Cooperation in Wireless Networks

Ivana Marić and Ron Dabora

Stanford University

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Objectives

- The case for cooperation
- Types of cooperation
- Performance measures
- Cooperation schemes
  - Performance, limitations
  - Building blocks
- Small networks, large scale networks
- Fundamental tradeoffs
- Introduce recent results

Outline

- Introduction
- Relaying strategies
- Conferencing and feedback
- Cooperation in networks with multiple communicating pairs
- Cooperation in fading channels
- Cooperation in large-scale networks
- Summary

Introduction

Introduction

- Motivation
- Basic measures
  - Capacity
  - Scaling laws
- Channel models
  - Static channels
  - Time-varying channels
    - Diversity

Challenges

- Higher data rates and better coverage
  - USIIA 2007, RIAA 2006
- Dynamic nature: time-varying channel, users' mobility, stochastically varying traffic
- Efficient spectrum allocation and coexistence of users
- Security and privacy
- Energy efficiency
- Operating large ad hoc networks
- Guaranteed rate (Quality-of-Service)

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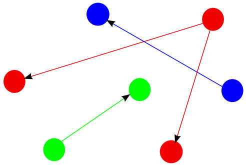
Traditional Approach: A Network is a Collection of Point-to-Point Links

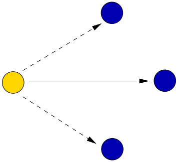
- Current wireless networks (cellular networks, Wi-Fi) are viewed as a collection of point-to-point links
  - To increase data rates the point-to-point rate is increased



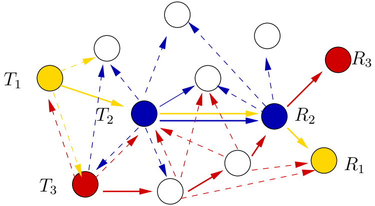
- What happens when this approach is exhausted (too expensive, approaching the fundamental limits)?

$\Rightarrow$ Need to find methods to significantly increase data rate for the same PtP link performance

Current View: Interference is Harmful



- Wireless networks are inherently broadcast
  - Any transmission is overheard by neighbouring nodes



Interference is extremely harmful for existing wireless network designs

Addressing the Challenges via Cooperation

- Nodes which are not the source or destination of a given message help communicating the message
- Different types of cooperation
  - Relaying
  - Conferencing (iterative decoding)
  - Feedback

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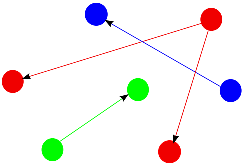

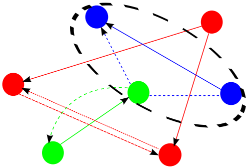

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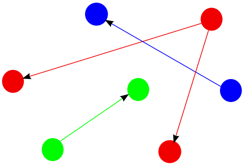

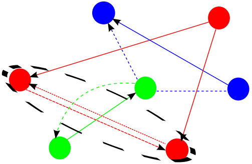

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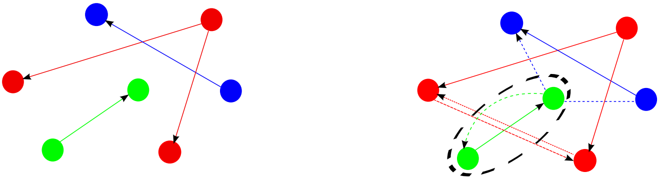

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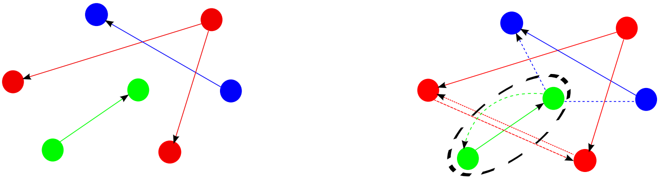

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- Different types of cooperation
  - Relaying
  - Conferencing (iterative decoding)
  - Feedback
- Future applications
  - Ad-hoc networks
  - Sensor networks
- Cooperation takes advantage of the broadcast nature of the wireless channel

Not Just Theoretical

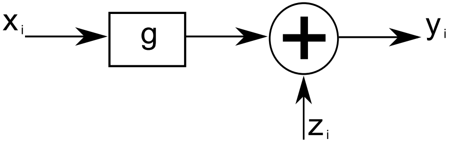
- Dlink High Speed 2.4GHz (802.11g) Wireless Range Extender

- Under development for the 802.16 (WirelessMAN/WiMAX)
  - j - multihop relay specification
  - m - advanced air interface

Let's begin

Memoryless Point-to-Point Channels

- Gaussian channel



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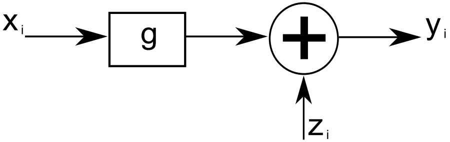
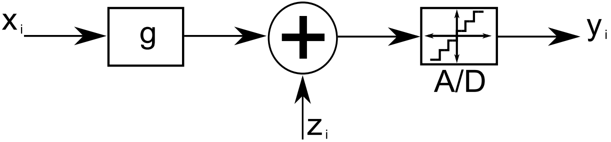


- z_i - bandlimited AWGN, i.i.d., $E\{|z_i|^2\} = N$
- Discrete channel: $x_i \in \mathcal{X}$, $y_i \in \mathcal{Y}$



The Memoryless Point-to-Point Channel Model

- A channel is characterized by the conditional distribution of its output at time i:

$$p(y_i|y^{i-1}, x^i), \qquad x_i \in \mathcal{X} \text{ and } y_i \in \mathcal{Y},$$

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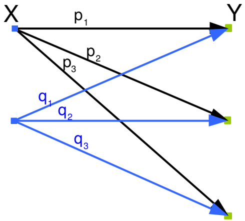
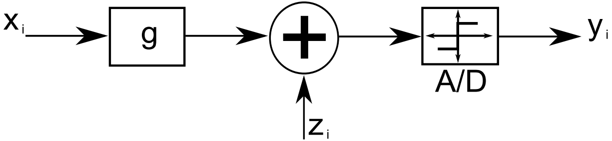
  $i = 1, 2, \ldots$
- $p(y_i|y^{i-1}, x^i)$ takes into account all the effects of signal processing: time synchronization, frequency synchronization, PLL, equalizer,...
- A channel is called memoryless if $p(y_i|y^{i-1}, x^i) = p(y_i|x_i)$



The Memoryless Point-to-Point Channel: BSC

- $|\mathcal{X}| = 2$, $|\mathcal{Y}| = 2$
- x_i - BPSK signal
- Decoding takes place after a 2-level quantization at the receiver with threshold at zero



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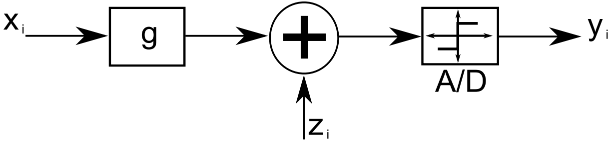
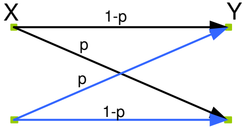
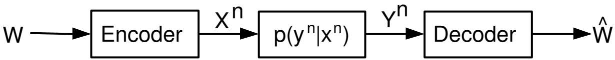


$\Leftrightarrow$ Binary symmetric channel



Channel Capacity



- R denotes the information rate in bits/symbol
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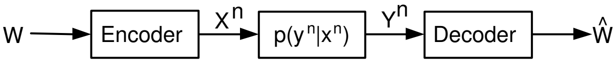

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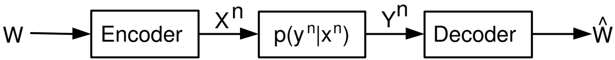

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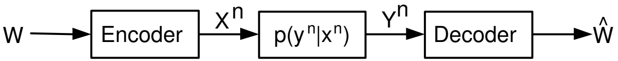

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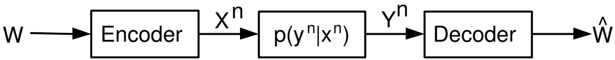

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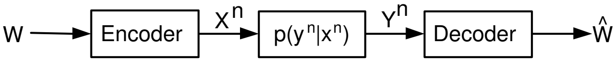
  $\Rightarrow$ $\forall \epsilon > 0$ we can find n large enough s.t. $\exists$ at least one codebook for which $P_e^{(n)} \leq \epsilon$

Channel Capacity: Converse



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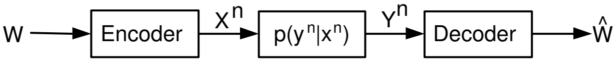

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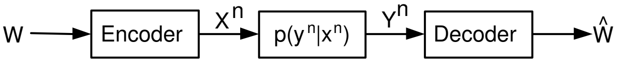

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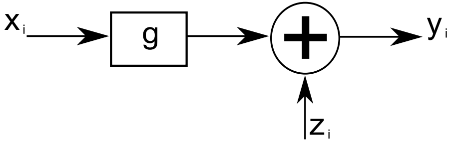
- Conversely, if $R > \max_{p(x)} I(X;Y)$ then the average probability of error achieved by *any* code is bounded away from zero for any n
- Definition: The *Capacity* of a channel is the maximal rate for which reliable communication can be achieved

AWGN Channel Capacity



- Let P be an average power constraint on the channel input:

$$\frac{1}{n}\sum_{i=1}^{n} |x_i(w)|^2 \leq P, \qquad w \in \mathcal{W}$$

- The problem: *Find the input distribution* $p(x)$ *that maximizes I(X;Y) subject to average input power constraint* P
- The solution: $X \sim \mathcal{CN}(0, P)$
- The capacity:

$$C = \log_2\left(1 + |g|^2 \frac{P}{N}\right) \text{ bits/transmission}$$

Network Capacity

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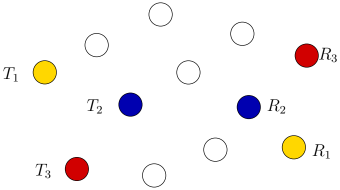
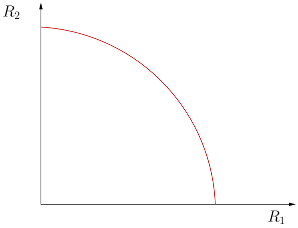
  - $\hat{W}_k^j, \;\; j \in \mathcal{D}_k$
- The probability of error for network transmission is

$$P_e^{(n)} = \Pr\left(\bigcup_{k=1}^{K} \bigcup_{j \in \mathcal{D}_k} \left\{\hat{W}_k^j \neq W_k\right\}\right)$$

Network Capacity Region

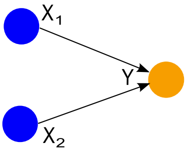
- The capacity region is the set of all rate vectors $(R_1, R_2, ..., R_K)$ such that the probability of error $P_e^{(n)}$ can be made arbitrarily small by taking n large enough
- Why it is important?
  1. It is the theoretical upper bound
  2. Determines optimal communication strategies
  3. Leads to practical designs

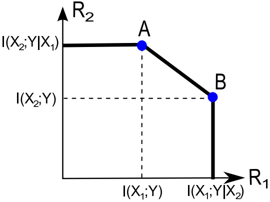




- Very hard to find

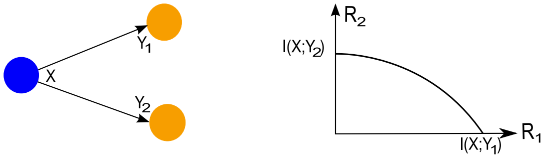
Multiuser Networks: The Multiple Access Channel





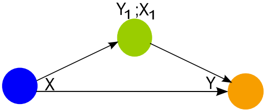
- The MAC: $p(y|x_1, x_2)$
  - $p(x_1, x_2) = p(x_1)p(x_2)$
  - Introduced by Shannon in 1961
  - Capacity known for both discrete and Gaussian channels
    - Capacity [Ahlswede'71, Liao'72]
    - MIMO [Telatar'99]
    - Fading [Gallager'94, Shamai & Wyner'97, Tse & Hanly'98]

Multiuser Networks: The Broadcast Channel



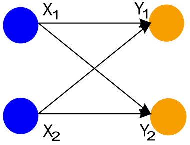
- The BC: $p(y_1, y_2|x)$
  - Introduced by Cover in 1972
  - Capacity known only for special cases
    - Degraded channels [Bergmans'73,74, Gallager'74]
    - General BC with degraded message sets [Körner & Maron'77]
    - MIMO BC [Weingarten, Steinberg & Shamai'06]
  - Best achievable region due to Marton'79
  - Best upper bound due to Nair & El-Gamal'07

