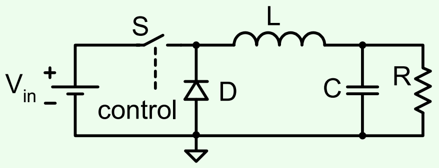
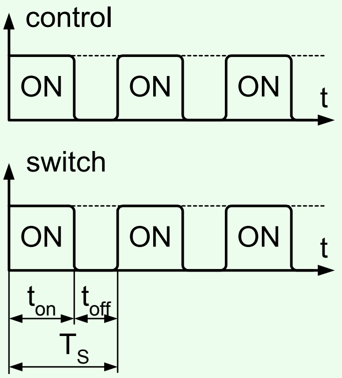
BUCK, BOOST, BUCK-BOOST, DCM

- 2.1 Buck converter
    - 2.1.1 Operation modes
    - 2.1.2 Voltage transfer function
    - 2.1.3 Current modes (CCM, DCM)
    - 2.1.4 Capacitor current
- 2.2 Boost converter
    - 2.2.1 Operation modes
    - 2.2.2 Voltage transfer function
- 2.3 Buck-Boost converter
- 2.4 Comparison between topologies
- 2.5 Simulation of SMPS
    - 2.5.1 The simulations problem
    - 2.5.2 Basics of average model of SMPS
    - 2.5.3 Example: Boost average model simulations

Buck Converter Constant Switching Frequency





Switch frequency: $f_s = \frac{1}{T_s}$

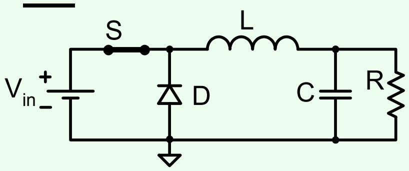
Duty Cycle:

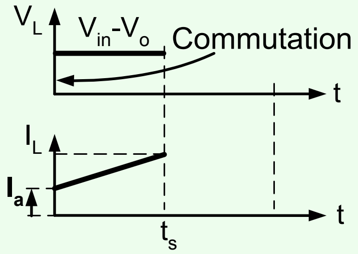
$$\frac{t_{on}}{T_s} = D_{on} \qquad \text{or} \to D$$

$$\frac{t_{off}}{T_s} = D_{off} \qquad \to 1 - D$$

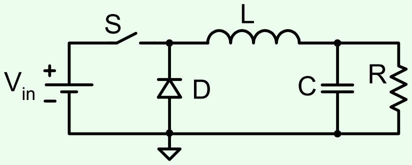
Operation modes

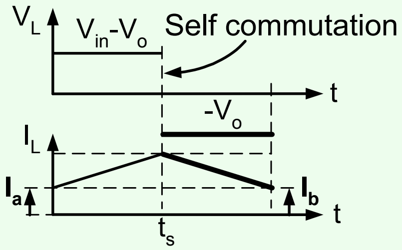
On





Off

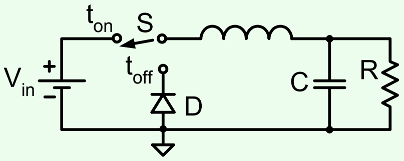




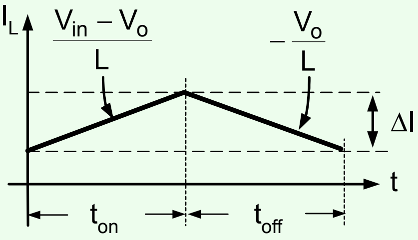
At steady state I_a=I_b

Buck

In this case



Inductor current waveform at steady state

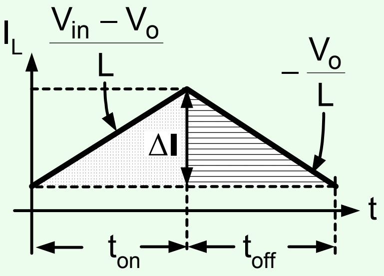


Voltage transfer function

The ΔI method

Left triangle

$$\Delta I = \frac{V_{in} - V_o}{L} \cdot t_{on}$$



Right triangle

$$\Delta I = \frac{V_o}{L} \cdot t_{off}$$

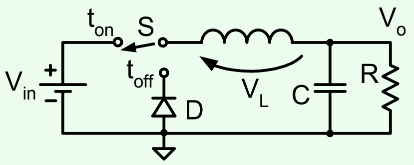
$$\left(\frac{V_{in} - V_o}{L}\right) t_{on} = \frac{V_o}{L} t_{off}$$

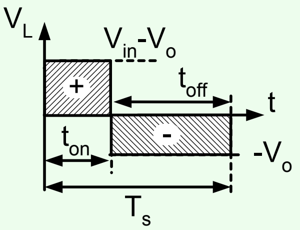
$$\frac{V_o}{V_{in}} = \frac{t_{on}}{t_{on} + t_{off}} = \frac{t_{on}}{T_s} = D_{on}$$

Independent of L !

Voltage transfer function

The average voltage method





At steady state, over one switching cycle:

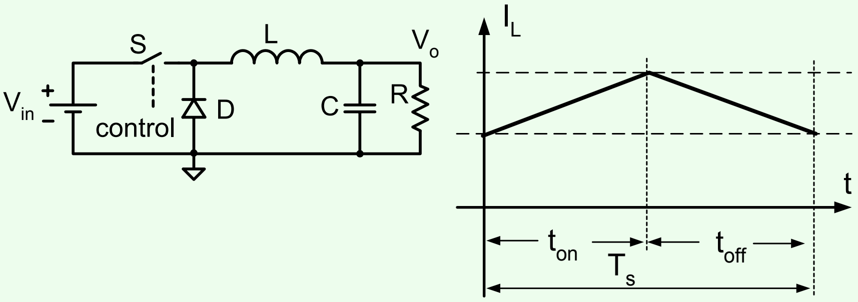
$$\overline{V_L} = 0;$$

$$S_+ = (V_{in} - V_o) \cdot t_{on};$$

$$S_- = (-V_o) \cdot t_{off};$$

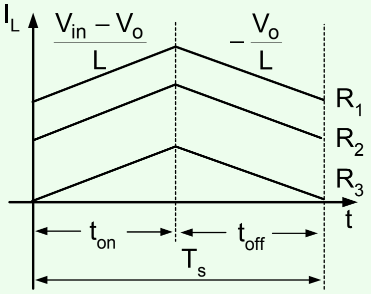
$$S_+ + S_- = 0 \Rightarrow \frac{V_o}{V_{in}} = D_{on}$$

Load Change with Fixed D



How will I_L change if R is getting smaller?

