Relaying in the Presence of Interference: Achievable Rates, Interference Forwarding, and Outer Bounds

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***Abstract*—The smallest network model that captures relaying in the presence of multiple communicating pairs causing interference to each other is the interference channel with a relay. In this paper, an achievable rate region for the interference channel with a relay is derived. Special cases of strong interference under which this region is the capacity region are presented. The results obtained demonstrate the benefits of *interference forwarding* at a relay. By forwarding interfering messages, the relay can improve their reception at unintended receivers and, thus, facilitate interference cancellation. We show that intentionally forwarding interfering messages can improve the achievable rates. The achievable rates and interference forwarding gains are also illustrated by numerical results in Gaussian channels. Finally, a sum-rate outer bound to the capacity region of the Gaussian interference channel with a relay is derived and compared with the achievable rate region. The cut-set bound for this channel is also derived and shown to be much looser than the new sum-rate outer bound.**

***Index Terms*—Capacity, decode-and-forward, interference cancellation, interference channels, interference-forwarding, network information theory, relaying.**

I. Introduction

COOPERATION via relays that forward information in wireless networks improves the performance in terms of rate, coverage, reliability, and energy efficiency. Cooperative strategies for the single-relay channel have been developed in [1]–[3], and further generalized to multi-relay channels (see, for example, [4] and [5]). Relay channel models typically consider a single communicating pair and, hence, do not capture cooperation for multiple source–destination pairs. And yet, wireless applications typically involve simultaneous communications from many sources to many destinations. Such scenarios bring in new elements not encountered in the classic relay channel: 1) the presence of interference caused by simultaneous transmissions from multiple sources; 2) the opportunity for joint encoding of messages at a relay; 3) increased interference at nodes caused by the relay forwarding desired information to others. These elements impact the optimal relaying schemes. The various aspects of relaying for multiple sources can be captured by considering the smallest such network, which we refer to as the interference channel with a relay (ICR) (see Fig. 1). The ICR model contains elements of relay, interference, broadcast and multiaccess channels (MACs), and thus, determining its capacity and associated encoding/decoding schemes is extremely challenging.

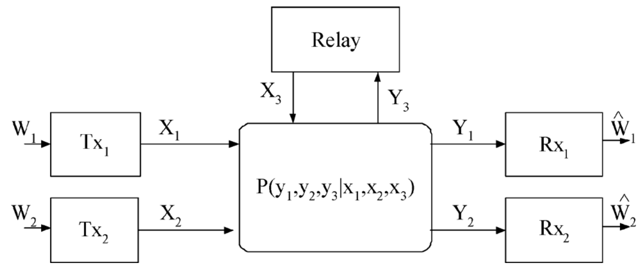

Fig. 1. The interference channel with a relay.

In this paper, we first derive an achievable rate region for the ICR. We then show that the obtained region is the capacity region in special cases of strong interference in which the receivers can decode both messages. We also present strong interference conditions for our encoding scheme. In the cooperative strategies of decode-and-forward and compress-and-forward [1], the relay forwards the *desired* message to the intended destination. In this study, we propose a new cooperative approach for relaying in the presence of multiple pairs. In this approach, the relay, in addition to forwarding desired messages, intentionally forwards an interfering signal to a destination. The motivation for this approach can be found by considering the interference channel [6], [7]: in the presence of multiple transmitters, decoding of an interfering message (or a part of it) improves the rates [7]. In strong interference, this is the optimal approach [8]. In cooperation, by intentionally forwarding interfering messages, the relay can increase the interference already present at the destinations, thus facilitating interference cancellation. We refer to this scheme as *interference forwarding*. We emphasize that interference forwarding is not a byproduct of message forwarding (received at another destination), but an objective in itself, intended to increase the achievable region beyond what is achieved with other schemes. This is in contrast to the work presented in [9] and [10]. We demonstrate the interference forwarding gains by considering special cases of

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the achievable rate region derived in this paper. We show that forwarding messages can improve rates even when it does not enhance their reception at a desired receiver, i.e., when they are only creating interference. We present scenarios in which a relay can help without forwarding any desired information. We also present numerical results of our derived achievable rate regions and outer bounds for Gaussian channels that illustrate the rate gains that can be obtained from interference forwarding.

We then derive a sum-rate outer bound on the capacity region of the Gaussian ICR. For the Gaussian interference channel, Kramer introduced the idea of a genie that provides a receiver with the minimum information necessary to decode both messages [11]. In particular, the receivers obtain a noisy sum of the source signals. This approach led to a new, improved outer bound for the interference channel. In this study, we apply this idea to the ICR. We propose a genie that gives the receiver a noisy observation of the source and the relay channel inputs. Unlike the interference channel in which the channel inputs are independent, in ICRs, the channel inputs at the relay and at each encoder are dependent. In our approach, the maximum entropy inequality will guarantee that the bound is maximized by jointly Gaussian inputs. Our bound also applies to the *cognitive* ICR, in which the relay knows *a priori* the messages sent by the sources. For the cognitive ICR, two outer bounds were developed in [12]. We show that the new bound presented in this paper can be tighter than existing cognitive ICR outer bounds. We also compare the new outer bound to the cut-set bound and to our achievable rate region.

A. Related Work

Inner bounds to the capacity of the ICR were first presented in [9]. We introduced the idea of interference forwarding and demonstrated its gains in [13], and subsequently in [14] and [15]. In these works, we derived inner bounds on the performance, and obtained capacity in the special case of strong interference. We showed that in some communication scenarios, there may be more benefit from increasing interference at the assisted destination than in the classic approach of forwarding desired information. The authors in [12], [16]–[19] considered a similar channel model under the assumption that the relay is cognitive, in the sense that it knows *a priori* the messages to be sent by the two sources. Strong interference conditions for the cognitive Gaussian case were presented in [16]. Inner and outer bounds were also presented in [20]. The capacity region of the fading ICR in the strong interference regime was determined in [21]. ICRs with in-band and out-of-band signaling to/from the relay were considered in [10]. A special case of the ICR was considered in the context of cellular networks with relays in [22]. The interference channel with a cognitive relay can also be viewed as a broadcast channel with two cognitive encoders [23]. This view led to a new achievable rate region [23]. We presented a sum-rate upper bound for the ICR in [24]. The MIMO ICR was considered in [25] and [26]. The results presented herein differ from the related works as here we introduce the idea of interference forwarding and demonstrate its benefits based on our obtained inner and outer bounds. The achievable rates and interference forwarding gains are also illustrated by numerical results in Gaussian channels.