Multiuser Networks: The Relay Channel



- The relay channel: $p(y, y_1|x, x_1)$
- The most basic form of cooperation
  - Introduced by van der Meulen in 1968
  - Capacity known only for special cases
    - Physically degraded channels [Cover & El-Gamal'79]
    - Gaussian relay channel with $SNR \rightarrow \infty$ [Kramer'05]
    - Stochastically degraded relay channel with deterministic link [Zhang'88]
  - Fundamental schemes introduced by Cover & El-Gamal'79
    - Decode-and-forward
    - Compress-and-forward

Multiuser Networks: The Interference Channel

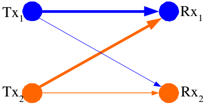


- The interference channel: $p(y_1, y_2 | x_1, x_2)$
- The building block for multiple-pairs communication
  - Introduced by Shannon in 1961
  - Capacity known only for special cases
    - Strong interference [Carleial'75, Sato'81, Costa & El-Gamal'87]
    - Gaussian IC with very weak interference [Shang, Kramer & Chen'08, Motahari & Khandani'08]
    - Cognitive Gaussian IC
    - No interference
  - Best achievable region due to Han & Kobayashi'81

The Interference Channel: Strong Interference vs. Weak Interference

Weak interference

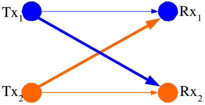
- At least one of the cross-links is worse than the its respective direct link



- Decoding W_1 at Rx_2 reduces the maximum rate of W_1
- No single-letter expression for the capacity region
- Capacity known for special cases

Strong interference

- The cross-links are better than the direct links



- Decoding W_1 at Rx_2 does not constrain the maximum rate of W_1
- The capacity achieving scheme is known [Sato'81], [Han and Kobayashi'81]

• The rates with weak interference are generally *less* than the rates with (very) strong interference

The Cut-Set Upper Bound

- A general tool for outer bounding the capacity region of a network
- Let $\mathcal{V} = \{1, 2, ..., K\}$ index the network nodes
- Let R^{ij} denote the information rate from node i to node j
- A cut is a partition of $\mathcal{V}$ into two sets $\mathcal{S}$ and $\bar{\mathcal{S}} = \mathcal{V} \setminus \mathcal{S}$

Theorem (∼Aref'80)

If the information rates $\{R^{ij}\}$ are achievable then there exists a joint distribution $p\left(x^{(1)}, x^{(2)}, ..., x^{(K)}\right)$ such that for every cut $(\mathcal{S}, \bar{\mathcal{S}})$

$$\sum_{i \in \mathcal{S}, j \in \bar{\mathcal{S}}} R^{ij} \leq I\left(\mathbf{X}^{(\mathcal{S})}; \mathbf{Y}^{(\bar{\mathcal{S}})} | \mathbf{X}^{(\bar{\mathcal{S}})}\right)$$

Operating Regimes: Half-Duplex vs. Full-Duplex

- We shall compare results for different operating regimes
- In *full-duplex* the nodes receive and transmit simultaneously
- In *half-duplex* a node can either receive or transmit
  - Often encountered in wireless systems in practice

Time-Varying Channels: Fast vs. Slow Fading

- When the channel is time varying, the received signal is given by
$$y_{r,i} = h_{tr,i} x_{t,i} + z_i$$
  - $h_{tr,i}$ is the channel gain between the transmitter and receiver at time i
- $h_{tr,i}$ models a Rayleigh fading channel: $h_{tr,i} \sim \mathcal{CN}(0,1)$
- There are three types of Rayleigh fading
  - Fast fading: $h_{tr,i} \sim \mathcal{CN}(0,1)$, i.i.d., $i = 1, 2, ..., n$
  - block fading: $h_{tr,i} = h_{tr}$, $i = 1, 2, ..., n$
  - Slow fading: $h_{tr,i} = h_{tr}$

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- There are three types of Rayleigh fading
  - Fast fading: $h_{tr,i} \sim \mathcal{CN}(0, 1)$, i.i.d., $i = 1, 2, ..., n$
  - block fading: $h_{tr,i} = h_{tr}$, $i = 1, 2, ..., n$
  - Slow fading: $h_{tr,i} = h_{tr}$

Time-Varying Channels: Channel State Information

- CSI is the knowledge a network node has on the channels
- Two types: transmitter CSI (CSIT) and receiver CSI (CSIR)
- Let H denote the random channel state and let the channel be defined by $p(y|x,h)$.
- There are four possible CSI configurations:

CSIT	CSIR	Capacity
No	No	$\max_{p(x)} I(X;Y)$
		$p(y|x) = \sum_h p(y|x,h)p(h)$
No	Yes	$\max_{p(x)} I(X;Y|H)$
Yes	No	varies
Yes	Yes	$E_H\{\max_{p(x|h)} I(X;Y|h)\}$

Time-Varying Channels: Outage Capacity

- Shannon's capacity measure is also called ergodic capacity
  - Assumes that the channel is information stable (ex. i.i.d. fading)
  - Application is delay tolerant
- For slow fading Rayleigh channels, the mutual information $I(X;Y|h)$ is a random variable
  - depends on the channel realization h
  - The channel is non-ergodic
- Note that for every $R > 0$, $\Pr\left(I(X;Y|h) < R\right) > 0$

  $\Rightarrow$ The Shannon capacity is zero
- The event $\{h : I(X;Y|h) < R\}$ is called outage
- Outage capacity is the maximum rate that can be guaranteed for a given outage probability P_{out}:

$$\sup_R \Pr(I(X;Y|h) < R) \leq P_{\text{out}}$$

Diversity

- Transmitting signals carrying the same information over different paths in time, frequency or space
- Cooperation diversity is achieved when nodes forward to other nodes information
  - Enhance desired information
  - Facilitate interference cancellation

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Diversity

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- Cooperation diversity is achieved when nodes forward to other nodes information
  - Enhance desired information
  - Facilitate interference cancellation
- Diversity reduces outage probability
- Transmitter cooperation, receiver cooperation
- When the transmitters cooperate and also the receivers cooperate the system resembles a MIMO system

  $\Rightarrow$ Distributed MIMO
- Differences between MIMO and distributed MIMO:
  - Messages known only at source nodes
  - Cannot perform antenna power allocation
  - Nodes may have half-duplex constraints

Time-Varying Channels: Finite-State Models

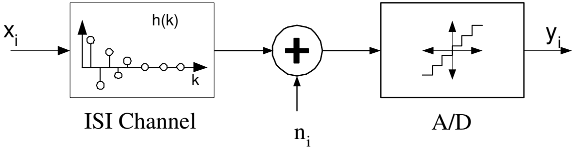
- Slow fading, fast fading are extreme cases
- An alternative model for time-varying channels with memory
  - Correlated fading, multipath
  - Filters (pulse shape, IF and RF filters)
  - AGC, Timing, PLL, equalizer
- The finite-state channel (FSC) was introduced as early as 1953 [McMillan'53]
- Time variations are represented by correlated states

Finite-State Channels

- Memory is captured by the state of the channel at the end of the previous symbol's transmission
  - S_i is the channel state at time i
  - s_0 is the initial channel state

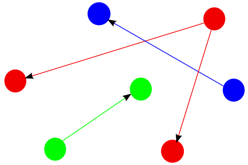
$$p(y_i, s_i | x_i, x^{i-1}, y^{i-1}, s^{i-1}, s_0) = p(y_i, s_i | x_i, s_{i-1})$$

  - S_{i-1} contains all the history information for time i
  - $\mathcal{S}$ is finite
- ISI channel: $S_{i-1} = \left(X_{i-1}, X_{i-2}, ..., X_{i-J}\right)$



Analysis of Large Networks: Scaling Laws

- Finding the capacity region of even small networks is a very difficult task
  - Many possibilities for cooperation
- Scaling laws allow us to obtain insights on the performance of large scale networks.
- Pioneering work of Gupta and Kumar'00
- Notation
  - $f(n) = \mathcal{O}(g(n)) \Leftrightarrow \lim_{n\to\infty} \left|\frac{f(n)}{g(n)}\right| < \infty$
  - $f(n) = \Theta(g(n)) \Leftrightarrow f(n) = \mathcal{O}(g(n))$ and $g(n) = \mathcal{O}(f(n))$



Analysis of Large Networks: Scaling Laws

- Definitions:
  - Assume a network that consists of n nodes that form m source-destination pairs.
  - Let d_l denote the distance between source and destination for pair $l \leq m$
  - The transport capacity is defined as

$$C_T = \sup_{(R_1, R_2, \ldots, R_m) \text{ feasible}} \sum_{l=1}^{m} R_l d_l$$

- The transport capacity
  - Provides a single number which summarizes what a network can deliver
  - Follows a scaling law such as

$$C_T(n) = \Theta(\sqrt{n}), \mathcal{O}(n) \qquad \text{bit-meters/second}$$

  - Does not provide explicit information on the individual rates

Some Important Questions

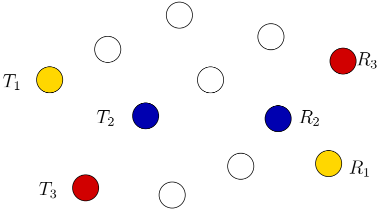
- How to incorporate relaying into the design of a network?
  - Compare performance of different schemes
  - Under what conditions capacity is achieved
  - What are the maximum rate gains we can expect from adding relays to the network?
- Different aspects of relaying that arise when considering multiple communicating pairs
  - Do not exist in the classic relay channel
- Understand the fundamental performance tradeoffs associated with node cooperation
- Analysis of cooperation in large scale networks

Cooperative Strategies

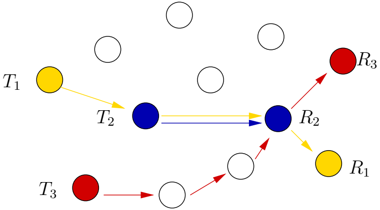
In This Section...

- Decode-and-forward
- Compress-and-forward
- Amplify-and-forward
- Capacity upper bound
- Performance comparison

Cooperation in Wireless Networks



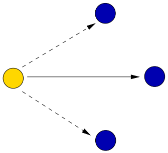
Cooperation in Wireless Networks



- Traditional approach: multihop routing
- Many point-to-point links
- Intermediate nodes store and forward packets

Broadcast

- Wireless networks are inherently broadcast
  - Any transmission is overheard by neighboring nodes



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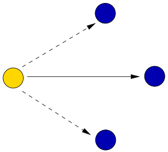
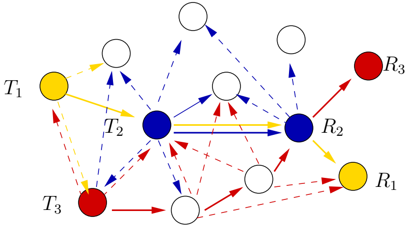
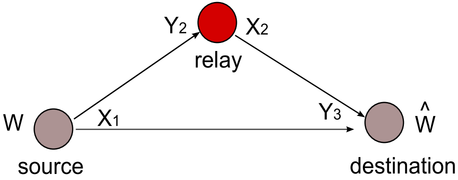




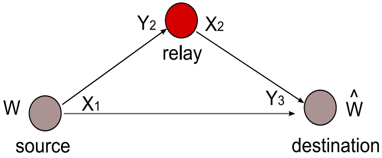
- Interference is harmful for current wireless network designs
- Cooperative strategies exploit broadcast

Relay Channel



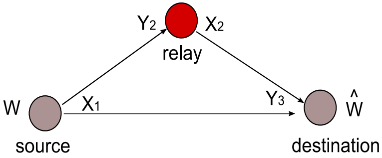
- Message $W \in \{1, \dots, M\}$ sent at rate R
- Encoding at the source: $X_1^n = f_1(W)$
- At the relay at time i: $X_{2,i} = f_{2,i}(Y_2^{i-1}),\ i = 2, \dots, n$
- Decoding: $\hat{W} = g(Y_3^n)$
- $R = \log_2 M/n$

Decode-and-Forward



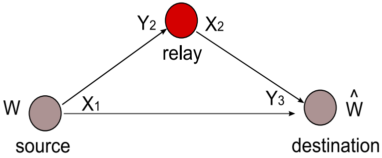
- Exploit broadcast transmission at the source

Decode-and-Forward



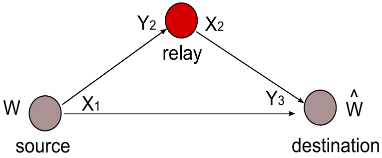
- Exploit broadcast transmission at the source
- Source and relay transmit simultaneously