Load Change

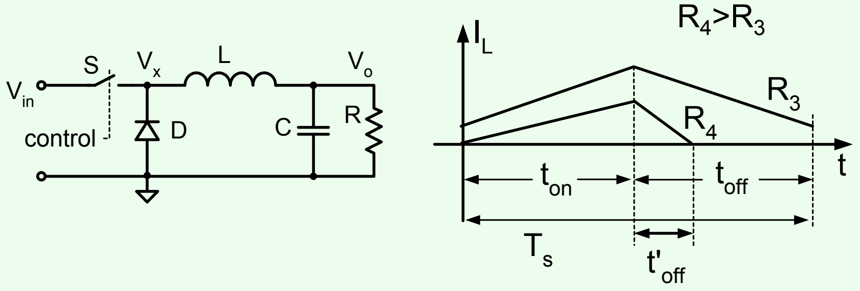


$R_1 < R_2 < R_3$

CCM - Continues Conductor Current Mode

DCM - Discontinues Conductor Current Mode

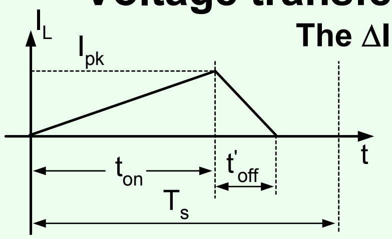
Discontinuous Inductor Current Mode (DCM)



- Different voltage transfer ratio ≠ D_{on}
- Higher ripple current

Voltage transfer function (DCM)

The ΔI method



$$I_{pk} = \frac{V_{in}-V_o}{L} t_{on} = \frac{V_o}{L} t'_{off}$$

$$D_{off} = \frac{(V_{in}-V_{out})D_{on}}{V_{out}}$$

$$I_{AV} = \frac{1}{T_S}\left(\frac{1}{2}\frac{V_{in}-V_o}{L}T_{on}\cdot T_S(D_{on}+D_{off})\right)$$

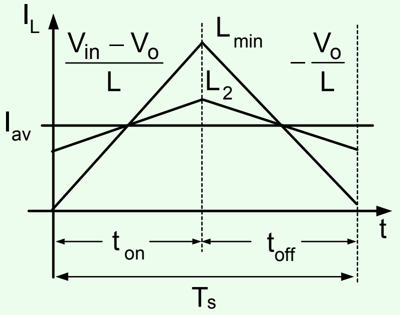
$$I_{AV} = \frac{1}{2}\frac{V_{in}-V_o}{L}T_{on}\cdot D_{on}(1+\frac{V_{in}-V_o}{V_o})$$

$$I_{AV} = \frac{V_o}{R}$$

$$\longrightarrow R(V_{in}-V_o)D_{on}^2 T_S V_{in} = 2LV_o^2$$

$$\frac{V_o}{V_{in}} = \frac{RD_{on}^2 T_s}{4L}\left(\sqrt{1+\frac{8L}{RD_{on}^2 T_s}}-1\right)$$

Boundary of CCM and DCM



- For CCM $L > L_{min}$
- In Buck $\frac{V_o}{L_{min}} t_{off} = I_{pk} = 2I_{av}$ $L_{min} = \frac{V_o D_{off}}{2I_{av} f_s} = \frac{R D_{off}}{2f_s}$

Example

A BUCK converter has a following characteristics:

Output voltage: $V_o = 5\,V$ Output current: $I_{out} = I_{av} = 10\,A$

Input voltage: $V_{in} = 10\,V$ Frequency: $f_s = 100\,kHz$

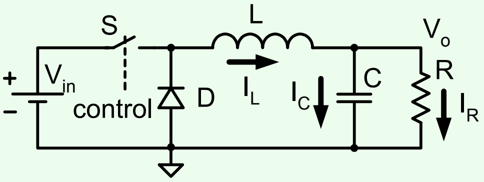
Current mode: CCM

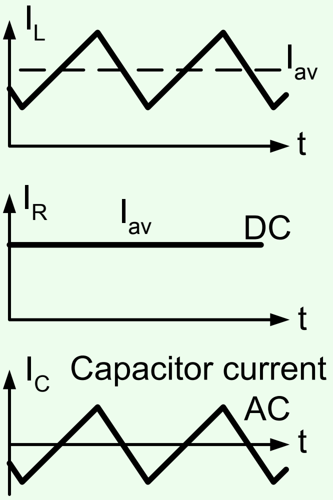
Find: L_{min}

$$\frac{V_o}{V_{in}} = D_{on} = 0.5 \xrightarrow{CCM} D_{off} = 1 - D_{on} = 0.5$$

$$L_{min} = \frac{V_o D_{off}}{2I_{av} f_s} = \frac{5 \cdot 0.5}{2 \cdot 10 \cdot 10^5} = 1.2\,\mu H$$

Capacitor current



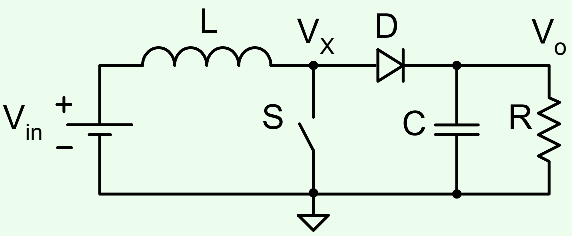


- Assumption:

  V_0 has small ripple

$$I_C = I_L - I_R$$

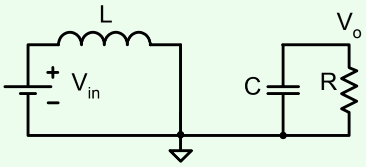
BOOST Step-Up

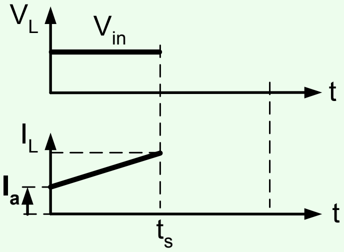


- $V_o > V_{in}$ **Why ??**

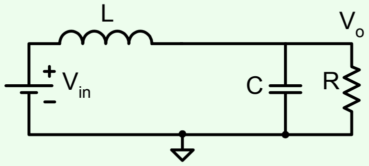
Operation modes Boost

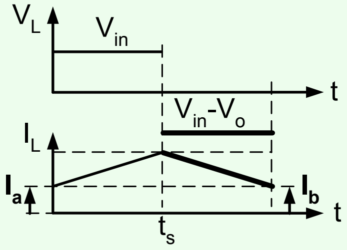
ON $V_L=V_{in}$





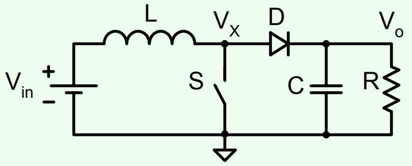
OFF $V_L=V_{in}-V_o$

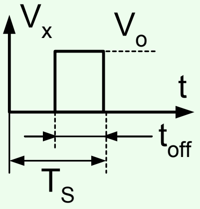




Voltage transfer function

The average voltage method





$$\overline{V}_L = 0; \quad \overline{V}_{in} - \overline{V}_x = 0; \quad \overline{V}_{in} = \overline{V}_x;$$

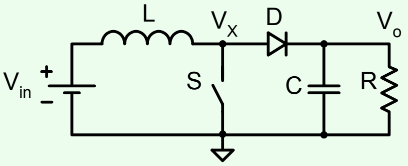
$$V_{in} = \overline{V}_{in};$$

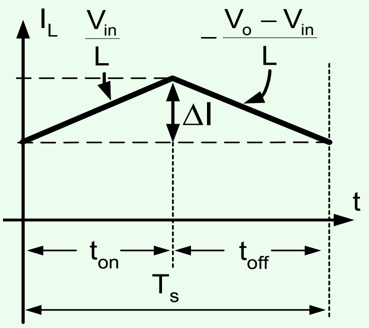
$$\overline{V}_x = \frac{V_o t_{off}}{T_s} = V_o D_{off};$$

$$V_{in} = V_o D_{off} \rightarrow \frac{V_o}{V_{in}} = \frac{1}{D_{off}}$$

Voltage transfer function

The ΔI method



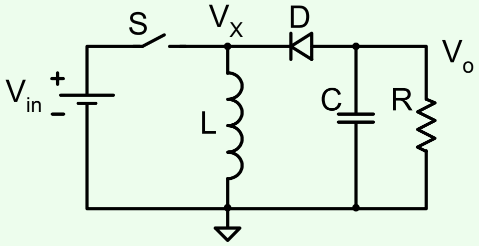


$$\frac{V_{in}}{L} \cdot t_{on} = \frac{V_o - V_{in}}{L} \cdot t_{off}$$

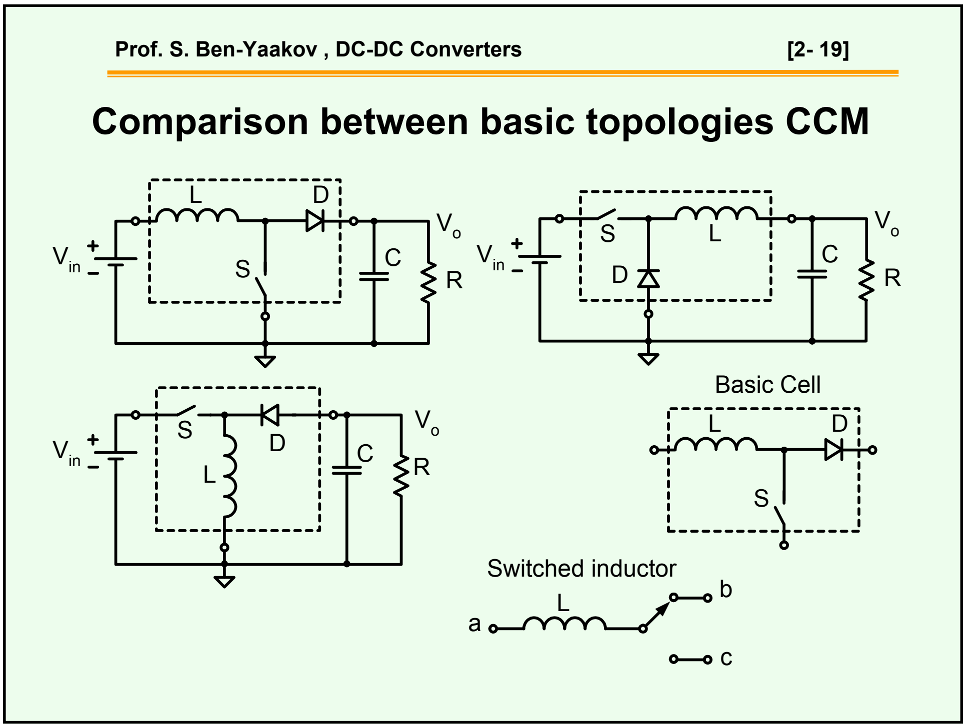
$$V_{in} \cdot (t_{on} + t_{off}) = V_o \cdot t_{off}$$

$$\frac{V_o}{V_{in}} = \frac{1}{D_{off}}$$

BUCK-BOOST
Step-Up Step-Down



- Find V_o/V_{in}
  Hint: Average of V_x ?