The remainder of this paper is organized as follows. We start by introducing the channel model in Section II. The achievable rate region and the capacity in strong interference are derived in Section III. Benefits of interference forwarding are discussed in Section IV. Numerical results in Gaussian channels are presented in Section V. A new sum-rate outer bound is presented in Section VII, including numerical comparisons between the new sum-rate outer bound, the cut-set bound, and an achievable rate region. Section VIII concludes the paper. The proofs are given in the appendix.

II. Channel Model

The discrete ICR consists of three finite input alphabets $\mathcal{X}_1, \mathcal{X}_2, \mathcal{X}_3$, three finite output alphabets $\mathcal{Y}_1, \mathcal{Y}_2, \mathcal{Y}_3$, and a probability distribution $p(y_1, y_2, y_3|x_1, x_2, x_3)$. Each encoder $t, t = 1, 2$, wishes to send a message $W_t \in \mathcal{W}_t \triangleq \{1, \ldots, 2^{nR_t}\}$ to decoder $t, t = 1, 2$ (see Fig. 1). We assume that the relay is full-duplex. The channel is memoryless and time invariant in the sense that

$$p(y_{1,i}, y_{2,i}, y_{3,i}|x_1^i, x_2^i, x_3^i, y_1^{i-1}, y_2^{i-1}, y_3^{i-1}, w_1, w_2) = p_{Y_1,Y_2,Y_3|X_1,X_2,X_3}(y_{1,i}, y_{2,i}, y_{3,i}|x_{1,i}, x_{2,i}, x_{3,i}). \quad (1)$$

We will also consider the Gaussian channel described by the following input–output relationship:

$$\begin{aligned} Y_1 &= X_1 + h_{12}X_2 + h_{13}X_3 + Z_1 \\ Y_2 &= h_{21}X_1 + X_2 + h_{23}X_3 + Z_2 \\ Y_3 &= h_{31}X_1 + h_{32}X_2 + Z_3 \end{aligned} \quad (2)$$

where h_{ij} is a real number representing the channel gain from node j to node i, $Z_t \sim \mathcal{N}(0, 1)$, $E[X_t^2] \leq P_t, t = 1, 2, 3$, and $\mathcal{N}(0, \sigma^2)$ denotes the normal distribution with zero mean and variance σ^2.

An (R_1, R_2, n) code for the ICR consists of two message sets $\mathcal{W}_1$, $\mathcal{W}_2$, an encoding function at each transmitter, $X_t^n = f_t(W_t)$, $t = 1, 2$, n encoding functions at the relay $X_{3,i} = f_{3,i}(Y_3^{i-1})$, $i = 1, 2, \ldots, n$, and a decoding function at each receiver $\hat{W}_t = g_t(Y_t^n)$, $t = 1, 2$. The average error probability of the code is given by $P_e = P\left[\{\hat{W}_1 \neq W_1\} \cup \{\hat{W}_2 \neq W_2\}\right]$. The capacity region of the ICR is the closure of the set of rate pairs (R_1, R_2) for which the receivers can decode their messages with an arbitrarily small positive error probability.

III. Achievable Rate Region and Capacity in Strong Interference

In this section, we present an achievable rate region for the ICR and conditions under which this region is the capacity region.

Theorem 1: Let $\mathcal{R}_{\text{ICR}}$ denote the rate region obtained by taking all nonnegative rate pairs (R_1, R_2) that satisfy

$$R_1 \leq I(X_1, X_3; Y_1|U_2, X_2, Q) \quad (3a)$$
$$R_2 \leq I(X_2, X_3; Y_2|U_1, X_1, Q) \quad (3b)$$
$$R_1 + R_2 \leq I(X_1, X_2, X_3; Y_1, Q) \quad (3c)$$
$$R_1 + R_2 \leq I(X_1, X_2, X_3; Y_2, Q) \quad (3d)$$
$$R_1 \leq I(X_1; Y_3|U_1, U_2, X_2, X_3, Q) \quad (3e)$$

$$R_2 \leq I(X_2; Y_3 | U_1, U_2, X_1, X_3, Q) \quad (3f)$$
$$R_1 + R_2 \leq I(X_1, X_2; Y_3 | U_1, U_2, X_3, Q) \quad (3g)$$

for some joint distribution that factors as

$$p(q)p(u_1, x_1|q)p(u_2, x_2|q)p(x_3|u_1, u_2, q) \times p(y_1, y_2, y_3|x_1, x_2, x_3). \quad (4)$$

Then, the region $\mathcal{R}_{\text{ICR}}$ is achievable for the ICR.

Proof: See Appendix A. ■

In order to simplify notation, the time-sharing variable is not considered in the rest of this paper. A time-sharing variable can be easily incorporated into the achievability results.

Remark 1: In the encoding strategy of Theorem 1, rate splitting is not used. Instead, each destination node jointly decodes the messages (W_1, W_2) as in the MAC [27]. Bounds (3a)–(3d) are rate constraints for decoding at the two destination nodes. Compared to the MAC rate constraints, the error in decoding the unwanted message at each destination node is ignored here. Consequently, there is one less rate constraint at each decoder, when compared to the MAC rate bounds. Bounds (3e)–(3g) are decoding constraints at the relay. Since the relay decodes both messages, possible error events at the relay are the same as in the MAC.

Remark 2: Special cases of Theorem 1 were obtained in [13, Theorem 2] and [14, Theorem 1].

Consider next the conditions

$$I(X_1, X_3; Y_1|X_2) \leq I(X_1, X_3; Y_2|X_2) \quad (5)$$
$$I(X_2, X_3; Y_2|X_1) \leq I(X_2, X_3; Y_1|X_1) \quad (6)$$

that hold for every distribution (4).

Under conditions (5) and (6), we have the following outer bound to the capacity of the ICR.

Theorem 2: Under conditions (5) and (6), the union of the set of rates (R_1, R_2) that satisfy

$$R_1 \leq I(X_1, X_3; Y_1|U_2, X_2) \quad (7a)$$
$$R_2 \leq I(X_2, X_3; Y_2|U_1, X_1) \quad (7b)$$
$$R_1 + R_2 \leq I(X_1, X_2, X_3; Y_1) \quad (7c)$$
$$R_1 + R_2 \leq I(X_1, X_2, X_3; Y_2) \quad (7d)$$
$$R_1 \leq I(X_1; Y_1, Y_3|U_2, X_2, X_3) \quad (7e)$$
$$R_2 \leq I(X_2; Y_2, Y_3|U_1, X_1, X_3) \quad (7f)$$
$$R_1 + R_2 \leq I(X_1, X_2; Y_1, Y_2, Y_3|X_3) \quad (7g)$$

for any distribution that factors as (4), is an outer bound to the capacity region of the ICR.