Decode-and-Forward



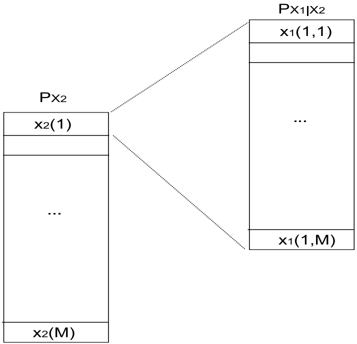
- Exploit broadcast transmission at the source
- Source and relay transmit simultaneously
- Messages sent in blocks: $w_1, w_2, \ldots, w_b, \ldots$

Decode-and-Forward



- Exploit broadcast transmission at the source
- Source and relay transmit simultaneously
- Messages sent in blocks: $w_1, w_2, \ldots, w_b, \ldots$
- Two random codebooks: x_1^n, x_2^n generated with $p(x_2)p(x_1|x_2)$

Superposition Coding



- Random codebooks x_1^n, x_2^n generated with $p(x_2)p(x_1|x_2)$

In block b:

- The source: $x_1^n(h_b(w_{b-1}), w_b)$ *block Markov encoding*
- The relay: $x_2^n(h_b(w_{b-1}))$

Decode-and-Forward Strategies

- Irregular encoding, successive decoding
  - Codebooks x_1^n, x_2^n have different sizes
- Regular encoding, sliding-window decoding
  - Decoding over two block
- Regular encoding, backward decoding
  - Decoding starts from the last received block

Decode-and-Forward

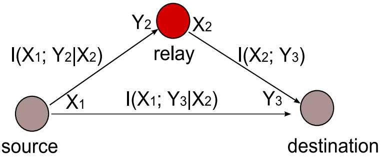
	block 1	block 2	block 3	block 4
source	$x_1(1,w_1)$	$x_1(w_1,w_2)$	$x_1(w_2,w_3)$	$x_1(w_3,1)$
relay	$x_2(1)$	$x_2(w_1)$	$x_2(w_2)$	$x_2(w_3)$

relay decodes w_1 (end of block 1)

destination decodes w_1 (end of block 2)

$$R \leq I(X_1; Y_2|X_2)$$
$$R \leq I(X_1; Y_3|X_2) + I(X_2; Y_3) = I(X_1, X_2; Y_3)$$

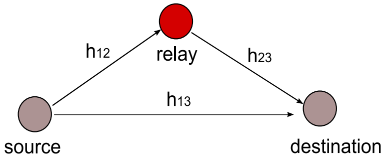
$$R = \max_{p(x_1,x_2)} \min\{I(X_1; Y_2|X_2), I(X_1, X_2; Y_3)\}$$



Decode-and-Forward in AWGN Channels

- Choose: Gaussian $p(x_1, x_2)$
  - $E[|X_1|^2] = P_1$, $E[|X_2|^2] = P_2$, $E[X_1 X_2^*] = \rho\sqrt{P_1 P_2}$
- Superposition codebook:
- Gen. symbols: $X_{10} \sim \mathcal{CN}(0, (1-\rho^2)P_1)$, $X_2 \sim \mathcal{CN}(0, P_2)$
- In block b: $x_{10}^n(w_b), x_2^n(w_{b-1})$

  $x_1^n(w_{b-1}, w_b) = x_{10}^n(w_b) + \rho\sqrt{\frac{P_1}{P_2}} x_2^n(w_{b-1})$

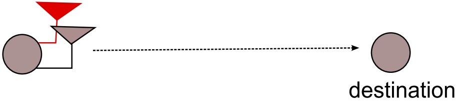


$$Y_2 = h_{12}X_1 + Z_2$$
$$Y_3 = h_{13}X_1 + h_{23}X_2 + Z_3$$

DF Rate in AWGN Channels

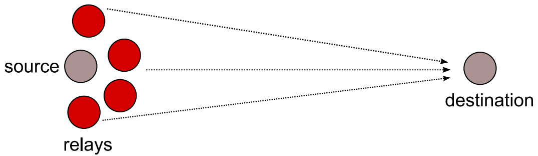
$$R = \max_{\rho} \min \left\{ \log_2 \left(1 + \frac{|h_{12}|^2 (1 - |\rho|^2) P_1}{N} \right), \right.$$
$$\left. \log_2 \left(1 + \frac{|h_{13}|^2 P_1}{N} + \frac{|h_{23}|^2 P_2}{N} + \frac{2\mathcal{R}e\{\rho h_{13} h_{23}^*\} \sqrt{P_1 P_2}}{N} \right) \right\}$$

- Signals *coherently-combined*
- Relay signal perfectly phase-aligned with the source signal
- Not practical
- Decoding constraint at the relay can be severe
- DF optimal for $|h_{12}| \to \infty$: source and relay act as two transmit antennas
- DF performs well when the relay is close to the source

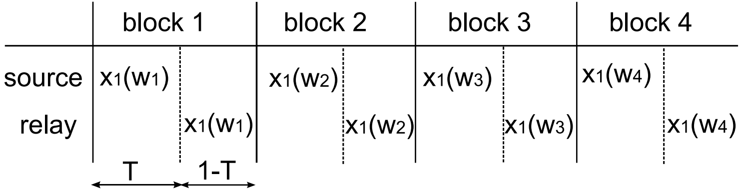


Antenna-Clustering Capacity

- Generalizes to multiple relays
- Relays act as a multiple-transmit antenna

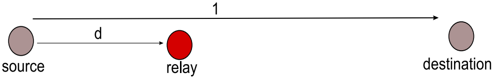


Classic Multihop



$$R = \min\left\{ T\log_2(1+\frac{|h_{12}|^2P_1}{TN}), \bar{T}\log_2(1+\frac{|h_{23}|^2P_2}{\bar{T}N}) \right\}$$

- For $\alpha = 2$, performs worse then using no relay at all
- Gains for $\alpha > 2$ and for half-duplex relays
    - α-path-loss exponent



DF in Half-Duplex Relay Channel

	block 1	block 2	block 3	block 4
source	$x_1(w_1)$	$x_1(w_1)$	$x_1(w_2)$	$x_1(w_2)$
relay		$x_2(w_1)$		$x_2(w_2)$

↑ relay decodes w_1 (end of block 1)

↑ destinations decodes w_1 (end of block 2)

- All nodes know a priori when a relay listens/talks

DF in Half-Duplex Relay Channel

	block 1	block 2	block 3	block 4
source	$x_1(w_1)$	$x_1(w_1)$	$x_1(w_2)$	$x_1(w_2)$
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relay decodes w_1

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- All nodes know a priori when a relay listens/talks
- Mode modulation: data modulates listen/talk interval

DF in Half-Duplex Relay Channel

	block 1	block 2	block 3	block 4
source	$x_1(w_1)$	$x_1(w_1)$	$x_1(w_2)$	$x_1(w_2)$
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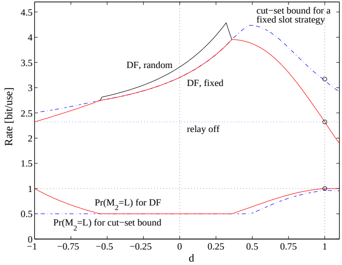

Compress-and-Forward

- Relay does not decode the source message
- Relay *quantizes* Y_2^n into quantization codeword $\hat{Y}_2^n$
  - By finding a jointly typical $\hat{y}_2^n$ with received y_2^n
- Three codebooks: $x_1^n(w_b), \hat{y}_2^n(s_{b-1}, z_b), x_2^n(s_{b-1})$

How does relay operate?

- In block b: knows s_{b-1}, decides on z_b thru quantization
- Obtains $\hat{y}_2(s_{b-1}, z_b)$

What does it send?

- Binning: each z randomly assigned to bin s
- In block $b+1$: sends $x_2(s_b)$ such that $z_b \in s_b$

Compress-and-Forward

	block 1	block 2	block 3	block 4
source	$x_1(w_1)$	$x_1(w_2)$	$x_1(w_3)$	$x_1(1)$
relay	$x_2(1)$	$x_2(s_1)$	$x_2(s_2)$	$x_2(s_3)$
relay determines	$\hat{y}_2(1,z_1)$	$\hat{y}_2(s_1,z_2)$	$\hat{y}_2(s_2,z_3)$	$\hat{y}_2(s_3,1)$

↑ destination decodes w_1 (at the end of block 2)

Destination in block $b+1$:

- Decodes s_b
- Determines $z_b \in s_b$
- Knows $\hat{y}_2(s_{b-1}, z_b), x_2(s_{b-1})$
- Decodes w_b using $(\hat{y}_2(s_{b-1}, z_b), y_{3,b})$

Compress-and-Forward

	block 1	block 2	block 3	block 4
source	$x_1(w_1)$	$x_1(w_2)$	$x_1(w_3)$	$x_1(1)$
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↑ destination decodes w_1 (at the end of block 2)

Destination in block $b+1$:

- Decodes s_b $\quad \rightarrow R_Q \leq I(X_2; Y_3)$
- Determines $z_b \in s_b$
- Knows $\hat{y}_2(s_{b-1}, z_b), x_2(s_{b-1})$
- Decodes w_b using $(\hat{y}_2(s_{b-1}, z_b), y_{3,b})$ $R \leq I(X_1; \hat{Y}_2, Y_3 | X_2)$

Compress-and-Forward Rate

$$R = I(X_1; \hat{Y}_2, Y_3 | X_2)$$

subject to

$$I(\hat{Y}_2; Y_2 | Y_3 X_2) \leq I(X_2; Y_3)$$

for $p(x_1)p(x_2)p(\hat{y}_2|x_2, y_2)p(y_1, y_2|x_1, x_2)$

- R is single-user rate when receiver has two antennas

Compress-and-Forward in AWGN Channels

- Choose: $\hat{Y}_2 = Y_2 + \hat{Z}_2 \quad \hat{Z}_2 \sim \mathcal{CN}(0, \hat{N}_2)$
- For smallest $\hat{N}_2$ choose: $I(Y_2; \hat{Y}_2 | X_2 Y_3) = I(X_2; Y_3)$

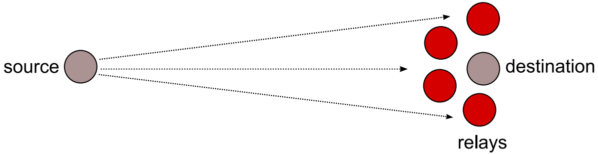
$$\hat{N}_2 = N \frac{P_1(|h_{12}|^2 + |h_{13}|^2) + N}{P_2 |h_{23}|^2}$$

$$R = \log_2 \left(1 + \frac{P_1 |h_{12}|^2}{N + \hat{N}_2} + \frac{P_1 |h_{13}|^2}{N} \right)$$

- Optimal for $|h_{23}| \to \infty$: relay and destination act as two-receive antenna
- CF performs well when the relay is close to destination

Antenna-Clustering Capacity

- Generalizes to multiple relays
- Relays act as a multiple-receive antenna



Antenna-Clustering Capacity

- Two closely spaced clusters: DF and CF
- Achieves optimal scaling behavior

Amplify-and-Forward

- In discrete channel: $X_{2,i} = Y_{2,i-1}, \quad \mathcal{Y} \subseteq \mathcal{X}$

Amplify-and-Forward

- In discrete channel: $X_{2,i} = Y_{2,i-1}, \quad \mathcal{Y} \subseteq \mathcal{X}$
- In Gaussian channel: $X_{2,i} = a_i Y_{2,i-1} \qquad i = 1, \ldots, n$
- a_i chosen to satisfy power constraint

Amplify-and-Forward

- In discrete channel: $X_{2,i} = Y_{2,i-1}, \quad \mathcal{Y} \subseteq \mathcal{X}$
- In Gaussian channel: $X_{2,i} = a_i Y_{2,i-1} \qquad i = 1, \ldots, n$
- a_i chosen to satisfy power constraint
- At the destination channel with ISI:

$$\begin{aligned} Y_{3,i} &= h_{13} X_{1,i} + h_{23} X_{2,i} + Z_{3,i} \\ &= h_{13} X_{1,i} + a h_{12} h_{23} X_{1,i-1} + Z'_{3,i} \end{aligned}$$

Amplify-and-Forward

- In discrete channel: $X_{2,i} = Y_{2,i-1}, \quad \mathcal{Y} \subseteq \mathcal{X}$
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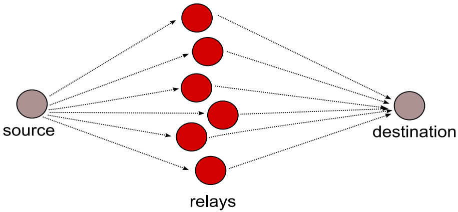
- Waterfilling optimization of the spectrum of X_1^n
  - Relay should not always transmit with maximum power
- In low-SNR: *bursty* AF improves performance

AF Scaling Capacity



- Optimal scaling as number of relays increases
- Requires coherent combining of relay signals