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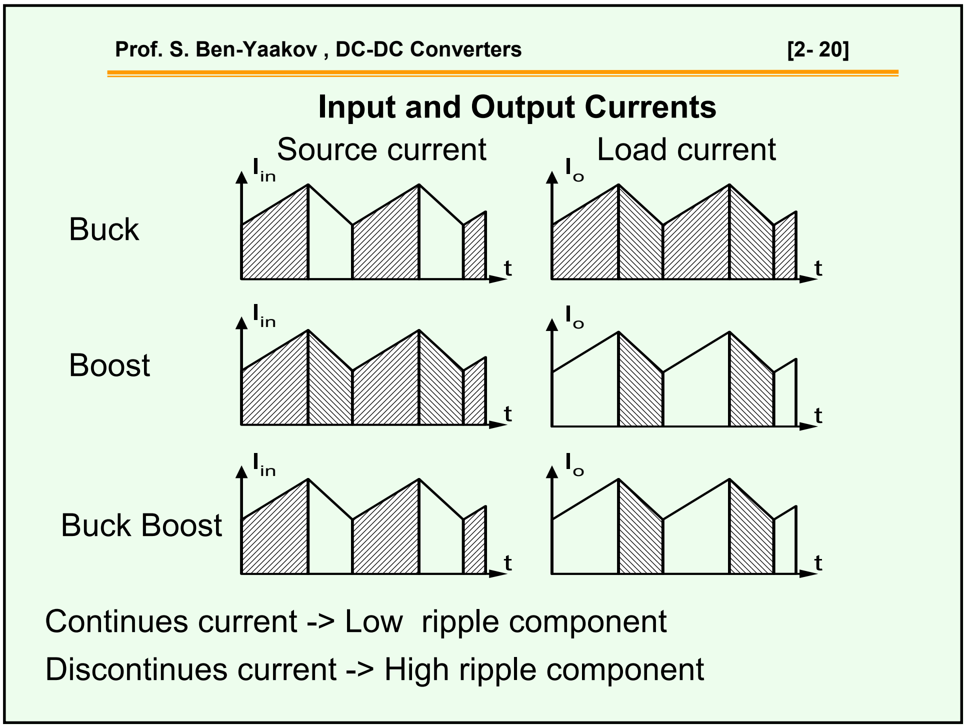
Comparison between basic topologies CCM



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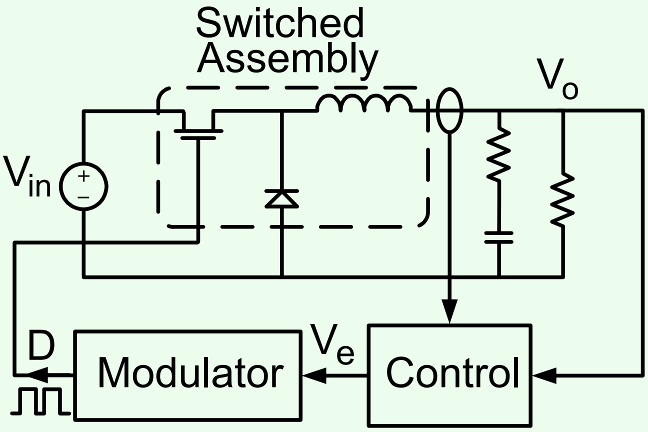
Input and Output Currents

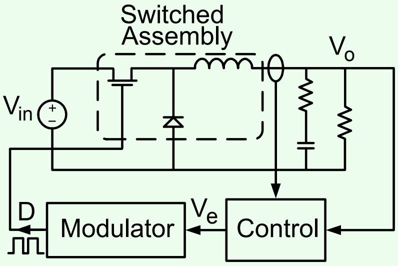
Continues current -> Low ripple component

Discontinues current -> High ripple component

The simulation problem

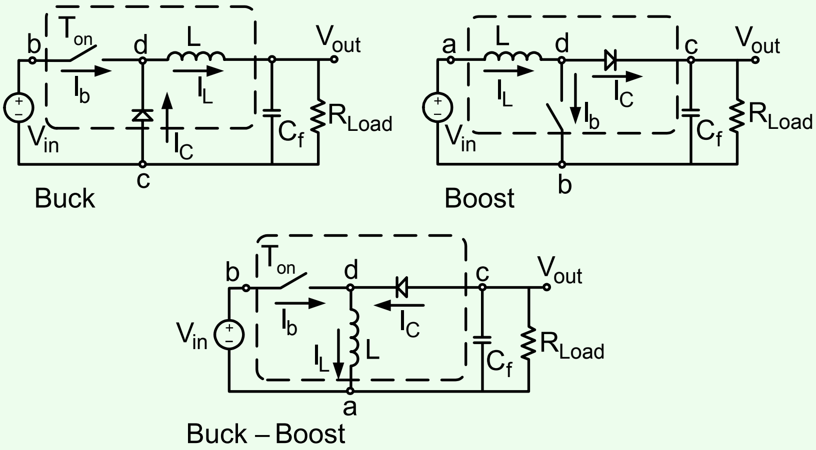


The simulation problem

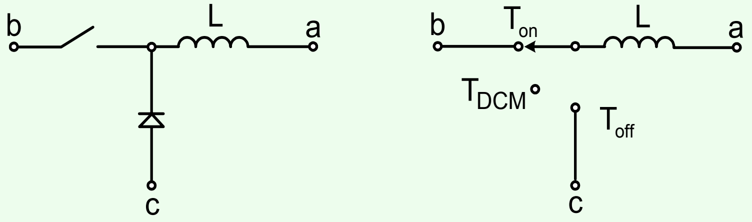


- The problematic part : Switched Assembly
- Rest of the circuit continuous - SPICE compatible
- Only possible simulation :
  Time domain (cycle-by-cycle) -Transient
- The objective : translate the
  Switched Assembly into an equivalent
  circuit which is SPICE compatible

Average Simulation of PWM Converters



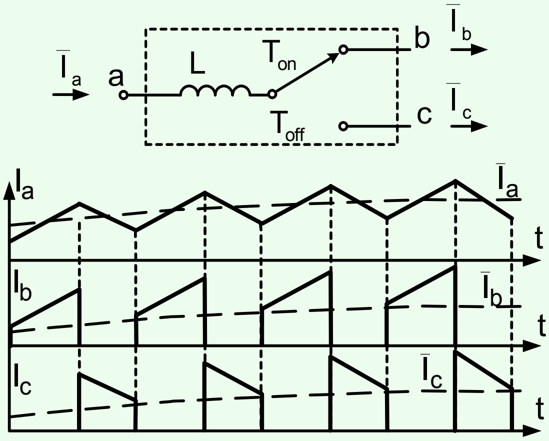
The Switched Inductor Model



T$_{on}$ - switch conduction time
T$_{off}$ - diode conduction time
T$_{DCM}$ - no current time (in DCM)

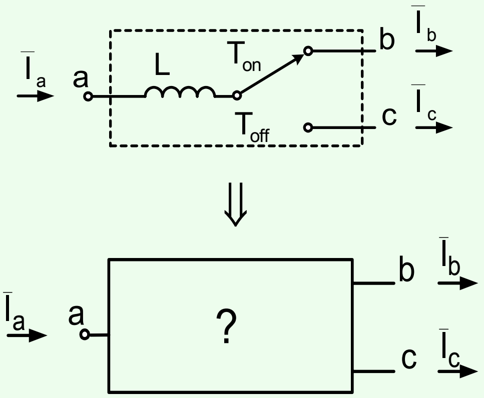
The Switched Inductor Model (SIM) (CCM)

