:

1. Core surface (after insulation is added), and
2. Wound core, assuming 40% winding factor.

- “Hot Spot” - Critical parameter

Hanna Curve

$$n = \frac{LI_{pk}}{A_e B_{max}}$$

$$H_n = \frac{HLI_{pk}}{A_e B_{max}}$$

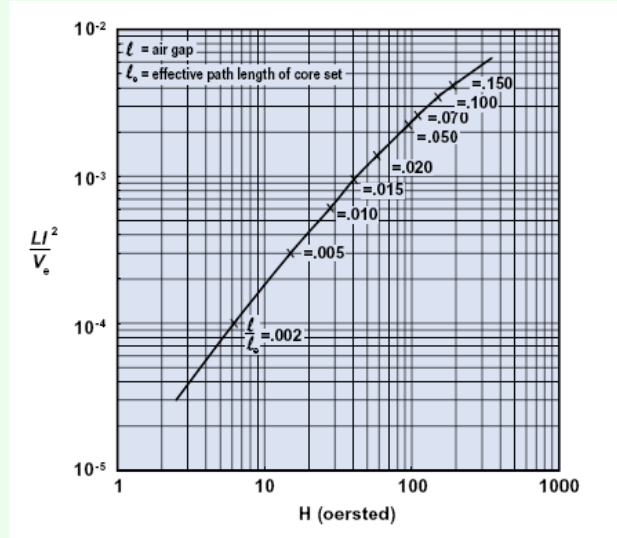
$$H_n = \frac{nI LI}{l_e A_e B_{max}}$$

$$H = \frac{LI^2}{V_e} \frac{1}{B_{max}}$$

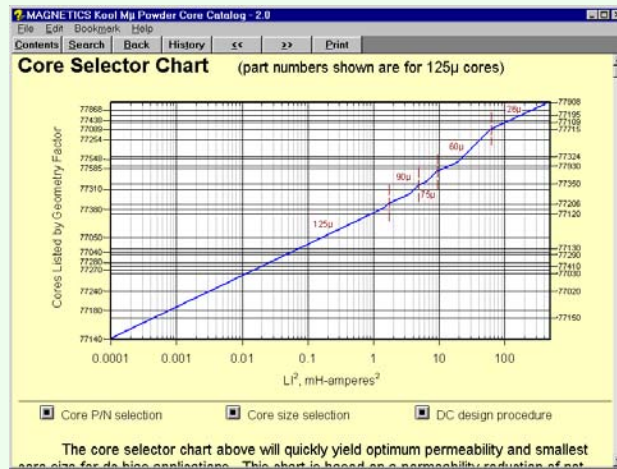
$$HB_{max} = \frac{LI^2}{V_e}$$

$$\frac{B_{max}}{H} = \mu = \mu_r \mu_0$$

Hanna Curve



Core Size Selection



Basic Design of Distributed Gap Core

1. Calculate Ll^2
2. Look up manufacturer data
3. Select Core
4. Calculate $n = 1000 \sqrt{\frac{L}{A_L(1000)}}$
5. Check L_{\min}
6. Calculate losses. Temp rise and $\mu = f\left(\frac{nl}{l_e}\right)$
7. Iterate