Proof: See Appendix B. ■

We observe that the gap between the achievable rate region (3a)–(3g) and the outer bound (7a)–(7g) is only due to decoding constraints at the relay.

Remark 3: Conditions (5) and (6) can be viewed as the *strong interference conditions* in the sense that under these conditions, the received interfering signals are strong so that the receivers can decode both messages without rate penalty.

Remark 4: If the relay is not present, the ICR reduces to the interference channel. To see what happens in that case, we can assume that the relay channel input X_3 is a constant which is known to the receivers. Decoding requirements at the relay (3e)–(3g) are not needed. Conditions (5) and (6) reduce to the strong interference conditions for the IC [8]

$$I(X_1; Y_1|X_2, X_3) \leq I(X_1; Y_2|X_2, X_3) \quad (8)$$
$$I(X_2; Y_2|X_1, X_3) \leq I(X_2; Y_1|X_1, X_3) \quad (9)$$

for any $p(x_1)p(x_2)p(x_3)p(y_1, y_2|x_1, x_2, x_3)$, and the region (3a)–(3d) reduces to the IC capacity region in strong interference [8].

In the following, we use Theorems 1 and 2 to prove two capacity results. We first determine the capacity in strong interference for the interference channel with a cognitive relay. In this case, the relay *a priori* knows source messages (W_1, W_2), and hence, the relay function is given by $X_3^n = f_3(W_1, W_2)$. This is the only difference from the channel model defined in Section II. Then, no block Markov encoding is needed in the encoding scheme, and therefore, the decoding constraints at the relay (3e)–(3g) can be omitted from the achievable region of Theorem 1, and we can also set $U_1 = X_1$ and $U_2 = X_2$. Rate constraints (3a)–(3g) now become

$$R_1 \leq I(X_1, X_3; Y_1|X_2) \quad (10a)$$
$$R_2 \leq I(X_2, X_3; Y_2|X_1) \quad (10b)$$
$$R_1 + R_2 \leq I(X_1, X_2, X_3; Y_1) \quad (10c)$$
$$R_1 + R_2 \leq I(X_1, X_2, X_3; Y_2) \quad (10d)$$

for a joint distribution

$$p(x_1)p(x_2)p(x_3|x_1, x_2)p(y_1, y_2, y_3|x_1, x_2, x_3). \quad (11)$$

Furthermore, the proof approach in Theorem 2 for bounds (7a)–(7d) still applies, thereby proving the converse. We hence proved the following lemma.

Lemma 1: Under conditions (5) and (6), the convex hull of the rates (10a)–(10d), evaluated for the joint distribution (11), is the capacity region of the interference channel with a cognitive relay.

We next consider the following *degradedness* condition:

$$p(y_1, y_2|y_3, x_3, x_1, x_2) = p(y_1, y_2|y_3, x_3), \quad (12)$$

i.e., the Markov chain $(X_1, X_2) \to (X_3, Y_3) \to (Y_1, Y_2)$ holds.

Condition (12) is a generalization of the physical degradedness condition in the single-relay channel [1]. In fact, for $t = 1, 2$, (12) implies

$$p(y_t|y_3, x_3, x_t) = p(y_t|y_3, x_3)$$

i.e., the Markov chain $X_t \to (X_3, Y_3) \to Y_t$ holds.

Focusing on the case in which the relay can observe only one source signal, i.e.

$$p(y_3|x_3, x_2, x_1) = p(y_3|x_3, x_2), \quad (13)$$

we have the following capacity result.

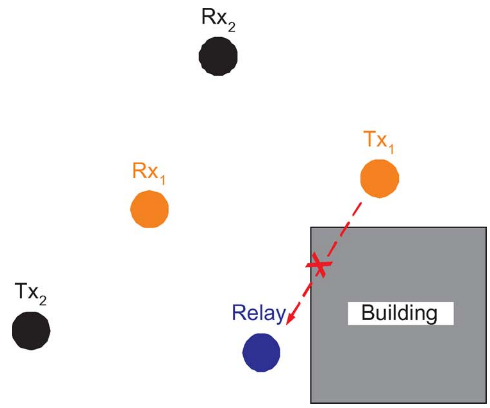

Fig. 2. ICR scenario in which the relay cannot receive signal from Tx_1.

Theorem 3: Under conditions (5) and (6) and conditions (12) and (13), the rates of Theorem 1 are the capacity region of the ICR.

Proof: When (13) holds, we can choose $U_1 = X_1$, $U_2 = X_3$ in Theorem 1. The proof follows immediately by evaluating the inner bound of Theorem 1 and the outer bound of Theorem 2 under conditions (12) and (13), and observing that they coincide. ∎

IV. Gains From Interference Forwarding

A. Comparison With Rate Splitting

To demonstrate the benefits of interference forwarding, we next focus on a special case of the ICR model. In particular, we show that there are scenarios in which the relay can improve the rate to a user *without* forwarding desired information. To show that, we consider a scenario in which the relay cannot observe X_1. Consequently, the relay cannot forward information about W_1. We also assume that receiver 2 cannot receive from the relay. Consequently, the relay cannot assist in sending W_2, and the relay can, thus, only increase the interference at receiver 1. In particular, the following is assumed:

A_1: Condition (13) holds, i.e., the relay observation Y_3 is independent of channel input X_1, given X_2 and X_3. Hence, we have a Markov chain $X_1 \rightarrow (X_2, X_3) \rightarrow Y_3$. In the Gaussian channel, this condition corresponds to $h_{31} = 0$. This situation can happen, for example, when there is heavy shadowing between source 1 and the relay in a wireless channel (see Fig. 2).

A_2: Y_2 is independent of the relay channel input X_3, given X_1, X_2:

$$p(y_2|x_1, x_2, x_3) = p(y_2|x_1, x_2). \tag{14}$$

Note that, due to condition (13), the relay can forward only information desired at Rx_2. Hence, from the perspective of Rx_1, the relay is only performing interference forwarding. We now show how such relaying can help the Tx_1–Rx_1 pair.