The concept of average signals

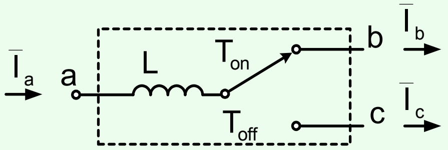


The SIM

Objective : To replace the switched part by a continuous network



Average current



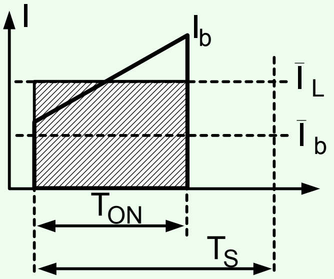
$$\bar{I}_b = \frac{\bar{I}_L T_{on}}{T_S} = \bar{I}_L D_{on}$$

$$D_{on} = \frac{T_{ON}}{T_S}$$

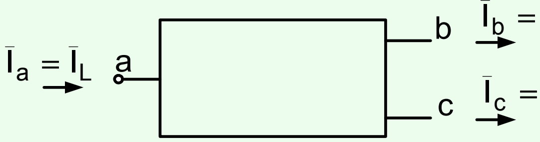
$$\bar{I}_a = \bar{I}_L$$

Similarly :

$$\bar{I}_c = \frac{\bar{I}_L T_{off}}{T_S} = \bar{I}_L D_{off}$$



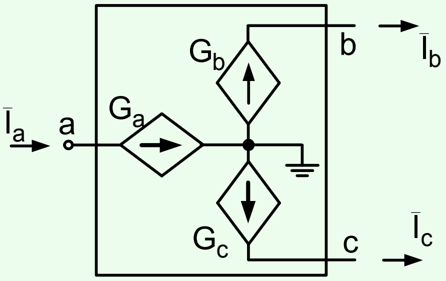
Toward a continuous model



$$\bar{I}_b = \bar{I}_L \cdot D_{on}$$

$$\bar{I}_c = \bar{I}_L \cdot D_{off}$$

$\Downarrow$



G_a, G_b, C_c - current dependent sources

$$G_a \equiv \bar{I}_L$$

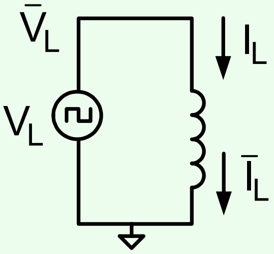
$$G_b \equiv \bar{I}_L \cdot D_{on}$$

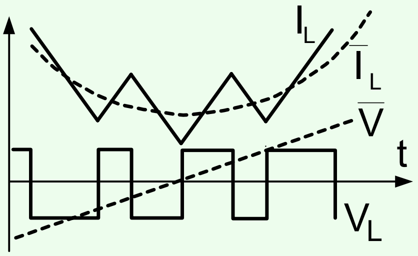
$$G_c \equiv \bar{I}_L \cdot D_{off}$$

Average inductor current

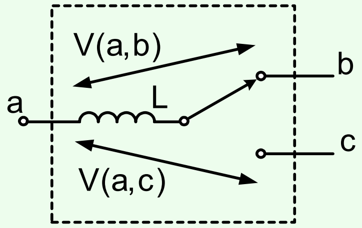
Deriving $\bar{I}_L$

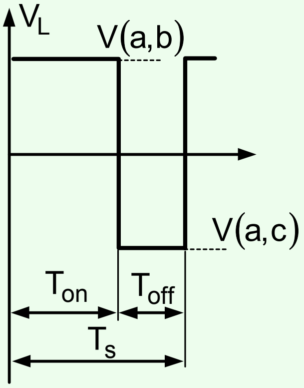
$$\frac{dI_L}{dt}=\frac{V_L}{L} \Rightarrow \frac{d\bar{I}_L}{dt}=\frac{\bar{V}_L}{L}$$





Average inductor current

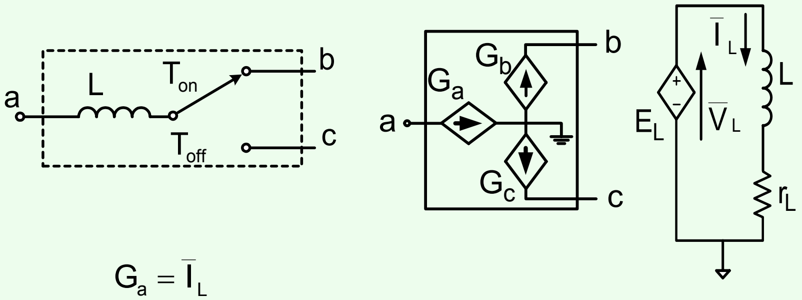




$$\bar{V}_L=\frac{V(a,b)\cdot T_{on}+V(a,c)\cdot T_{off}}{T_S}=$$

$$=V(a,b)\cdot D_{on}+V(a,c)\cdot D_{off}$$

The Generalized Switched Inductor Model (GSIM)



$G_a = \overline{I}_L$

$G_b = \overline{I}_L \cdot D_{on}$

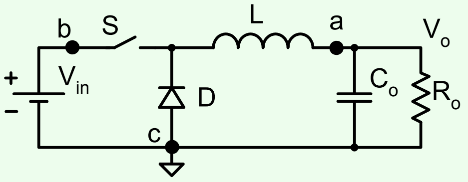
$G_c = \overline{I}_L \cdot D_{off}$

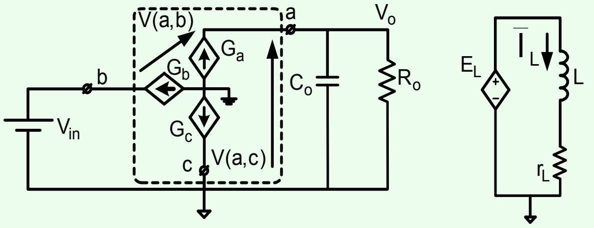
$E_L = V(a,b) \cdot D_{on} + V(a,c) \cdot D_{off}$

Topology independent !