Cut-Set Bound on Capacity

- Cut: partition of the set of nodes into two sets: $(\mathcal{S}, \bar{\mathcal{S}})$
- $W(\mathcal{S})$- set of messages with source in $\mathcal{S}$ and sink in $\bar{\mathcal{S}}$
- Choose encoders (inputs): P_X.
- Denote as $\mathcal{R}(P_X, \mathcal{S})$ set of rates that satisfies:

$$\sum_{w \in W(\mathcal{S})} R_w \leq I(X_{\mathcal{S}}; Y_{\bar{\mathcal{S}}} | X_{\bar{\mathcal{S}}}) \tag{1}$$

- Cut-set bound for fixed P_X:

$$\mathcal{R}(P_X) = \bigcap_{\mathcal{S}} \mathcal{R}(P_X, \mathcal{S})$$

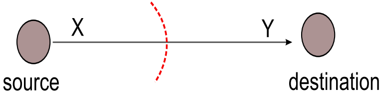
- Cut-set bound:

$$\bar{\mathcal{R}} = \bigcup_{P_X} \mathcal{R}(P_X)$$

Cut-Set Bound Examples

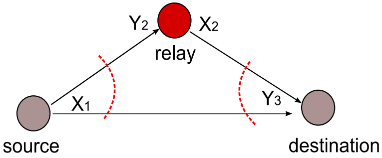
- Point-to-point channel

$$\bar{\mathcal{R}} = \bigcup_{P_X} I(X;Y)$$

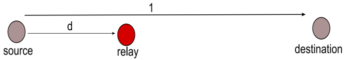


- Relay Channel

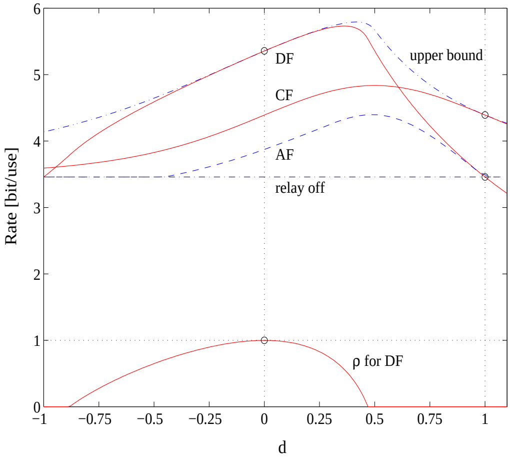
$$\bar{\mathcal{R}} = \bigcup_{P_{X_1 X_2}} \min\{I(X_1;Y_2,Y_3|X_2), I(X_1,X_2;Y_3)\}$$



Performance Comparison



$P_1 = P_2 = 10$, N=1,
$\alpha = 2$



Cooperative Strategies: Summary

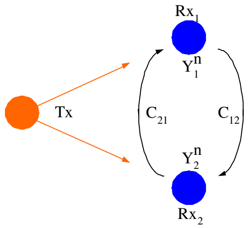
- DF: when relay is close to source
- CF: when relay is close to destination
- Generalize to multiple relays
- Capacity results are rare

Conferencing and Feedback

In This Section...

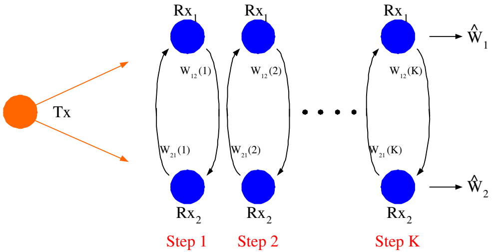
- Cooperation via conferencing
- Feedback
  - Fundamental results for memoryless channels
  - Finite-state channels
- Summary

Conferencing



- Conferencing refers to two users interactively helping each other decode their messages:
  - The transmission over the wireless medium is typically received by users in the vicinity of the target user
  - Users have dedicated (orthogonal) links between them, over which they communicate

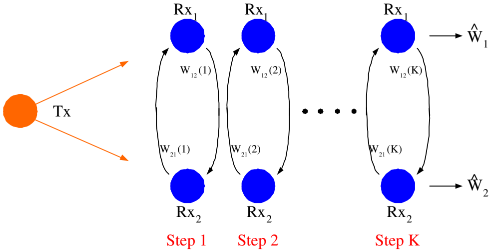
Multi-Step Conferencing



- A conference can span several cycles
  - At each cycle receivers use more refined knowledge on the other receiver's channel output
- Decoding takes place after the last cycle
- Admissible conference: the total rates of the conference messages is less than the capacity of the conference links

$$\frac{1}{n}\sum_{k=1}^{K}\log_2\left|\mathcal{W}_{ij}^{(k)}\right| \le C_{ij}, \qquad (i,j)\in\left\{(1,2),(2,1)\right\}$$

A Conference: Formal Definition



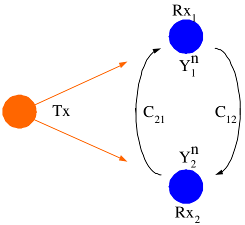
- An (C_{12}, C_{21})-admissible K-cycle conference between Rx_1 and Rx_2 consists of
  - K message sets from node i to node j, $(i,j) = \{(1,2),(2,1)\}$

$$\mathcal{W}_{ij}^{(k)} = \left\{1, 2, ..., 2^{nR_{ij}^{(k)}}\right\}, \qquad k = 1, 2, ..., K.$$

  - K pairs of mapping functions

$$\begin{aligned} h_{12}^{(k)} &: \mathcal{Y}_1^n \times \mathcal{W}_{21}^{(1)} \times \mathcal{W}_{21}^{(2)} \times \cdots \times \mathcal{W}_{21}^{(k-1)} \mapsto \mathcal{W}_{12}^{(k)} \\ h_{21}^{(k)} &: \mathcal{Y}_2^n \times \mathcal{W}_{12}^{(1)} \times \mathcal{W}_{12}^{(2)} \times \cdots \times \mathcal{W}_{12}^{(k)} \mapsto \mathcal{W}_{21}^{(k)} \end{aligned}$$

Full Cooperation



- Full cooperation: When each receiver sends his channel output to the other receiver
  - Y_1^n becomes available at Rx_2
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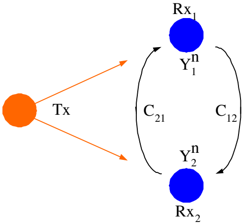
Full Cooperation



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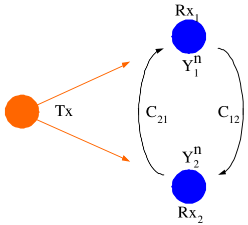

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  - Rate $I(X; Y_1, Y_2)$ is achievable at Rx$_1$

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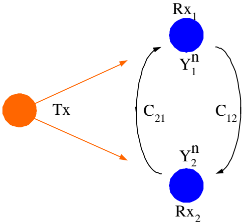

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  - Using a scheme by Slepian & Wolf'73
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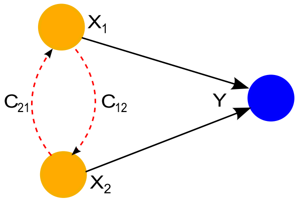
- We will focus on results for partial cooperation

Conferencing: MAC



- The encoders exchange messages prior to transmission
- The capacity region [Willems'83]

$$
\begin{aligned}
R_1 &\leq I(X_1; Y|X_2, U) + C_{12} \\
R_2 &\leq I(X_2; Y|X_1, U) + C_{21} \\
R_1 + R_2 &\leq \min\left\{I(X_1, X_2; Y|U) + C_{12} + C_{21}, I(X_1, X_2; Y)\right\}
\end{aligned}
$$

for $p(u)p(x_1|u)p(x_2|u)$

- This is achieved with a single conference step

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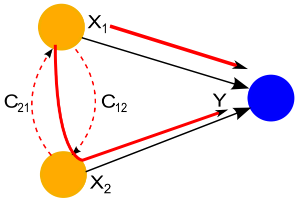

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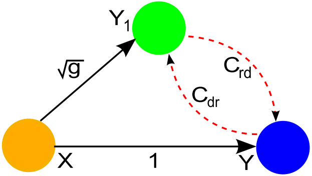
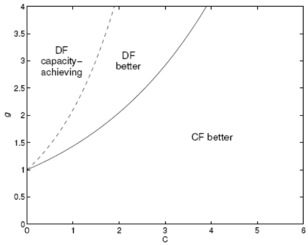
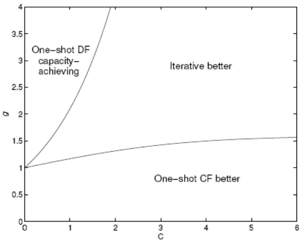
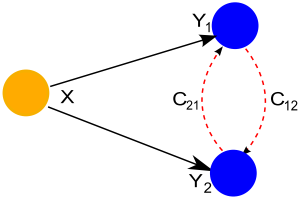


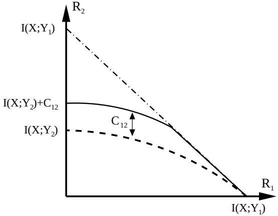
- Compare two schemes ($C = C_{rd} + C_{dr}$):
  - Single step (classic relaying, $C_{dr} = 0$)
  - Single cycle with
    - Step 1: CF from destination to relay
    - Step 2: DF from relay to destination



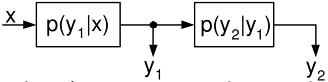


Conferencing: Broadcast Channel





- When the channel is physically degraded, a single conference step achieves capacity



  - It is enough to let the strong receiver assist the weak receiver

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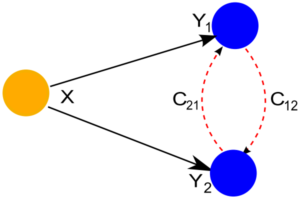

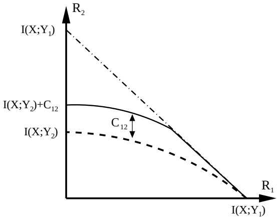

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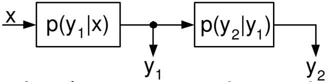
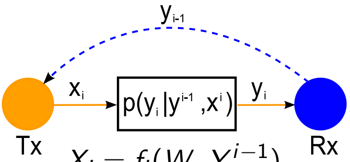


  - It is enough to let the strong receiver assist the weak receiver
- For the general BC
  - It is still an open question whether higher rates can be achieved with multiple steps
  - Can design a K-cycle conference using $K-1$ CF cycles and a final DF step

Feedback

- PtP channel: the receiver sends back information to the transmitter
  - Allows transmitter to adapt its signal to the channel

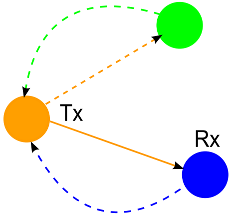


$$X_i = f_i(W, Y^{i-1})$$

- Network: the wireless medium is a broadcast medium
  - Signals received at nodes in the vicinity of the destination are correlated with the signal at the destination node

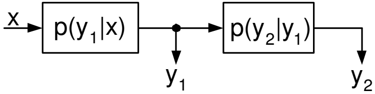
Feedback in Multiuser Scenarios

- In Multiuser scenarios feedback facilitates both direct and indirect cooperation
  - Direct: Feedback sent from the destination receiver
  - Indirect: Feedback sent from neighbouring receivers
- Consider for example the BC
  - Feedback from one receiver can increase the rate to both receivers



Memoryless Multiuser Scenarios

- Sometimes feedback does not help
  - The PtP DMC ($p(y^n|x^n) = \prod_{i=1}^{n} p(y_i|x_i)$)
  - The physically degraded DMBC [El-Gamal'78,81]



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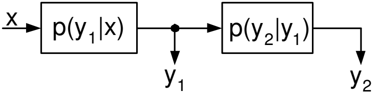

- Feedback does help in the following scenarios:
  - The discrete, memoryless MAC [Gaarder & Wolf'75]
  - The discrete, memoryless relay channel
    - Feedback achieves the cut-set bound [Cover & El-Gamal'79]
  - The general BC [Ozarow'79]
    - Including the stochastically degraded channel

Channels with Memory: Finite-State Channels

- The memory for time i is represented by the state S_{i-1}
  - The PtP-FSC:

$$p(y_i, s_i | x_i, x^{i-1}, y^{i-1}, s^{i-1}, s_0) = p(y_i, s_i | x_i, s_{i-1})$$

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  - The FSBC:

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Finite-State Channels

- Can model effects beyond the physical propagation medium
  - Filters, loops
- Example: to incorporate the effect of a K-tap equalizer, the state S_{i-1} can also be a function of Y_{i-K}^{i-1}
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Finite-State Channels: Capacity of the PtP-FSC