We assume that, when the relay does not help, the strong interference condition [8] given by (9) is not satisfied at Rx_1. Hence, receiver 1 cannot decode W_2 without decreasing the maximum achievable rate R_2. Receiver 2 is subject to strong interference, i.e., (8) holds. When an IC is not in strong interference, the highest known achievable rates are obtained by rate splitting [28]. To evaluate gains due to the relay, we compare the rates of Theorem 1 with the rates obtained via rate splitting. Because receiver 2 is in strong interference, no rate splitting is performed at encoder 1. The rate region can be obtained from [28] and [29] and is stated in the following lemma.

Lemma 2: Subject to assumptions A_1 and A_2, any rate pair (R_1, R_2) that satisfies

$$R_1 \leq I(X_1; Y_1|U_2, X_3) \tag{15a}$$
$$R_2 \leq I(X_2; Y_2|X_1) \tag{15b}$$
$$R_1 + R_2 \leq I(X_1, U_2; Y_1|X_3) + I(X_2; Y_2|X_1, U_2) \tag{15c}$$
$$R_1 + R_2 \leq I(X_1, X_2; Y_2) \tag{15d}$$
$$2R_1 + R_2 \leq I(X_1, U_2; Y_1|X_3) + I(X_1, X_2; Y_2|U_2) \tag{15e}$$

for some joint probability distribution that factors as $p(x_1)p(u_2, x_2)p(x_3)p(y_1, y_2, y_3|x_1, x_2, x_3)$ is achievable.

Proof: The encoding and decoding procedures are the same as those used in [29, Lemma 3], with the modifications we now describe. First, note that the message w_1 is encoded using a single codebook, generated according to $p(x_1)$, and the message w_2 undergoes rate splitting and is encoded using a superposition codebook, generated according to $p(u_2, x_2)$. Next, note that since the relay does not help, then it sends a predefined sequence, x_3^n, generated i.i.d. This sequence is known *a priori* at the receivers. Due to A_2 and the underlying distribution chain, we have that

$$\begin{aligned} p(y_1, x_1, x_2, u_2|x_3) &= p(y_2|x_1, x_2, x_3)p(x_1, x_2, u_2|x_3) \\ &= p(y_2|x_1, x_2)p(x_1, x_2, u_2) \\ &= p(y_2, x_1, x_2, u_2). \end{aligned}$$

Therefore, in the decoding rule at Rx_2, there is no need to include x_3^n in the joint-typicality tests, which are based on the received signal y_2^n. The rate constraints due to decoding at Rx_2 are, thus, given by (53)–(57) in [29, Lemma 3], such that the indices 1 and 2 swapped, and additionally we set $W_1 = X_1$ and $W_2 = U_2$. Decoding at Rx_2, therefore, leads to four constraints. When decoding at Rx_1, we use the fact that x_3^n is known by combining it with the received signal: let $\tilde{Y}_1 = (Y_1, X_3)$. Then, the rate constraints for decoding at Rx_1 are obtained from (53)–(57) in [29, Lemma 3] by replacing Y_1 with $\tilde{Y}_1$, and setting $W_1 = X_1$ and $W_2 = U_2$. Then, note that due to the independence of X_3 and (X_1, X_2, U_2), the chain rule for mutual information can be used to simplify the constraints such that X_3 is moved to the conditioning of all mutual information expressions. Also note that as there is no private rate from Tx_1, then we set $T_1 = R_1$ and $S_1 = 0$ in the equations of [29, Lemma 3]. This implies that the constraint [29, eq. (54)] can be omitted, as it accounts for decoding error of only the interference at Rx_1. We, therefore, obtain three rate constraints due to decoding at Rx_1.

Finally, the rate constraints of (15) are obtained by applying Fourier–Motzkin elimination to the seven constraints for decoding at Rx_2 and at Rx_1, obtained as described above. ∎

We denote by $\mathcal{R}_{RS}$ the convex hull of rates that satisfy (15a)–(15e). Consider the following two conditions satisfied for all $p(x_1)p(x_2,x_3)$:

$$I(X_2;Y_2|X_1) \le I(X_3;Y_1) \tag{16}$$
$$I(X_2;Y_2|X_1) \le I(X_2;Y_3|X_3). \tag{17}$$

We have the following proposition.

Proposition 1: For the considered ICR with (13) and (14), and under conditions (16) and (17), we have

$$\mathcal{R}_{RS} \subset \mathcal{R}_{\text{ICR}}.$$

Proof: See Appendix C. ■

Proposition 1 shows that relaying only an interfering message to an unintended receiver can improve the rate region obtained without relaying, and hence demonstrates rate gains from interference forwarding only.

Remark 5: Condition (16) implies strong link relay-receiver 1; (17) implies strong link transmitter 2-relay.

Remark 6: Proposition 1 generalizes to the case when condition (14) is not satisfied. The proof follows the same steps.

B. Strong Interference

We next show how the presence of the relay can change the interference conditions in opposite ways, namely, adding a relay to an IC can switch an IC that is in strong interference to be outside the strong interference regime, or can switch an IC that is not in strong interference to be inside the strong interference regime. These effects have to be taken into account when adding a relay into a network.

To see that, we compare the strong interference conditions with and without the relay at receiver 1.

- To observe that the strong interference conditions at receiver 1 can change in the presence of the relay, we compare (6) and (9) under $p(x_1)p(x_2)p(x_3)$. Specifically, suppose that (9) is satisfied, i.e., $I(X_2;Y_2|X_1,X_3) \le I(X_2;Y_1|X_1,X_3)$ for all $p(x_1)p(x_2)p(x_3)$. Therefore, without a relay, receiver 1 observes strong interference. Now, assume that

$$\begin{aligned} &I(X_3;Y_2|X_1) - I(X_3;Y_1|X_1) \\ &\quad \ge I(X_2;Y_1|X_1,X_3) - I(X_2;Y_2|X_1,X_3) \end{aligned} \tag{18}$$

  holds for all input distributions such that $p(x_1,x_2) = p(x_1)p(x_2)$. We observe that (18) in conjunction with the chain rule for mutual information implies that $I(X_2,X_3;Y_2|X_1) \ge I(X_2,X_3;Y_1|X_1)$, i.e., in the ICR, the strong interference condition (6) is not satisfied, and thus, Rx_1 cannot decode W_2 without reducing R_2. Hence, the relay "pushes" Rx_1 out of strong interference. We observe from (18) that this happens when $I(X_3;Y_2|X_1)$ is large enough so that (18) is satisfied, implying that the link from the relay to Rx_2 is strong. This allows the relay to increase the rate R_2 and prevents Rx_1 from decoding W_2 without constraining R_2.
- The opposite can also happen: Assume that (9) does not hold, namely without a relay receiver 1 does not observe strong interference. Now, we add a relay and assume that

$$\begin{aligned} &I(X_3;Y_1|X_1) - I(X_3;Y_2|X_1) \\ &\quad \ge I(X_2;Y_2|X_1,X_3) - I(X_2;Y_1|X_1,X_3) \end{aligned} \tag{19}$$

  holds for all input distributions such that $p(x_1,x_2) = p(x_1)p(x_2)$. Observe that (6) holds, i.e., in the ICR, receiver 1 observes strong interference. Therefore, Rx_1 moves into strong interference due to interference forwarding, and decoding W_2 does not constrain R_2. From (19), we observe that this happens when $I(X_3;Y_1|X_1)$ is large, i.e., the link from the relay to Rx_1 is strong. This allows the relay to forward enough interference information to receiver 1 so that the receiver can decode it.