<u>Example</u> Implementation in Buck Topology

1. The formal approach



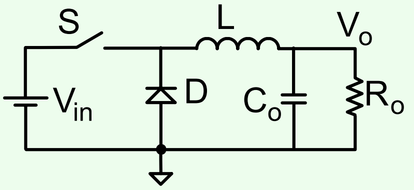


$G_a = \overline{I}(L) \quad G_b = \overline{I}(L) \cdot D_{on} \quad G_c = \overline{I}(L) \cdot D_{off}$

$E_L = [V_0 - V_{in}] \cdot D_{on} + [0 - V_0] \cdot D_{off}$

Implementation in Buck Topology

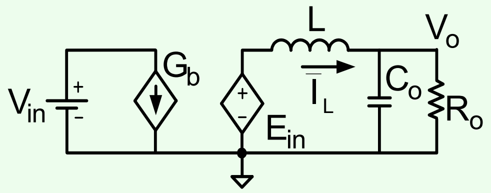
2. The intuitive approach - by inspection



$E_{in} = V_{in} \cdot D_{on}$

$G_b = \overline{I}_L \cdot D_{on}$

$E_{in} - V_o \rightarrow \overline{V}_L$



Polarity: (voltage and current sources) selected by inspection

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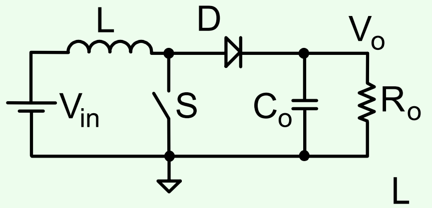
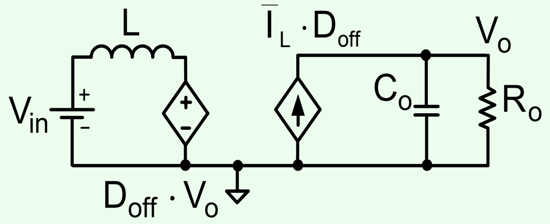
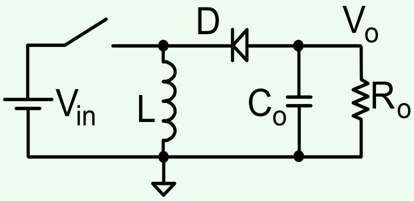
Boost

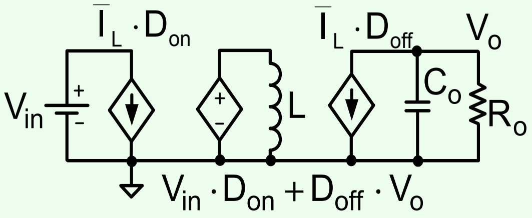




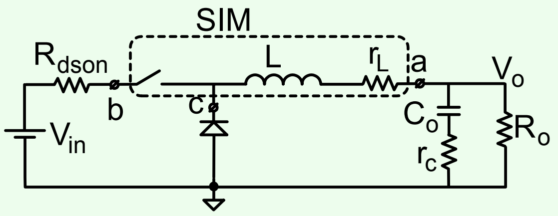
- Emulate average voltage on inductor
- Create $\overline{I}_L$ dependent current sources

Buck-Boost

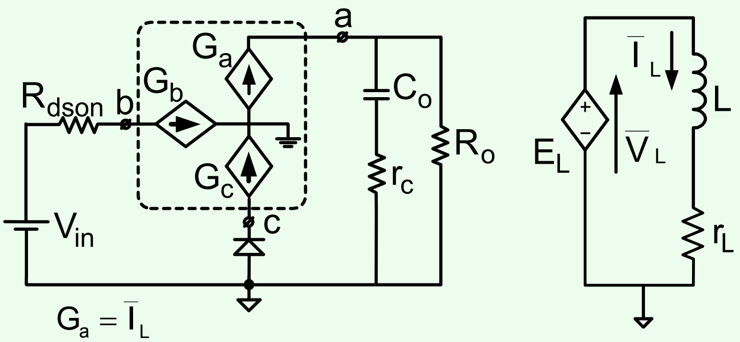




Partially accounting for parasitics

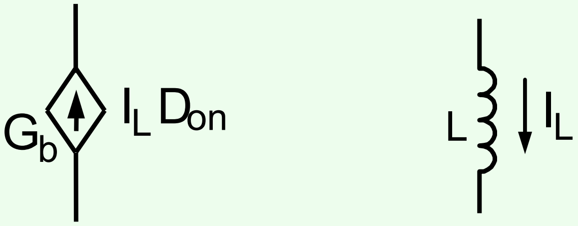


Modified Average Model



$$G_a = \overline{I}_L$$
$$G_b = \overline{I}_L \cdot D_{on}$$
$$G_c = \overline{I}_L \cdot D_{off}$$
$$E_L = (V_a - V_b) \cdot D_{on} + (V_a - V_c) \cdot D_{off}$$

Making the model SPICE compatible

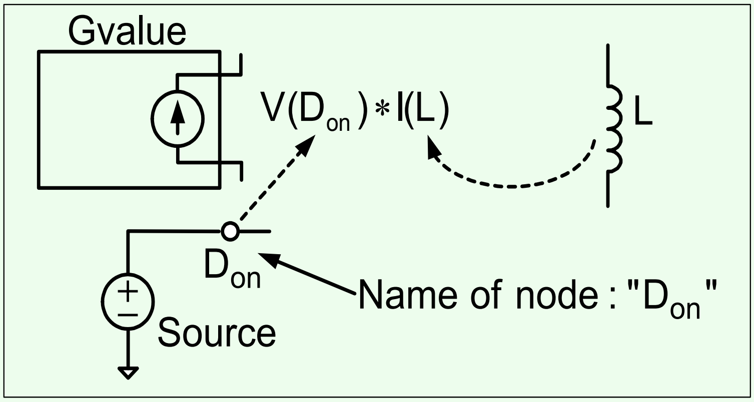


I_L and D_{on} are time dependent variables $\{I_L(t), D_{on}(t)\}$

D_{on} is not an electrical variable

In SPICE environment

$\Downarrow$



D_{on} is coded into voltage

Simulation

Running SPICE simulation

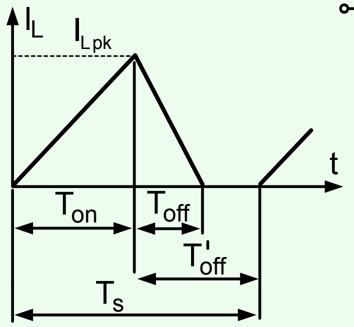
DC (steady state points) - as is

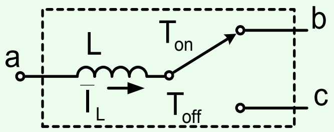
TRAN (time domain) - as is

AC (small signal) - as is

* Linearization is done by simulator !