- The capacity of a channel with memory is usually given by a limiting expression as the blocklength $n \to \infty$
  - We must verify that the limit exists and is finite
  - Otherwise the channel does not support reliable communication in the Shannon sense
- We assume no CSI
- Capacity without feedback [Gallager'68]

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$$C_{FB} = \lim_{n\to\infty} \max_{\prod_{i=1}^{n} p(x_i|x^{i-1}, y^{i-1})} \min_{s_0 \in \mathcal{S}} \frac{1}{n} I(X^n \to Y^n | s_0)$$

Remark

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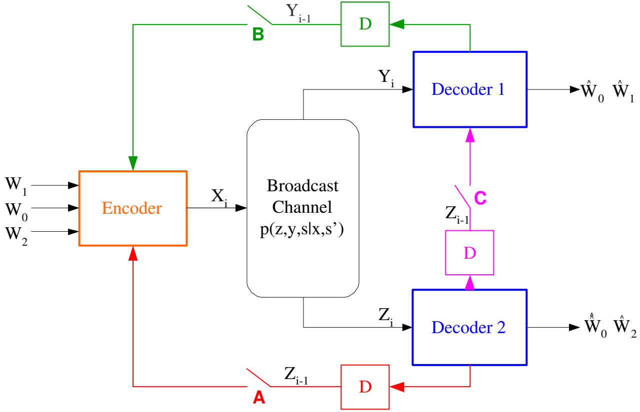
- Feedback increases the capacity of the PtP-FSC [Permuter et al.'08]
  - In contrast to the DMC

The Finite-State Broadcast Channel with Feedback and Cooperation



- 8 possible configurations
- Switch C facilitates full cooperation

The Finite-State Broadcast Channel with Feedback and Cooperation: Conclusions

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- Finite-state channels allow modelling of propagation as well as implementation aspects

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  - Channel Capacity
  - Transport capacity
    - Scaling laws

Summary First Half

- Relaying
  - Decode-and-forward
  - Compress-and-forward
  - Amplify-and-forward

Summary First Half

- Relaying
  - Decode-and-forward
  - Compress-and-forward
  - Amplify-and-forward
- Conferencing
  - Interactive decoding

Summary First Half

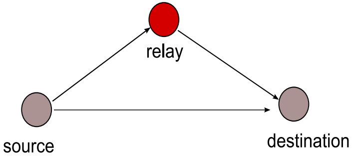
- Relaying
  - Decode-and-forward
  - Compress-and-forward
  - Amplify-and-forward
- Conferencing
  - Interactive decoding
- Feedback
  - Cooperation between transmitters and receivers
  - Useful in network scenarios
    - Feedback does not have to come from the target receiver

Networks with Multiple Source-Destination Pairs

In This Section...

- Differences when relaying for multiple pairs
- Cooperation in interference channel with a relay
- Cooperation in cognitive radio networks

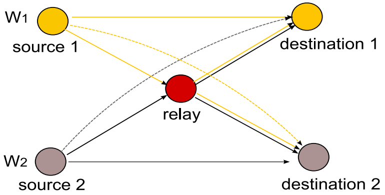
Relaying



- Relay strategies well developed
  - decode, compress, amplify -and-forward
- Capture broadcast
- No interference
  - One flow

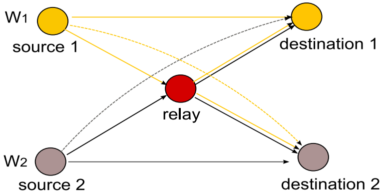
Relaying for Multiple Sources?

- The smallest network: interference channel with a relay



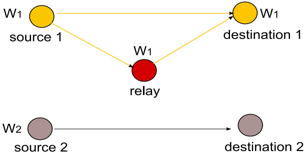
Relaying for Multiple Sources?

- The smallest network: interference channel with a relay

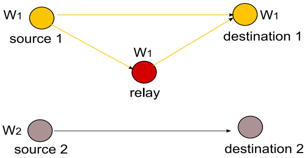


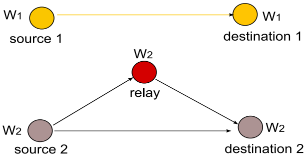
- Simple approach: multihop routing
- Relay time-shares in helping sources

Multihop

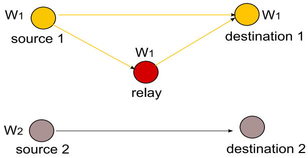

W1
source 1
W1
destination 1
W1
relay
W2
source 2
destination 2

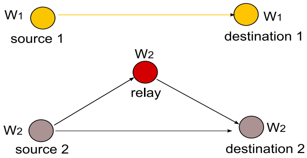
Multihop


W1
source 1
W1
relay
W1
destination 1
W2
source 2
destination 2


W1
source 1
W2
relay
W1
destination 1
W2
source 2
W2
destination 2

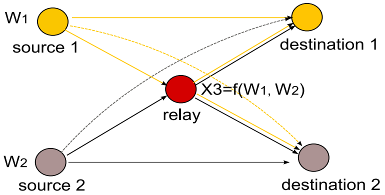
Multihop





- How can we do better?
- No combining of bits, symbols or packets at the relay

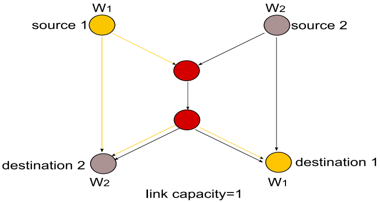
Generalized Relaying



- Joint encoding and forwarding of multiple data streams

Network Coding

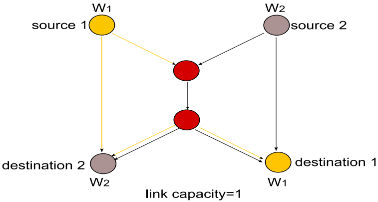
Butterfly network:



Routing achieves
$(R_1, R_2) = (\beta, 1 - \beta)$,
for any $\beta \in [0, 1]$

Network Coding

Butterfly network:



Routing achieves
$(R_1, R_2) = (\beta, 1 - \beta)$,
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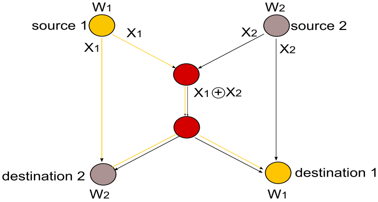
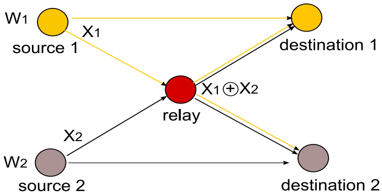


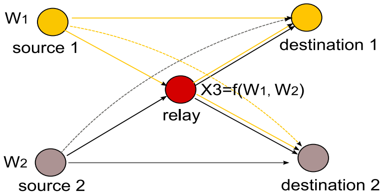
Network coding: relay combines packets. Achieves $(R_1, R_2) = (1, 1)$

Joint Encoding of Messages

Network Coding idea:



Generalized relaying:



Encoding Elements From...

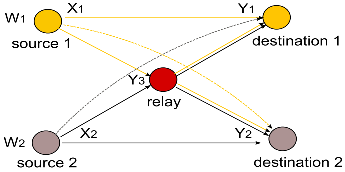
- Relay channel: generalized amplify, quantize, decode -and-forward
- MAC channel: interference cancellation
- Interference channel: rate-splitting
- Broadcast channel: binning, dirty paper coding

- Many encoding strategies can be applied
- Evaluation is difficult
- Goal: Develop strategies that can be applied to larger networks and can bring gains

Simple Joint Encoding Strategies: Gaussian Channel

$$Y_3 = h_{13}X_1 + h_{23}X_2 + Z_3$$

$$Y_j = \sum_{i=1}^{3} h_{ij}X_i + Z_j$$



- Amplify-and-Forward (analog network coding):

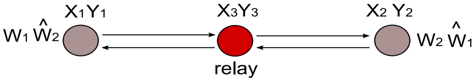
$$X_3 = cY_3 = c(h_{13}X_1 + h_{23}X_2 + Z_3)$$

- Decode-and-Forward:

$$X_3 = \sqrt{P_3}(\sqrt{c}V_1(W_1) + \sqrt{\bar{c}}V_2(W_2))$$

- Can outperform time-sharing

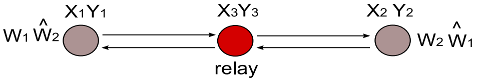
DF with Network Coding



$$Y_1 = h_{31}X_3 + Z_1$$
$$Y_2 = h_{32}X_3 + Z_2$$
$$Y_3 = h_{13}X_1 + h_{23}X_2 + Z_3$$

- MAC to relay, BC to sources

DF with Network Coding



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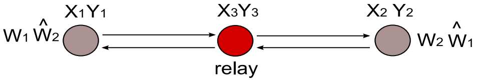
- MAC to relay, BC to sources

Relay broadcasts:

- For $R_1 = R_2 : x_3^n(w_1 \oplus w_2)$

DF with Network Coding



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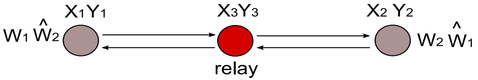
Relay broadcasts:

- For $R_1 = R_2 : x_3^n(w_1 \oplus w_2)$
- For $R_1 \geq R_2 : x_3^n(w_{11}, w_{12} \oplus w_2)$
  where he splits $w_1 = (w_{11}, w_{12})$ at (R_1', R_2)

DF with Network Coding



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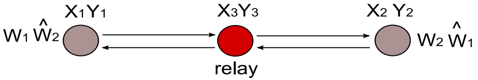
- AF: $x_3 = a(h_{13}x_1 + h_{23}x_2 + z_3)$, under power constraint

DF with Network Coding



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- MAC to relay, BC to sources

Relay broadcasts:

- For $R_1 = R_2 : x_3^n(w_1 \oplus w_2)$
- For $R_1 \geq R_2 : x_3^n(w_{11}, w_{12} \oplus w_2)$
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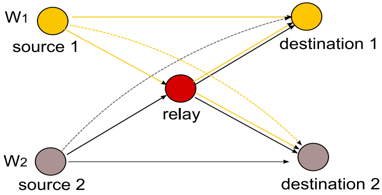
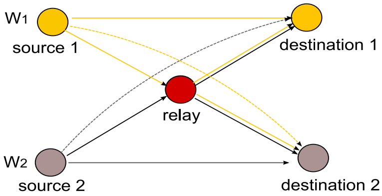
- AF: $x_3 = a(h_{13}x_1 + h_{23}x_2 + z_3)$, under power constraint
- $x_3^n(w_1, w_2)$

Differences when Relaying for Multiple Sources

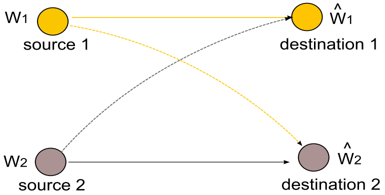


Differences when Relaying for Multiple Sources



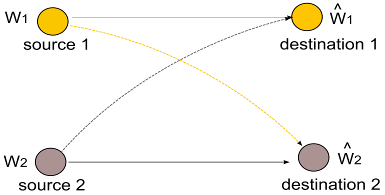
- Joint relaying of multiple data streams
- Interference:
- Sources create interference
- Relaying one message increases interference to other users

Interference Channel



- No relay
- Capacity region unknown

Interference Channel

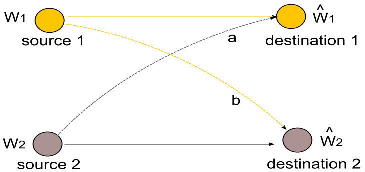


- No relay
- Capacity region unknown
- except...