For the Gaussian channel (2), condition (18) evaluates to

$$h(X_2+h_{23}X_3+Z_2|X_1) \ge h(h_{12}X_2+h_{13}X_3+Z_1|X_1). \tag{20}$$

Evaluating (20) with Gaussian inputs yields

$$\begin{aligned} &P_2 + h_{23}^2P_3 + h_{23}\rho_{23}\sqrt{P_2P_3} - h_{23}^2\rho_{13}^2P_3 \\ &\quad \ge h_{12}^2P_2 + h_{13}^2P_3 + h_{12}h_{13}\rho_{23}\sqrt{P_2P_3} - h_{13}^2\rho_{13}^2P_3 \end{aligned} \tag{21}$$

where ρ_{13} and ρ_{23} are respective correlation coefficients between (X_1,X_2) and (X_2,X_3). The condition (21) has to hold for any value of correlation coefficients ρ_{13} and ρ_{23}.

V. Interference Forwarding in Gaussian Channels

We next illustrate gains from interference forwarding in Gaussian channels. In particular, we evaluate the region (3a)–(3g) for Gaussian inputs chosen as

$$\begin{aligned} &U_1 \sim \mathcal{N}(0,1), X_{10} \sim \mathcal{N}(0,1) \\ &X_1 = \sqrt{\bar{\alpha}P_1}X_{10} + \sqrt{\alpha P_1}U_1 \\ &U_2 \sim \mathcal{N}(0,1), X_{20} \sim \mathcal{N}(0,1) \\ &X_2 = \sqrt{\bar{\beta}P_2}X_{20} + \sqrt{\beta P_2}U_2. \end{aligned} \tag{22}$$

Thus, the encoders 1 and 2 split their power between sending a new message (respectively with $\bar{\alpha}P_1$ and $\bar{\beta}P_2$) and cooperating with the relay in sending the message from the previous block. The power at the relay is split between forwarding messages W_1, W_2 from the previous block as

$$X_3 = \sqrt{\gamma P_3}U_1 + \sqrt{\bar{\gamma}P_3}U_2 \tag{23}$$

where $0 \le \alpha,\beta,\gamma \le 1$. Parameter γ determines how the relay splits its power for forwarding W_1, W_2. A higher γ results in more power dedicated for forwarding W_1.

The region (3a)–(3g) evaluates to

$$\begin{aligned} R_1 &\le C(P_1 + h_{13}^2\gamma P_3 + 2h_{13}\sqrt{\alpha P_1\gamma P_3}) \\ R_2 &\le C(P_2 + h_{23}^2\bar{\gamma}P_3 + 2h_{23}\sqrt{\beta P_2\bar{\gamma}P_3}) \\ R_1 + R_2 &\le C(P_1 + h_{12}^2P_2 + h_{13}^2P_3 + 2h_{13}\sqrt{\alpha P_1\gamma P_3} \\ &\qquad + 2h_{12}h_{13}\sqrt{\beta P_2\bar{\gamma}P_3}) \\ R_1 + R_2 &\le C(h_{21}^2P_1 + P_2 + h_{23}^2P_3 + 2h_{21}h_{23}\sqrt{\alpha P_1\gamma P_3} \\ &\qquad + 2h_{23}\sqrt{\beta P_2\bar{\gamma}P_3}) \end{aligned}$$

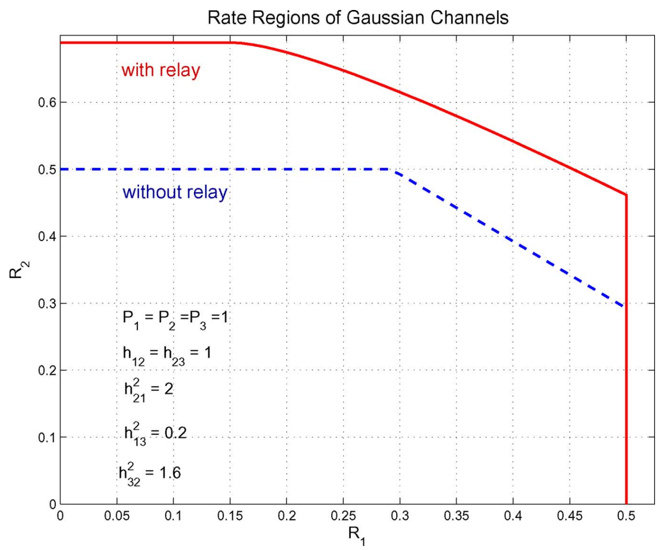

Fig. 3. Rate region of the Gaussian channel without the relay (dashed line) and with the relay (solid line) for $h_{31} = 0$.