Discontinuous Model (DCM)



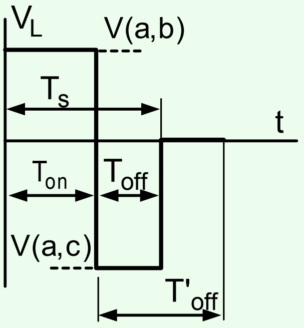


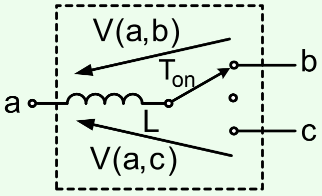
$$T'_{off} = T_s - T_{on}$$

$$D'_{off} = \frac{T_s - T_{on}}{T_s} = 1 - D_{on}$$

Combining CCM / DCM

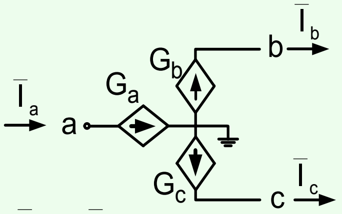
1.The average inductor current in DCM

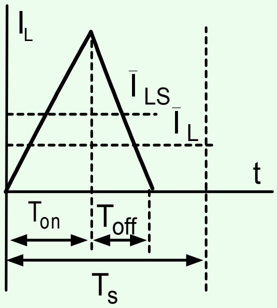




$$\overline{V}_L = V(a,b)D_{on} + V(a,c)D_{off} \quad \text{as in CCM}$$

Combining CCM / DCM

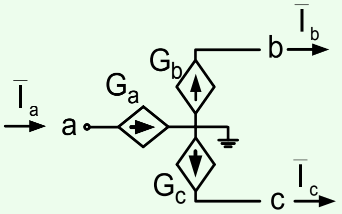


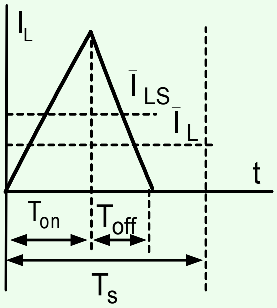


- $*$ $\bar{I}_a$ is $\bar{I}_L$
- $*$ $\bar{I}_b$ is sampledduring T_{on}
- $*$ $\bar{I}_c$ is sampledduring T_{off}
- $*$ $\bar{I}_b$ is $\bar{I}_c$ are sampling $\bar{I}_{Ls}$

$$\bar{I}_{Ls} = \frac{\bar{I}_L T_s}{T_{on} + T_{off}} = \frac{\bar{I}_L}{D_{on} + D_{off}}$$

Combining CCM / DCM





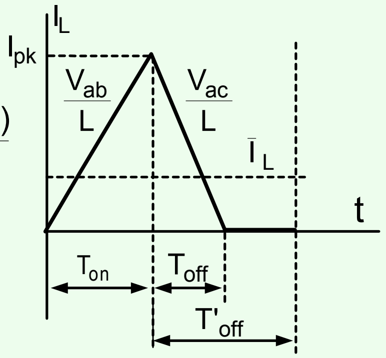
$$G_a = \bar{I}_L$$

$$G_b = \frac{\bar{I}_L D_{on}}{D_{on} + D_{off}}$$

$$G_c = \frac{\bar{I}_L D_{off}}{D_{on} + D_{off}}$$

in CCM: $(D_{on} + D_{off}) = 1$

Doff in DCM



$$I_{pk} = \frac{V(a,b)T_{on}}{L}$$

$$\bar{I}_L = \frac{1}{2}\left(\frac{V(a,b)T_{on}}{L}\right)\frac{(T_{on}+T_{off})}{T_S}$$

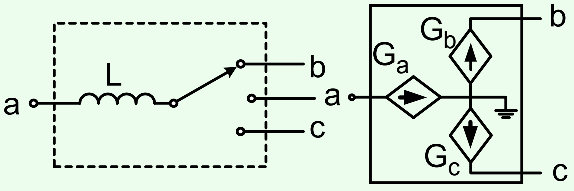
$$\bar{I}_L = \frac{V(a,b)D_{on}}{2Lf_s}(D_{on}+D_{off})$$

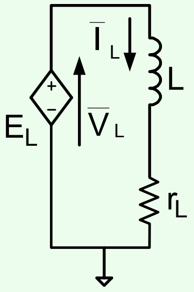
$$D_{off} = \frac{2\bar{I}_L L f_s}{V(a,b)D_{on}} - D_{on}$$

$$D'_{off} = 1 - D_{on}$$

$$D_{off} \le 1 - D_{on}$$

The combined DCM / CCM mode





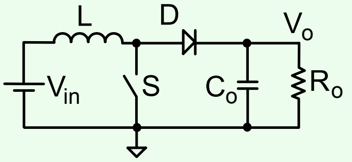
$$G_a \equiv \bar{I}_L$$

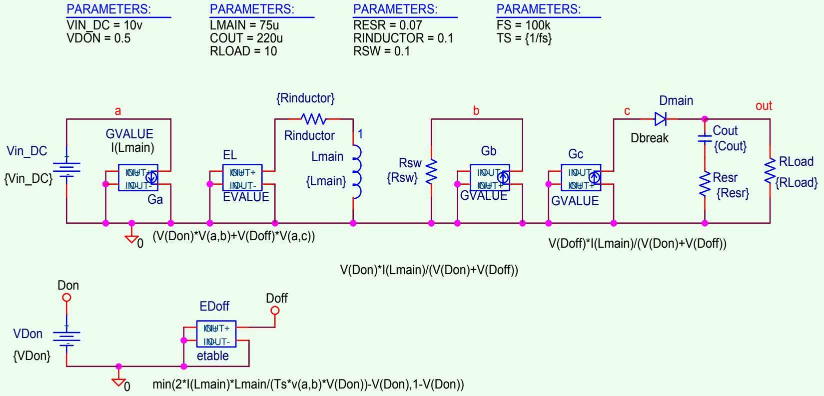
$$G_b \equiv \frac{\bar{I}_L D_{on}}{D_{on}+D_{off}} \qquad G_c \equiv \frac{\bar{I}_L D_{off}}{D_{on}+D_{off}}$$