In Strong Interference

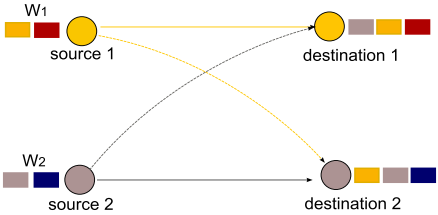
Gaussian channel:

$$Y_1 = X_1 + aX_2 + Z_1$$
$$Y_2 = bX_1 + X_2 + Z_2$$



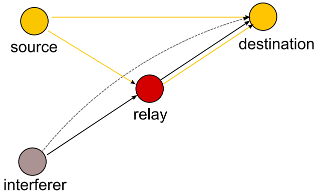
- Cross-link is 'stronger' than direct: $a, b \geq 1$
- Optimal: jointly decode both messages
- Multiaccess channel to each receiver
- Gains from interference cancellation

In General: Rate-Splitting



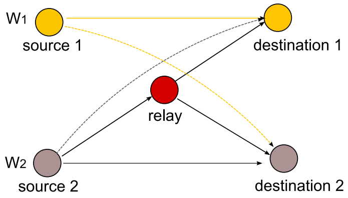
- If interference not strong: unwanted messages cannot be decoded
- To reduce interference: partial decoding
- An encoder splits message into two messages
- Decoder decodes one unwanted message and cancels interference

Interference Forwarding



- Relay observes signals from both sources
- Relay can use some of its power to forward interference
- Increase interference to cancel it

Special Case Scenario



- No source1-relay link
- Can forwarding interference W_2 help both receivers?
- Increases rate R_2 but increases interference at destination 1

Encoding

- No rate-splitting nor binning
- Block-Markov, regular encoding
- Decoding: sliding-window or backward

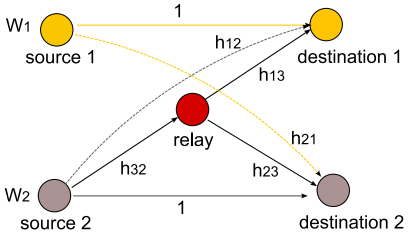
	block 1	block 2	block 3	block 4
source 1	$x_1(w_1(1))$	$x_1(w_1(2))$	$x_1(w_1(3))$	$x_1(w_1(4))$
source 2	$x_2(1,w_2(1))$	$x_2(w_2(1),w_2(2))$	$x_2(w_2(2),w_2(3))$	$x_2(w_2(3),1)$
relay	$x_3(1)$	$x_3(w_2(1))$	$x_3(w_2(2))$	$x_3(w_2(3))$

↑ relay decodes $w_2(1)$

↑ destinations decide $w_1(1)$, $w_2(1)$

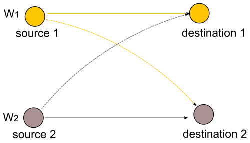
Gaussian Channel

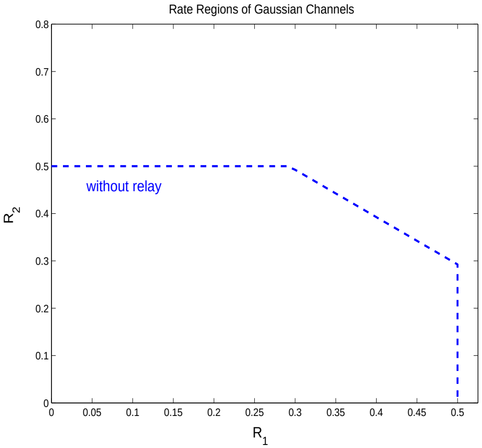
$$Y_1 = X_1 + h_{12}X_2 + h_{13}X_3 + Z_1$$
$$Y_2 = h_{21}X_1 + X_2 + h_{23}X_3 + Z_2$$
$$Y_3 = h_{32}X_2 + Z_3$$



No Relaying

- No relay: strong interference regime
-

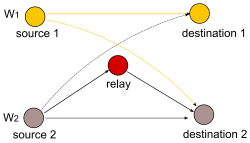


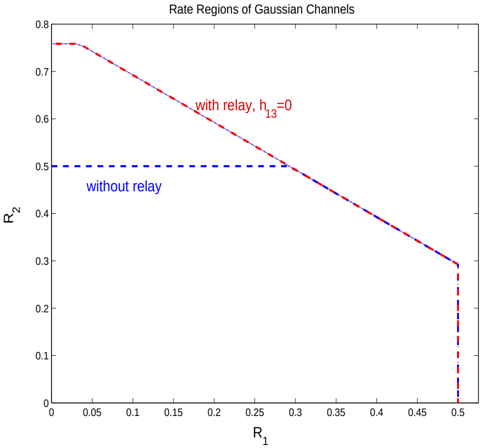


$h_{12} = 1, h_{21}^2 = 2, h_{23}^2 = 0.15, h_{32}^2 = 12$

Relaying

- No relay: strong interference regime
- With relay, no interference forwarding

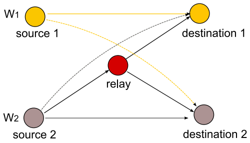


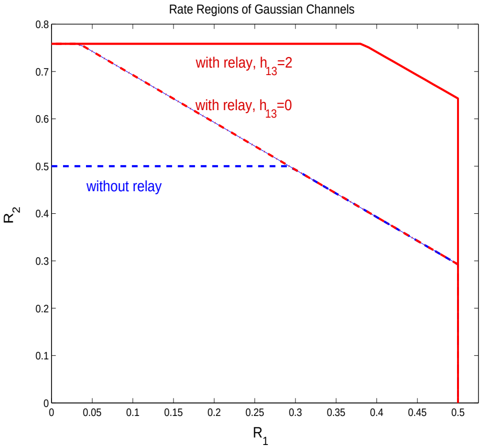


$h_{12} = 1, h_{21}^2 = 2, h_{23}^2 = 0.15, h_{32}^2 = 12$

Relaying and Interference Forwarding

- No relay: strong interference regime
- With relay, and interference forwarding





$$h_{12} = 1, h_{21}^2 = 2, h_{23}^2 = 0.15, h_{32}^2 = 12$$

Interference Forwarding

Relay can...

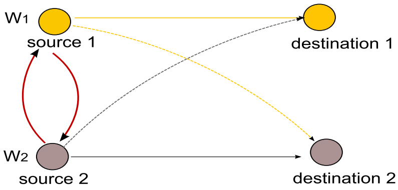
- help decoder by interference forwarding
  - Interference cancelation
- hurt decoder by increasing interference
  - Interference rate becomes too large

Interference forwarding:

- through decode,compress -and-forward
- More general schemes

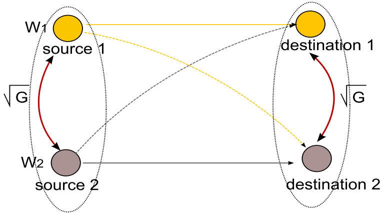
Virtual (Distributed) MIMO

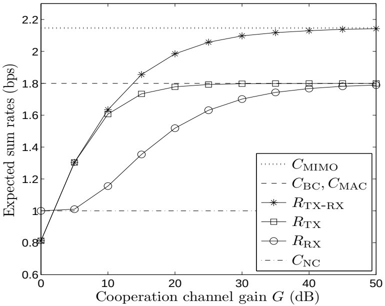
- No dedicated relay
- Transmitter cooperation
- Transmitters need knowledge about each other's messages
- Obtained through:
  1. Cooperative strategies
  2. Dedicated orthogonal links; conferencing
  3. Feedback
  4. Cognition



Gains From Virtual MIMO

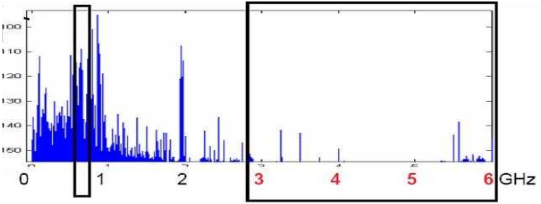
- Orthogonal links for cooperation





Cognitive Radio Networks

- Motivation: bandwidth gridlock
- Wireless spectrum is crowded
- Licensed band not efficiently used
- Its inefficient use led to *spectrum holes*

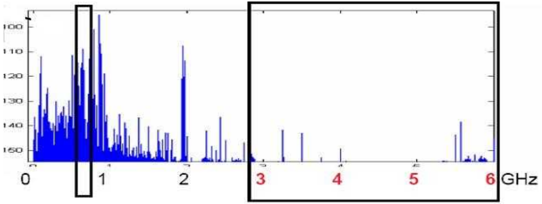


From slides by B. Brodersen, BWRC cognitive radio workshop

Cognitive Radios

- Co-exist with oblivious users without impacting their service
- Sense the environment
- Use the obtained side information to adaptively transmit

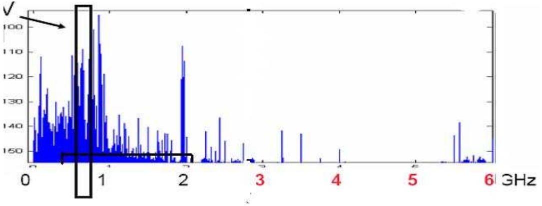
Interweave (Opportunistic) Approach



From slides by B. Brodersen, BWRC cognitive radio workshop

- Dynamic spectrum access
- Sense the environment
- Transmit in a spectrum hole

Underlay Approach

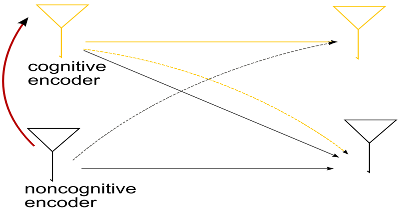


- Share the bandwidth; created interference below a threshold

Cognition and Cooperation

- Why not use obtained information for cooperation?
- In cooperation: a helper needs knowledge about relayed message
  - Assistance of the source node
  - Listening to the channel
- Cognitive node can obtain similar information through cognition
- Overlay paradigm: share the band and compensate for interference by cooperation

Overlay Approach



- What is the optimal cognitive strategy?

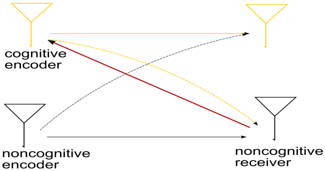
It All Hinges on...Side Information

- Interweave: users' activity
- Underlay: channel gains
- Overlay: channel gains, codebooks and (partial) messages

How Can Side Information be Obtained?

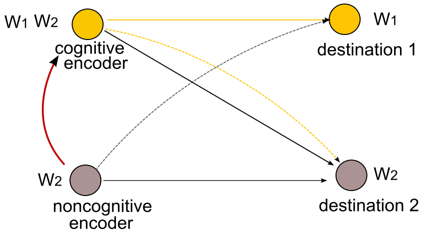
- Interweave: users' activity
  - Detection of spectrum holes
  - Holes common to the transmitter and receiver

- Underlay: channel gains
  - If there is a channel reciprocity or feedback



- Overlay: channel gains, codebooks and (partial) messages
  - Codebooks: through protocol
  - Messages via: retransmission; cooperation; decoding

Cognitive Radio Channel Model



- Two messages: $W_k \in \{1, \ldots, M_k\}$ sent at rates R_k
- Encoding: $X_1^n = f_1(W_1, W_2)$, $X_2^n = f_2(W_2)$
- Decoding: $\hat{W}_k = g_k(Y_k^n)$

Elements of Cognitive Encoding Strategy

- Opportunistic approach: interference avoidance

Elements of Cognitive Encoding Strategy

- Opportunistic approach: interference avoidance
- Utilize techniques developed from many canonical models

1. Cooperative strategies
   To increase rate at oblivious receiver
2. Rate-splitting
   To allow oblivious decoder to cancel part of interference
3. Precoding against interference
   To eliminate interference at cognitive receiver

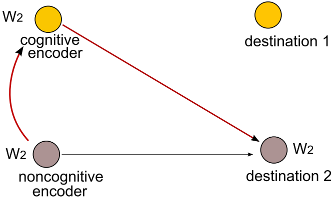
Cooperation

- To increase rate for the oblivious receiver

- Cognitive radio acts as a relay

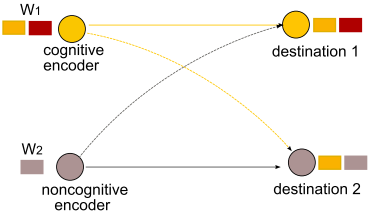
$$X_1^n = f_1(W_1, W_2)$$

- Dedicates some power to transmit the other user's message
- Increases interference to its own receiver



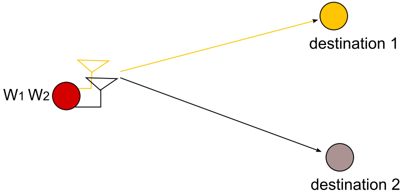
Rate-Splitting

- To reduce interference
- Without cognition: interference channel

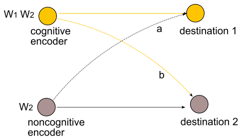


Precoding against Interference

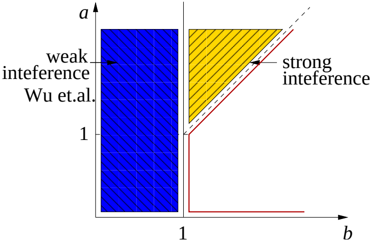
- To eliminate interference at cognitive receiver
- Full cognition: MIMO broadcast channel
- Strategy: precoding against interference *[Gel'fand and Pinsker, 1979]*
- Gaussian channels: Dirty-paper coding (DPC) *[Costa, 1981]*
  - Achieves capacity



Capacity Results for Gaussian Channels



$$Y_1 = X_1 + aX_2 + Z_1$$
$$Y_2 = bX_1 + X_2 + Z_2$$



- Regions for which capacity is known:
- Strong interference, $a > b > 1$
  Cooperation achieves capacity
- Weak interference, $b \leq 1$
  Dirty paper coding and cooperation achieve capacity

Insights

1. Orthogonalizing transmissions is suboptimal
2. Canceling strong interference is beneficial
3. Rate-splitting can be used for partially canceling interference

Insights

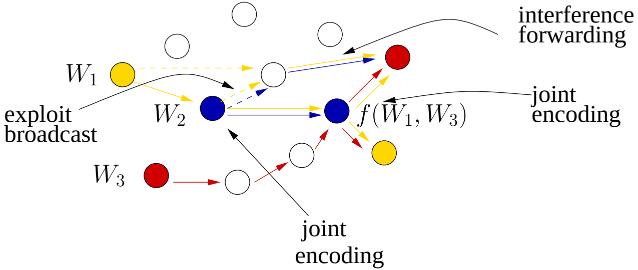
- Side information in cognitive radio networks can be used for:

- Cooperation
- Precoding against interference

- In considered network: cooperation and GP precoding capacity-achieving in some regimes
- Delay should be considered

Relaying for Multiple Sources

- Jointly encode messages
- Exploit broadcast
- Relays forward messages and interference
- Create virtual MIMO



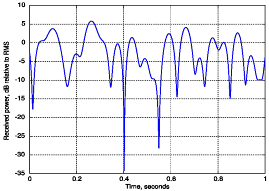
Cooperation in Fading Channels

In This Section...