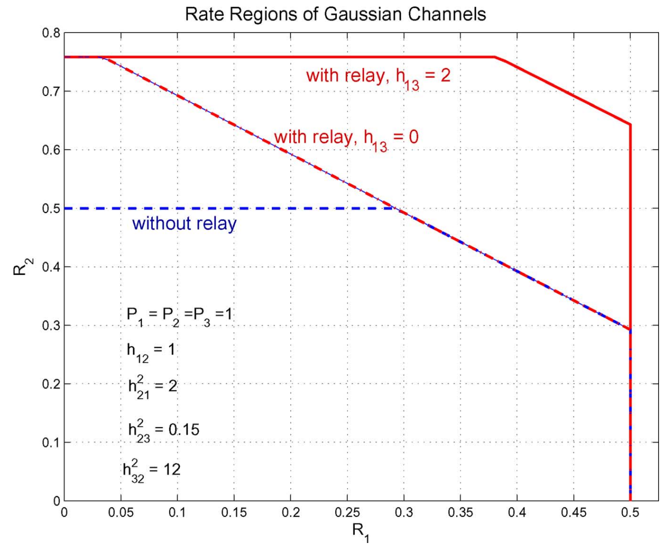

Fig. 4. Rate region of the Gaussian channel without the relay (dashed line) and with the relay (solid line) for $h_{31} = 0$. The dot-dashed region shows the rates for $h_{13} = 0$, i.e., when the relay does not perform interference forwarding. The difference between two regions with the relay illustrates the gains of interference forwarding.

$$
\begin{aligned}
R_1 &\le C(h_{31}^2 \bar{\alpha} P_1) \\
R_2 &\le C(h_{32}^2 \bar{\beta} P_2) \\
R_1 + R_2 &\le C(h_{31}^2 \bar{\alpha} P_1 + h_{32}^2 \bar{\beta} P_2)
\end{aligned} \tag{24}
$$

where $C(x) = 0.5\log(1+x)$.

To demonstrate that there are scenarios in which a relay can help by forwarding only undesired message information, we first evaluate rate region (24) under assumption (13), i.e., $h_{31} = 0$. Then, the signal sent by the relay simultaneously enhances signal reception at receiver 2 and increases interference at receiver 1.

The region (24) is shown by the solid line in Figs. 3 and 4 for two different sets of channel gains. Also shown are rates for the interference channel without the relay, by the dashed line. Without the relay, the strong interference conditions [8] hold, and hence, the latter region is the IC capacity region. With the help of the relay, R_2 increases. From the plots, we can also observe the mechanism in which interference forwarding helps transmission. The relay (via solely interference forwarding) helps receiver 1 to achieve a single-user rate, $R_1 = C(P_1)$ for a larger range of values of R_2 than with no relay. In effect, the relay increases the "strong interference" regime for receiver 1. To emphasize gains from interference forwarding, Fig. 4 shows rates (dot-dashed) for $h_{13} = 0$ when interference forwarding is not possible. The relay only sends undesired message information to receiver 2.

In the previous scenario, the relay could only observe one message and hence only forward interference to receiver 1. We are further interested in investigating whether the relay—being able to forward both the desired message and the interfering message to a destination—should ever allocate power to forward interference. For that reason, we assume $h_{31} > 0$ and $h_{23} = 0$, so that in this case, the relay can decode both the desired and the interfering message, but can forward them only to receiver 1 and cannot help receiver 2. We next show that forwarding W_2 can still be beneficial for decoder 1. Therefore,

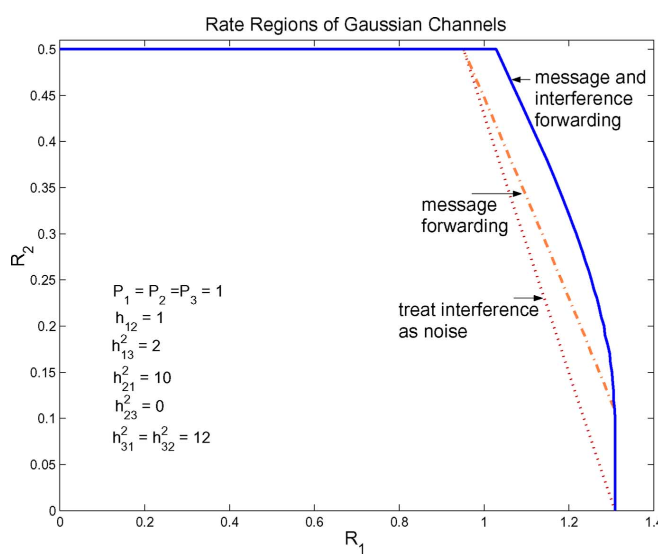

Fig. 5. Rate regions of a Gaussian ICR channel with and without interference forwarding are shown with respective solid and dot-dashed lines. The difference between the two regions illustrates the gains of interference forwarding. The dotted region shows the rates achievable when decoder 1 treats interference as noise. In this example, $h_{12} = 1$.

it is not always optimal for the relay to use all power to forward the desired message to the destination; the relay should allocate some portion of its power for sending the interference. We consider strong interference at destination 2, i.e., we assume $h_{21} > 1$. The rate region is shown in Figs. 5 and 6 for $P_1 = P_2 = P_3 = 1$. Both solid and dot-dashed lines show region (24), but the dot-dashed lines show the case when the relay only forwards message W_1 (i.e., $\gamma = 1$).

Since the encoders do not perform rate splitting, the receivers cannot partially decode unwanted messages. Thus, another decoding option is for decoders to treat the signal carrying the unwanted message as noise. Since we consider the case of strong

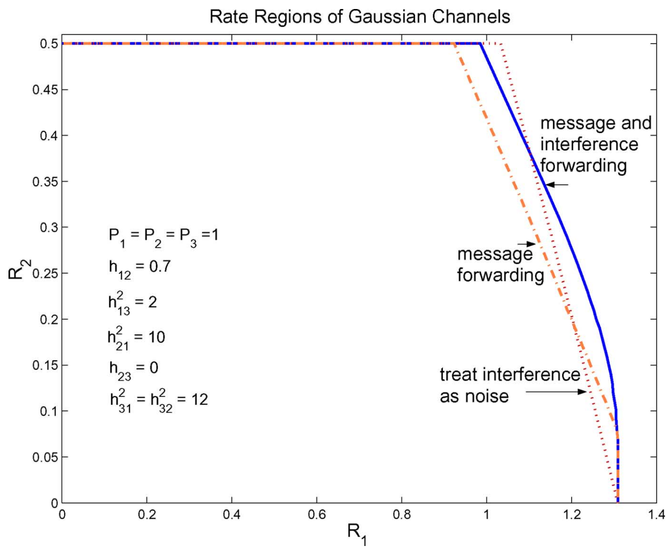

Fig. 6. Rate regions of a Gaussian ICR channel with and without interference forwarding shown with respective solid and dot-dashed lines. The difference between the two regions illustrates the gains of interference forwarding. The dotted region shows the rates achievable when decoder 1 treats interference as noise. In this example, $h_{12} = 0.7$.