$$E_L = V(a,b)\cdot D_{on} + V(a,c)\cdot D_{off}$$

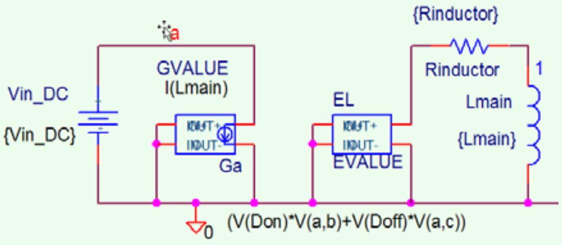
$$D_{off} = \min\left\{(1-D_{on}),\ \left(\frac{2\bar{I}_L L f_s}{V(a,b)D_{on}} - D_{on}\right)\right\}$$

Example: Boost average model simulation

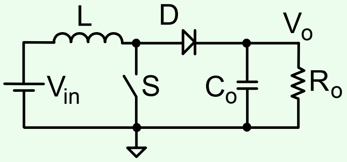


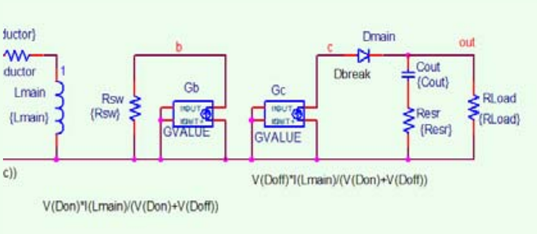


Example: Boost average model simulation

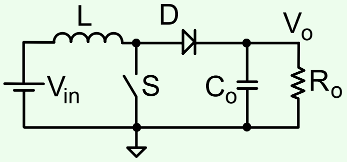


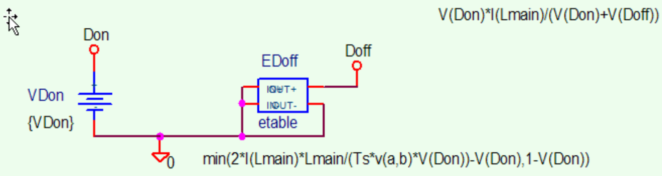
Example: Boost average model simulation





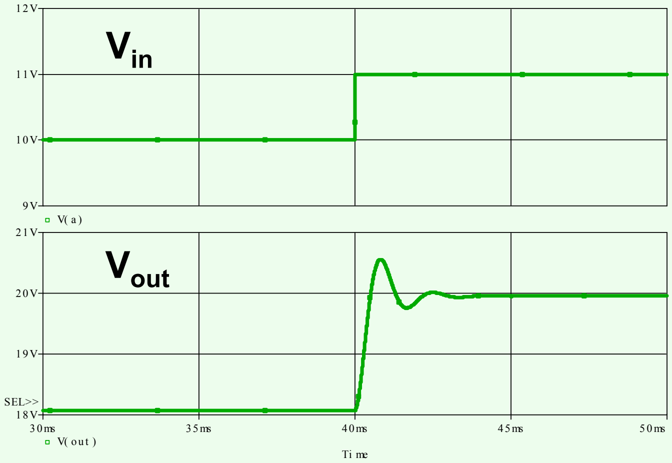
Example: Boost average model simulation



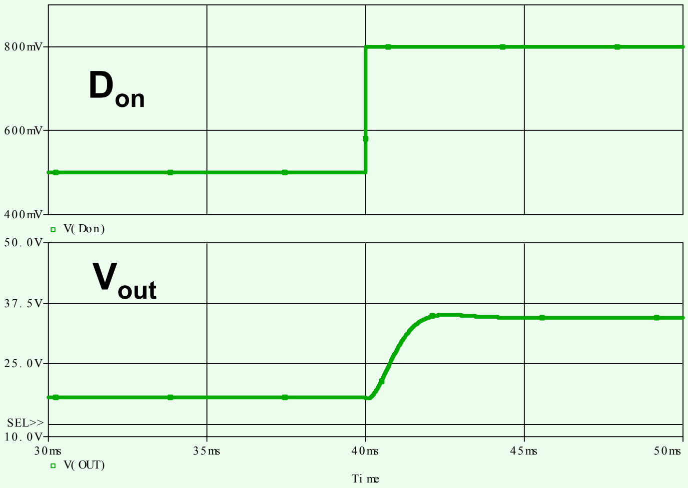


Boost: Response to step of input voltage

(average model simulation)

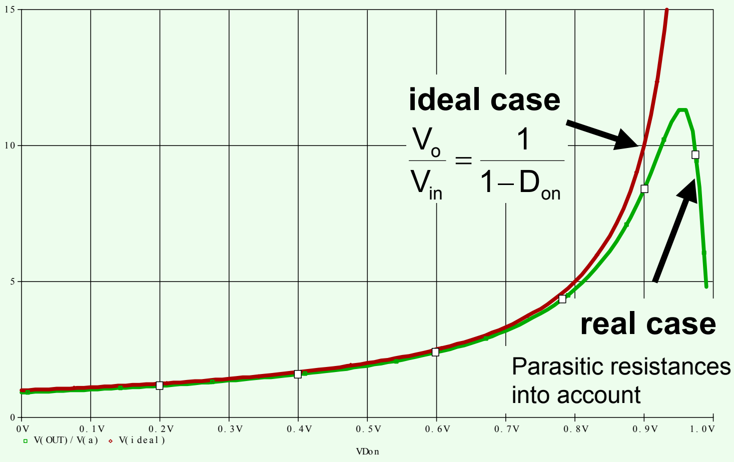


Boost: Response to step of duty cycle



Boost transfer function (CCM)

DC Sweep simulation



ideal case

$$\frac{V_o}{V_{in}} = \frac{1}{1-D_{on}}$$

real case

Parasitic resistances are taken into account