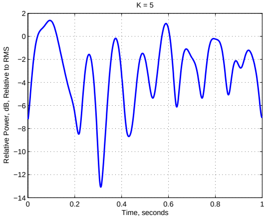
- Examples
- Diversity-multiplexing tradeoff for the PtP MIMO channel
- DMT for cooperative systems
- Summary

Fading Channels

- Rayleigh fading: Many scatterers, no LOS
  - Communication in dense urban areas

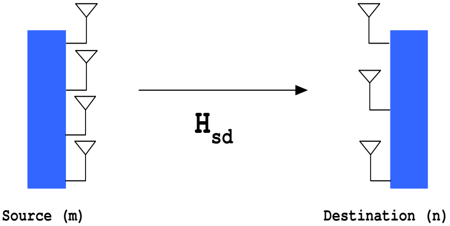


- Rician Fading: Many scatterers with LOS
  - Satellite communications



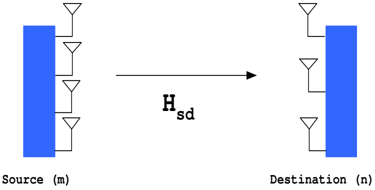
Channel Model

Consider a point-to-point (PtP) MIMO channel:



- m transmit antennas
- n receive antennas
- Consider codes with blocklength l

Channel Model



- Block-fading model: **H** is constant for the entire block of length l.
- $$\mathbf{Y} = \sqrt{\frac{\text{SNR}}{m}}\mathbf{H}\mathbf{X} + \mathbf{Z}$$

  $\mathbf{X} \in \mathcal{C}^{m\times l}$, $\mathbf{H} \in \mathcal{C}^{n\times m}$, $\mathbf{Y} \in \mathcal{C}^{n\times l}$, $\mathbf{Z} \in \mathcal{C}^{n\times l}$
- $h_{i,j} \sim \mathcal{CN}(0,1)$, i.i.d.; $z_{i,j} \sim \mathcal{CN}(0,1)$, i.i.d.
- Power constraint: $2^{-Rl}\sum_{i=1}^{2^{Rl}} ||\mathbf{X}(i)||_F^2 \leq ml$

MIMO Systems in Fading Environments: Diversity Gain

- Diversity: Sending the same information through several paths.
- Each path is subject to independent fading $\Rightarrow$ increase the reliability of reception.
  - $m = 1 \Rightarrow P_e(SNR) \doteq SNR^{-n}$
- Definition: Diversity gain d

$$d = - \lim_{\text{SNR} \to \infty} \frac{\log P_e(SNR)}{\log SNR}$$

- For i.i.d. Rayleigh fading the maximal diversity gain is mn
- Probability of error dominated by the outage event

MIMO Systems in Fading Environments: Multiplexing Gain

- Spatial Multiplexing: Sending independent information over parallel spatial channels.
- Each Tx-Rx pair is fading independently thus creating a parallel channel.
  - At high SNR, under i.i.d. Rayleigh fading assumption, the (ergodic) channel capacity is given by

$$C(\mathsf{SNR}) = \min\{m, n\} \log \mathsf{SNR} + \mathcal{O}(1)$$

- Definition: Multiplexing gain r

$$r = \lim_{\mathsf{SNR} \to \infty} \frac{R(SNR)}{\log SNR}$$

- For i.i.d. Rayleigh fading the maximal multiplexing gain is $\min\{m, n\}$

Example: Diversity-Multiplexing Relationship for Alamouti's Scheme

- 2×2 system
- $R = r \log \mathsf{SNR}$
- $\mathbf{X} = \begin{bmatrix} x_1 & -x_2^* \\ x_2 & x_1^* \end{bmatrix}$
- $\mathbf{Y} = \sqrt{\frac{\mathsf{SNR}}{2}}\mathbf{HX} + \mathbf{Z}$
- Using Alamouti's scheme we arrive at the equivalent channel

$$y_i = \sqrt{\frac{\mathsf{SNR}||\mathbf{H}||_F^2}{2}} x_i + w_i$$

- $P_{\text{out}} \doteq \Pr\left(||\mathbf{H}||_F^2 \leq \mathsf{SNR}^{-(1-r)^+}\right)$
- $d(r) = 4(1-r)^+$

Remarks

- The blocklength l is fixed
- Analysis for SNR goes to infinity
- A *scheme* $\mathcal{S}$ is a collection of codes $\{\mathbb{C}(\text{SNR})\}$, one for each SNR
  - Rate is $R(\text{SNR})$
- Both data rate and error probability scale with SNR
- Each *scheme* is characterized by the two parameters (d, r)
- Define for a fixed r the function $d^*(r)$ as

$$d^*(r) \triangleq \sup_{\mathcal{S} \text{ with the same } r} d$$

Diversity Gain vs. Multiplexing Gain - The Fundamental Result

- Characterized by Zheng and Tse in 2003.

Theorem (Zheng & Tse'03)

When $l \geq m+n-1$, the optimal tradeoff curve $d^*(r)$ is the piecewise linear function connecting the points $(k, d^*(k))$, $k = 0, 1, ..., \min\{m, n\}$, where

$$d^*(k) = (m-k)(n-k).$$

- $d^*_{\max} = mn$
- $r^*_{max} = \min\{m, n\}$

Diversity Gain vs. Multiplexing Gain - The Fundamental Result

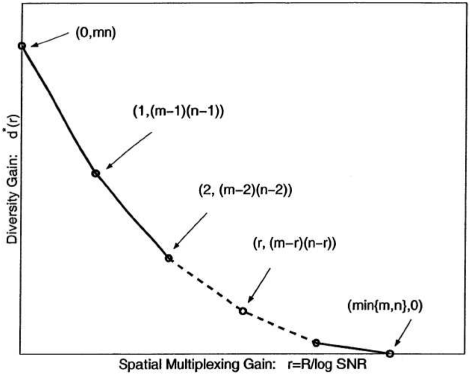
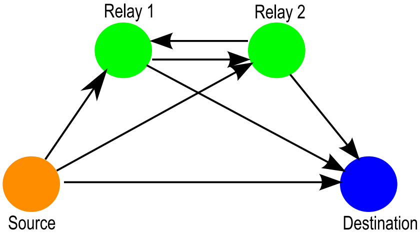


Fig. 1. Diversity–multiplexing tradeoff, $d^*(r)$ for general m, n, and $l \geq m + n - 1$.

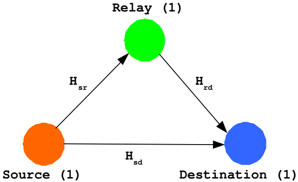
- Figure from Zheng & Tse'03.

Cooperative Networks



- Cooperation can be used to create a virtual MIMO:
  - Several single antenna nodes cooperate in sending/receiving information
  - CSI assumptions
- Which cooperation strategy is DMT superior?

Half Duplex Relay Channel

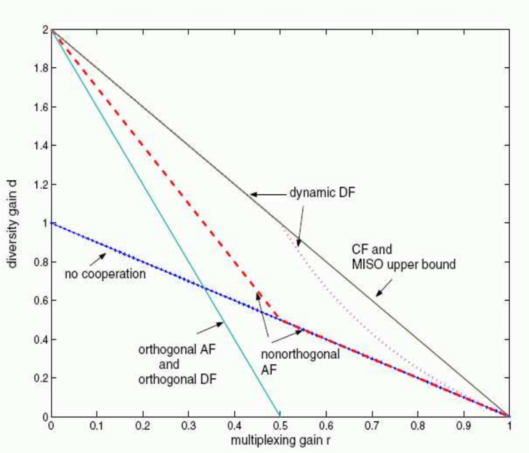


- MISO upper bound: $d^*(r) = 2(1-r)$
- Orthogonal DF: $d^*(r) = 2 - 4r$
- Dynamic DF

$$d^*(r) = \begin{cases} 2(1-r), & ,0 \leq r \leq 0.5 \\ \frac{1-r}{r} & ,0.5 \leq r \leq 1 \end{cases}$$

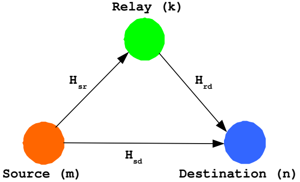
- Non-orthogonal AF: $d^*(r) = (1-r) + (1-2r)^+$
- CF with relay CSIT: $d^*(r) = 2(1-r)$

Half Duplex Relay Channel



- Figure from Kramer, Maric & Yates'06.

Full Duplex MIMO Relay Channel: The Optimal DMT



$$\mathbf{Y}_r = \mathbf{H}_{sr}\mathbf{X}_s + \mathbf{Z}_r$$
$$\mathbf{Y}_d = \mathbf{H}_{sd}\mathbf{X}_s + \mathbf{H}_{rd}\mathbf{X}_r + \mathbf{Z}_d$$

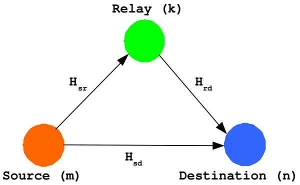
Theorem (Yuksel & Erkip'07)

The optimal DMT is equal to

$$d^*(r) = \min\{d_{m(n+k)}(r), d_{(m+k)n}(r)\}.$$

- The best DMT is achieved by the CF scheme
- $d^*_{\max} = \min\{m(n+k), (m+k)n\}$
- $r^*_{\max} = \min\{\min\{m, (n+k)\}, \min\{n, (m+k)\}\}$
  $= \min\{m, n\}$

Full Duplex MIMO Relay Channel: DMT Analysis of DF



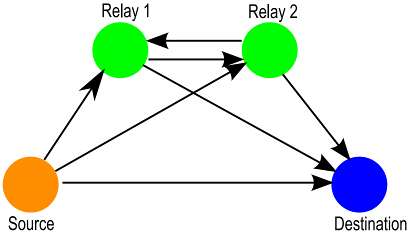
Theorem (Yuksel & Erkip'07)

The DMT achieved by DF is given by

$$d^*_{DF}(r) = \begin{cases} \min\{d_{(m+k)n}(r), \\ \qquad d_{mn}(r) + d_{mk}(r)\}, 0 \le r \le \min\{m,n,k\} \\ d_{mn}(r) \qquad\qquad , \min\{m,n,k\} \le r \le \min\{m,n\} \end{cases}$$

- If $m = 1$ or $n = 1$, DF is optimal
- if $k < \min\{m, n\}$ the relay cannot help at high r
  - Relay cannot decode if the multiplexing gain is too high
  - Additional outage event due to decoding at the relay

Multiple Relays: Non-Clustered

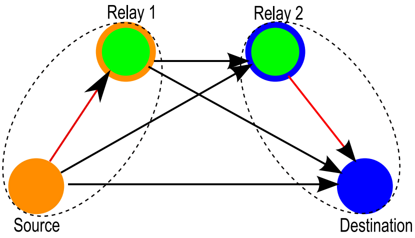


- Single antenna nodes
- Optimal DMT:

$$d^*(r) = d_{13}(r)$$

- DMT is achieved using DF at both relays

Multiple Relays: Clustered

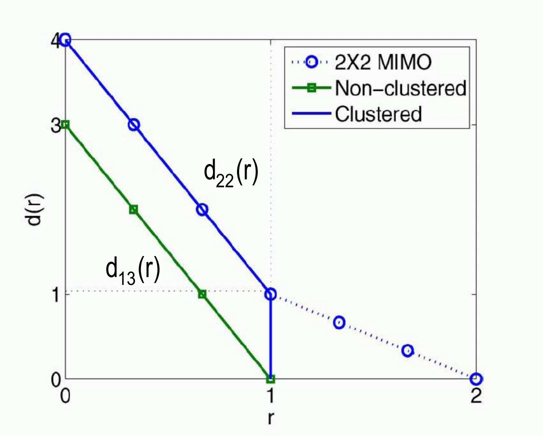


- Single antenna nodes
- Clustered nodes: the channel is AWGN (no fading)
- Optimal DMT:

$$d^*(r) = \begin{cases} d_{22}(r) & , r \leq 1 \\ 0 & , r > 1 \end{cases}$$

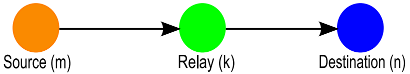
- DMT is achieved using DF at Relay 1 and CF at Relay 2

Clustered vs. Non-Clustered



- Figure from Yuksel & Erkip'07.