interference at decoder 2, i.e., $h_{21} > 1$, this approach would result in a lower rate R_2. Therefore, we compare rates (24) to the case when only decoder 1 treats the signal received from encoder 2 as noise. In this case, the relay forwards only the desired message W_1 (i.e., $\gamma = 1$). We have $\beta = 0$ since the power split at the encoders is only used for facilitating cooperation with the relay. The achievable rates are given by

$$\begin{aligned} R_1 &\le C\left(\frac{P_1 + h_{13}^2 P_3 + 2h_{13}\sqrt{\alpha P_1 P_3}}{1 + h_{12}^2 P_2}\right) \\ R_2 &\le C(P_2) \\ R_1 + R_2 &\le C(h_{21}^2 P_1 + P_2) \\ R_1 &\le C(h_{31}^2 \bar{\alpha} P_1) \\ R_1 + R_2 &\le C(h_{31}^2 \bar{\alpha} P_1 + h_{32}^2 P_2). \end{aligned} \tag{25}$$

The region is shown with a dotted line in Figs. 5 and 6. Note that both regions shown by dot-dashed and dotted lines are obtained when the relay performs message forwarding. The difference is in decoding: in the first case, receiver 1 decodes both messages whereas in the second case, it treats the undesired message as noise. The benefit of one versus the other strategy depends on the interference level at decoder 1 (i.e., on h_{12}) as illustrated in Figs. 5 and 6. The difference between the two figures is in the interference level experienced at decoder 1. When interference is weak (i.e., h_{12} is small), decoding it is less beneficial and gains of interference forwarding are smaller. In particular, the first sum-rate bound in (24), which comes from a constraint for decoding both (W_1, W_2) at decoder 1, becomes smaller as it depends on h_{12}. Then, treating interference as noise at decoder 1 performs better. As h_{12} and, thus, interference increases, the gains from interference forwarding increase.

Remark 7: The operation at the relay in Figs. 5 and 6 is the most pronounced manifestation of interference forwarding. Note that this situation differs from the case considered in [9] and [16] where the interference is increased as a byproduct to message forwarding. We do not refer to such strategy as interference forwarding.

VI. Special Case: Decoding Both Messages

We next derive sufficient conditions that allow the decoders to decode both messages without decreasing the rates when signaling with inputs (22) and (23).

When achievable rates (R_1, R_2) are used for signaling, receiver 1 can decode W_1, form $U_1^n(W_1)$, and with one block delay (due to block Markov encoding) also form $X_1^n(W_1)$ in order to evaluate

$$\hat{Y}_1 = Y_1 - X_1(1 - h_{21}) - \sqrt{\gamma P_3} U_1 (h_{13} - h_{23}). \tag{26}$$

From (2), (23), and (26), we obtain

$$\begin{aligned} \hat{Y}_1 &= h_{21}X_1 + h_{12}X_2 + h_{23}\sqrt{\gamma P_3}U_1 \\ &\quad + h_{13}\sqrt{\bar{\gamma} P_3}U_2 + Z_1 \\ &= h_{21}X_1 + h_{12}\sqrt{\bar{\beta} P_2}X_{20} + h_{23}\sqrt{\gamma P_3}U_1 \\ &\quad + (h_{12}\sqrt{\beta P_2} + h_{13}\sqrt{\bar{\gamma} P_3})U_2 + Z_1 \\ Y_2 &= h_{21}X_1 + X_2 + h_{23}\sqrt{\gamma P_3}U_1 + h_{23}\sqrt{\bar{\gamma} P_3}U_2 + Z_2 \\ &= h_{21}X_1 + \sqrt{\bar{\beta} P_2}X_{20} + h_{23}\sqrt{\gamma P_3}U_1 \\ &\quad + (\sqrt{\beta P_2} + h_{23}\sqrt{\bar{\gamma} P_3})U_2 + Z_2. \end{aligned} \tag{27}$$

By comparing $\hat{Y}_1$ and Y_2 in (27), we conclude that receiver 1 obtains a less noisy signal U_2 than receiver 2 if the condition

$$|h_{12}\sqrt{\beta P_2} + h_{13}\sqrt{\bar{\gamma} P_3}| \ge |\sqrt{\beta P_2} + h_{23}\sqrt{\bar{\gamma} P_3}| \tag{28}$$

is satisfied for every β. For $\beta = 1, \bar{\gamma} = 0$, this condition implies $h_{12} \ge 1$ and hence also X_{20} is received with a higher power at receiver 1 than at receiver 2. Therefore, since decoder 2 can decode W_2, so can receiver 1. Similarly, receiver 2 can decode W_1 when

$$|h_{21}\sqrt{\alpha P_1} + h_{23}\sqrt{\gamma P_3}| \ge |\sqrt{\alpha P_1} + h_{13}\sqrt{\gamma P_3}| \tag{29}$$

for every α. Therefore, in this scenario, the receivers can decode each other's message under conditions (28) and (29). We emphasize that these conditions apply only for the case of inputs (22) and (23), and thus, they are not general strong interference conditions.

In order to further understand how far the obtained achievable rates are from the capacity in Gaussian channels, we next present an outer bound to the rate performance. In particular, we derive a sum-rate outer bound for the Gaussian ICR and compare it to the obtained achievable rate region (24).

VII. Sum-Rate Outer Bound for Gaussian Channels

A. Genie-Aided Approach

We next focus on the Gaussian ICR (2) and present a genie-based outer bound on the sum rate. As in [11], the minimum information that allows decoding of both messages (W_1, W_2) is provided to a receiver. We let

$$Y_{1g} = d_1 X_1 + d_2 X_2 + d_5 X_3 + d_3 Z_1 + d_4 \tilde{Z}_1 \quad (30)$$

where d_i, $i = 1, \ldots, 5$ are real numbers, and $\tilde{Z}_1$ is zero-mean Gaussian random variable with unit variance, independent of other random variables. As in [11], we assume that signal (30) is given to receiver 1. After processing the signal and its own output, receiver 1 will be able to decode both messages (W_1, W_2). This will hold *regardless* of what the relay channel input is and yield the sum-rate bound given by the following theorem:

Theorem 4: The capacity region of the Gaussian ICR (2) is contained in the set of rate pairs (R_1, R_2) satisfying

$$R_1 + R_2 \leq \min_{\{d_i\}_{i=1}^5} I(X_1, X_2, X_3; Y_1, Y_{1g}) \quad (31)$$

where the mutual information is evaluated for jointly Gaussian inputs of the form $p(x_1)p(x_2)p(x_3|x_1, x_2)$ and parameters $d_i, i = 1, 2, \ldots, 5$, that satisfy

$$(1/h_{12} + \beta(d_3 - d_2/h_{12}))^2 + (\beta d_4)^2 \leq 1 \quad (32)$$
$$d_5 = (h_{23} - \alpha h_{13})/\beta \quad (33)$$
$$\alpha = (1 - \beta d_2)/h_{12} \quad (34)$$

for some real numbers α and $\beta \neq 0$.