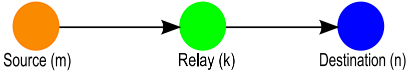
DMT of Multi-hop Relaying



- Full duplex: DMT achieved by DF

$$d^*_{\text{FD}}(r) = \min\left\{d_{mk}(r), d_{kn}(r)\right\}$$

DMT of Multi-hop Relaying



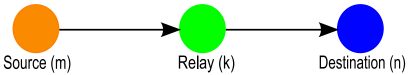
- Full duplex: DMT achieved by DF

$$d^*_{\text{FD}}(r) = \min\left\{d_{mk}(r), d_{kn}(r)\right\}$$

- Half duplex with fixed time allocation $(\alpha, 1-\alpha)$

$$d^*_{\text{HD-fixed}}(r) = \min\left\{d_{mk}\left(\frac{r}{\alpha}\right), d_{km}\left(\frac{r}{1-\alpha}\right)\right\}$$

DMT of Multi-hop Relaying



- Full duplex: DMT achieved by DF

$$d^*_{\text{FD}}(r) = \min \left\{ d_{mk}(r), d_{kn}(r) \right\}$$

- Half duplex with fixed time allocation $(\alpha, 1-\alpha)$

$$d^*_{\text{HD-fixed}}(r) = \min \left\{ d_{mk}\left(\frac{r}{\alpha}\right), d_{km}\left(\frac{r}{1-\alpha}\right) \right\}$$

- Half duplex DDF $(m, k, n) = (2, 2, 2)$

$$d^*_{\text{HD}}(r) = \begin{cases} \frac{8-10r}{2-r} & , r \in [0, 1/2) \\ \frac{3-4r}{1-r} & , r \in (1/2, 2/3) \\ 4\frac{1-r}{2-r} & , r \in (2/3, 1] \end{cases}$$

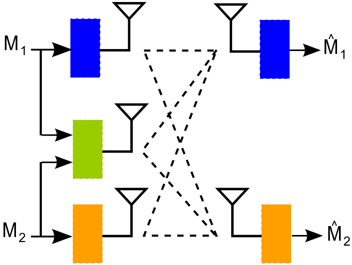
Degrees of Freedom of Cooperative Networks

- The term degrees of freedom is sometimes used instead of multiplexing gain.
  - The capacity at high SNR:

$$C(\text{SNR}) = \min\{m, n\} \log \frac{\text{SNR}}{m} + \sum_{i=|m-n|+1}^{\max\{m,n\}} E\left\{ \log \chi^2_{2i} \right\} + o(1)$$

- For a MIMO channel, when the matrix is full rank, we achieve the maximal degrees of freedom.
  - For Rayleigh fading with CSIR the MIMO channel matrix is full rank
- Cooperation does not increase the DOF

Degrees of Freedom with Cognitive Cooperation



Cognitive Relaying

- When cooperation is based on a cognitive relay node then each pair can achieve DOF 1
- The sum-rate DOF is 2

$\Rightarrow$ A cognitive relay increases the DOF of the system

- Achieved by *instantaneous* interference cancellation

Summary

- DMT is a performance metric for fading channels
  - Asymptotically high SNR
    - Outage is the dominating error event
  - Finite blocklength
- Soft information (CF) is important for achieving the maximum DMT
- With a single relay:
  - CF achieves the optimal DMT
  - With full-duplex single antenna nodes: DF also achieves the optimal DMT
- Clustering improves diversity but does not improve DOF
  - Source cluster $\Rightarrow$ use DF
  - Destination cluster $\Rightarrow$ use CF

Scaling Laws

Large Network Analysis

- Initiated by Gupta and Kumar

Large Network Analysis

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- n source-destinations: n-dimensional capacity region

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- Assumption:
- Each pair wants to communicate at rate: $R(n)$ *bits/s*
- Total throughput: $T(n) = nR(n)$

Large Network Analysis

- Initiated by Gupta and Kumar
- n source-destinations: n-dimensional capacity region
- We cannot solve even for n=2!
- Assumption:
- Each pair wants to communicate at rate: $R(n)$ *bits*/*s*
- Total throughput: $T(n) = nR(n)$
- Max achievable scaling?

Scaling in Multihop Networks

Upper bound: $T(n) \leq \mathcal{O}(\sqrt{n})$

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Communication scheme:

- Multihop routing
- Treat all unwanted signals as noise
- Achievability proved using percolation theory

Scaling in Multihop Networks

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Communication scheme:

- Multihop routing
- Treat all unwanted signals as noise
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- Can cooperative encoding schemes change the scaling?

Is Multihop Optimal?

- Dense vs. extended networks

Extended Networks:

For $\alpha > 4$: multihop is order-optimal

- α-path-loss exponent
- Attenuation is too large for cooperation to help

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For $\alpha \leq 4$?

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- Dense vs. extended networks

Extended Networks:

For $\alpha > 4$: multihop is order-optimal

- α-path-loss exponent
- Attenuation is too large for cooperation to help

For $\alpha \leq 4$?

- $2 \leq \alpha \leq 3$: $n^{2-\alpha/2}$
  $\alpha > 3$: $n^{1/2}$
- For $\alpha = 2$: linear scaling
- For $\alpha > 3$: multihop is optimal

Is Multihop Optimal?

- Dense vs. extended networks

Extended Networks:

For $\alpha > 4$: multihop is order-optimal

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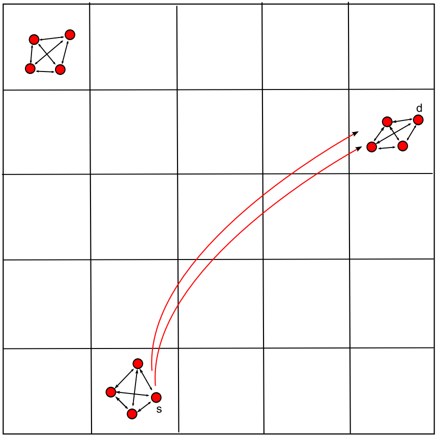
For $\alpha \leq 4$?

- $2 \leq \alpha \leq 3$: $n^{2-\alpha/2}$
  $\alpha > 3$: $n^{1/2}$
- For $\alpha = 2$: linear scaling
- For $\alpha > 3$: multihop is optimal
- Dense networks, $\alpha \geq 2$: $T(n) = \mathcal{O}(n^{1-\epsilon})$ is achievable
- Arbitrarily close to linear scaling

Motivation for Cooperative Strategy

- Nearest neighbor communications are not enough
- Linear scaling: in MIMO system
- With n transmit and receive antennas, in high-SNR: $n \log(SNR)$
- We already saw gains from clustering and mimicking MIMO
- This requires cooperation within clusters

Three-Phase Cooperative Scheme



- Form M-node clusters
- Sources in cluster cooperate
- MIMO long-range transmissions
- Destinations in cluster cooperate

Within Each Stage

- Transmit cluster: information bits exchanged between sources
- Receive cluster: quantized observations exchanged between destinations
- Spatial reuse: non-adjacent clusters send simultaneously
- TDMA long-range communications between clusters

Hierarchial Cooperation

- Perform multiple stages of the three-phase scheme
- Each stage improves throughput
- After h stages:

$$T(n) = \mathcal{O}(n^{h/(h+1)})$$

- Choose h s.t.

$$\frac{h}{h+1} \geq 1 - \epsilon$$

to obtain

$$T(n) = \mathcal{O}(n^{1-\epsilon})$$

Summary

- Lots of progress
- Different behavior for dense vs. extended networks
- Cooperation can change scaling
- For extended networks:
- For $\alpha \geq 3$: $\sqrt{n}$, multihop is optimal
- For $2 \leq \alpha \leq 3$: $n^{2-\alpha/2}$
  - Linear scaling only for $\alpha = 2$
- Dense networks: capacity scales linearly
- Design of practical systems requires more detailed analysis

Summary and Challenges

Summary

Relaying for Multiple Sources

- Gains from simple joint encoding strategies
- Canceling strong interference is beneficial
- Relays forward messages and interference
- Gains from virtual MIMO
- Gains from cognitive encoding techniques

Cooperation brings diversity-multiplexing gains

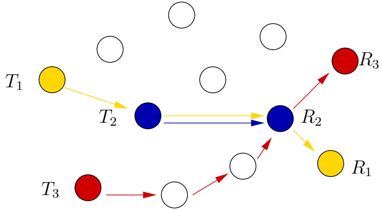
- Compress-and-forward achieves the optimal DMT in single relay channel

Cooperative communications can change capacity scaling

Conventional Network Architecture

- Network protocol layers
- Store-and-forward routing via a sequence of links
- Point-to-point transmissions on the path

- Network layer: decides on the next node, modifies the header
- PHY/Link layer: discards a packet in error



Network Architecture: Cooperative Protocols

Exploit broadcast

- Nodes collect erroneous packets
- A link is not necessarily point-to-point

Allow for encoding at the nodes

- Relaying, joint encoding of messages, network coding

Network Architecture: Cooperative Protocols

- Decoder: soft combining of packets
- Protocol: provide for relaying and routing

Network Architecture: Cooperative Protocols

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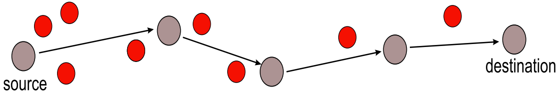
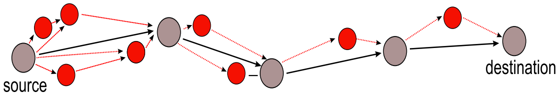
- For example:
- Routing on the network layer



Network Architecture: Cooperative Protocols

- Decoder: soft combining of packets
- Protocol: provide for relaying and routing
- For example:
- Routing on the network layer
- Sequence of cooperative links



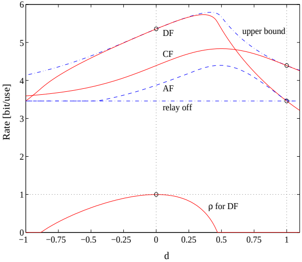
Demonstrated Gains from Cooperative Communications

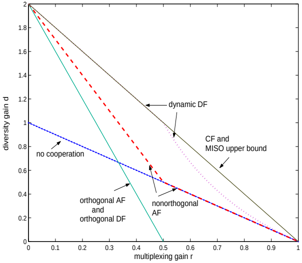
For small networks

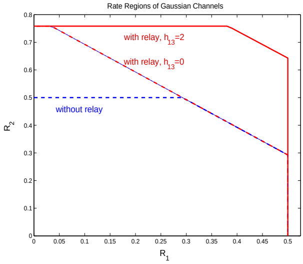
- Rates in relay channel
- Diversity-multiplexing gains
- Rate regions for multiple sources

For large networks

- Scaling law $\mathcal{O}(\sqrt{n}) \rightarrow \mathcal{O}(n)$

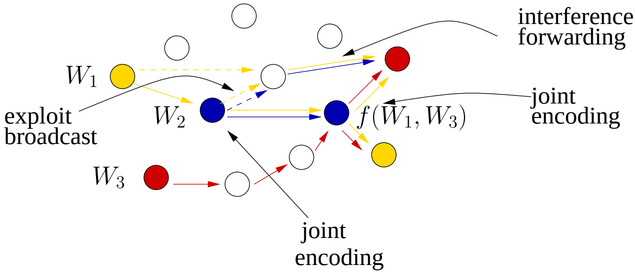






Challenges

- Capacity results for canonical models
- Many encoding possibilities at the relays
- Practical cooperative schemes
- Cooperative protocols



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