Proof: The proof is given in Appendix D. The proof adapts the approach of [11]. ■

A corresponding sum-rate bound can be obtained by letting a genie assist the other receiver. By minimizing the mutual information expression in (31) with respect to d_1, we obtain the optimum value of d_1 as

$$\begin{aligned} d_1^* = \Big[&(P_1 + h_{13}\rho_{13}\sqrt{P_1P_3})(h_{12}d_2P_2 + d_5h_{13}P_3 \\ &+ (h_{12}d_5 + h_{13}d_2)\rho_{23}\sqrt{P_2P_3} + d_3) \\ &- d_5\rho_{13}\sqrt{P_1P_3}(h_{12}^2P_2 + h_{13}^2P_3 \\ &+ h_{13}\rho_{13}\sqrt{P_1P_3} + 2h_{12}h_{13}\rho_{23}\sqrt{P_2P_3} + 1)\Big] \\ &\times \Big(P_1(h_{12}^2P_2 + h_{13}^2P_3 + 2h_{12}h_{13}\rho_{23}\sqrt{P_2P_3} \\ &- h_{13}^2\rho_{13}^2P_3 + 1)\Big)^{-1}. \end{aligned}$$

Remark 8: The bound of Theorem 4 applies also to the cognitive Gaussian ICR.

Remark 9: Evaluated for Gaussian inputs, the sum-rate bound (31) depends on the covariance matrix of sources and relay inputs.

Remark 10: The bound can be made more general by allowing the genie signal to depend also on the noise at the relay. This would introduce one more parameter that can be optimized in order to obtain a tighter sum-rate bound.

Remark 11: For $P_3 = 0$, the bound reduces to the bound in [11].

B. Cut-Set Bound

We next derive the cut-set bound [27, p. 445] for the ICR, and compare it to the sum-rate bound presented in Theorem 4.

Lemma 3: For the ICR, the cut-set bound is given by

$$\tilde{\mathcal{R}} = \bigcup_{p(x_1)p(x_2)p(x_3|x_1,x_2)} \mathcal{R}(p(x_1)p(x_2)p(x_3|x_1,x_2)) \quad (35)$$

where $\mathcal{R}(p(x_1)p(x_2)p(x_3|x_1,x_2))$ denotes the set of rate pairs that satisfy

$$\begin{aligned} R_1 &\leq \min\{I(X_1, X_3; Y_1|X_2), I(X_1; Y_1, Y_3|X_2, X_3)\} \\ R_2 &\leq \min\{I(X_2, X_3; Y_2|X_1), I(X_2; Y_2, Y_3|X_1, X_3)\} \\ R_1 + R_2 &\leq \min\{I(X_1, X_2, X_3; Y_1, Y_2), \\ &\qquad I(X_1, X_2; Y_1, Y_2, Y_3|X_3)\} \end{aligned} \quad (36)$$

evaluated for a distribution of the form $p(x_1)p(x_2)p(x_3|x_1,x_2)$.

For the Gaussian ICR (2), all the terms in (36) are maximized by jointly Gaussian inputs [27]. By comparing (31) and (36), we observe that the bound of Theorem 4 is always at least as tight as the first term in the sum rate of the cut-set bound, $I(X_1, X_2, X_3; Y_1, Y_2)$. This is because the genie gives only the minimum information that receiver 1 needs in order to decode both messages (W_1, W_2). We next compare the two bounds numerically.

C. Numerical Results

Comparison of the bound in Theorem 4 and the cut-set bound (36) is shown in Fig. 7. The figure demonstrates an improvement of the sum-rate bound over the sum-rate cut-set bound for a specific choice of channel gains and powers. Fig. 8 shows a comparison of the bound in Theorem 4 with the outer bounds developed for the cognitive ICR in [12, Th. 2 and Th. 3], as well as with the cut-set bound (36). In all plots, the sum-rate bound (31) is evaluated together with the individual cut-set bounds on R_1 and R_2 given in (36).

Because the genie enables the receivers to decode both messages, we expect the outer bound to be close to the achievable rates in the regimes in which such decoding is actually possible, i.e., when the receivers experience strong interference. This behavior is illustrated in Fig. 9. The figure shows the achievable rate region (24) and the derived outer bound. The gap between the achievable rates and the outer bound in this regime is due to constraint (33) imposed in the outer bound. This constraint may not always be necessary in order to allow receivers to decode both messages. Rather, this constraint comes from the proof approach

Remark 12: In general, in the regime of strong interference, there will be a gap between the achievable rate region and our outer bound because, in Theorem 4, constraint (33) does not allow for $\beta = 0$ (i.e., to turn OFF the genie.)

VIII. Conclusions and Future Work

This paper considers cooperative strategies for multiple communicating pairs. In such networks, the presence of interference impacts relaying schemes. We derived an achievable rate region for the general ICR. Special cases in which this region

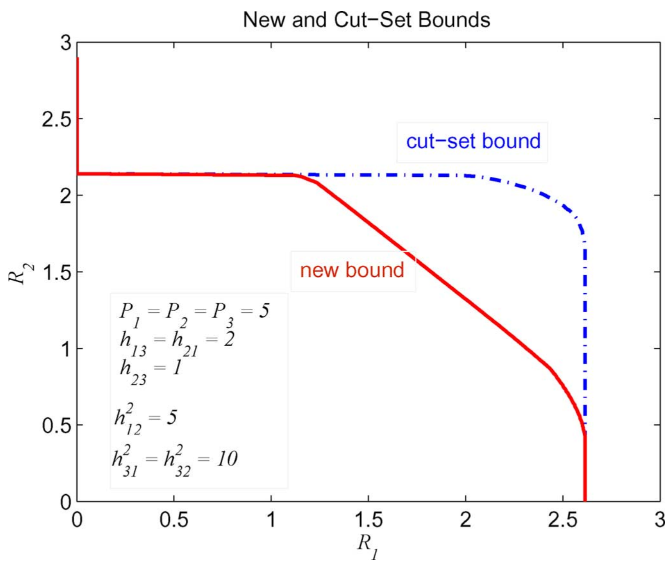

Fig. 7. Comparison of the cut-set outer bound of Lemma 3 with the outer bound of Theorem 4.

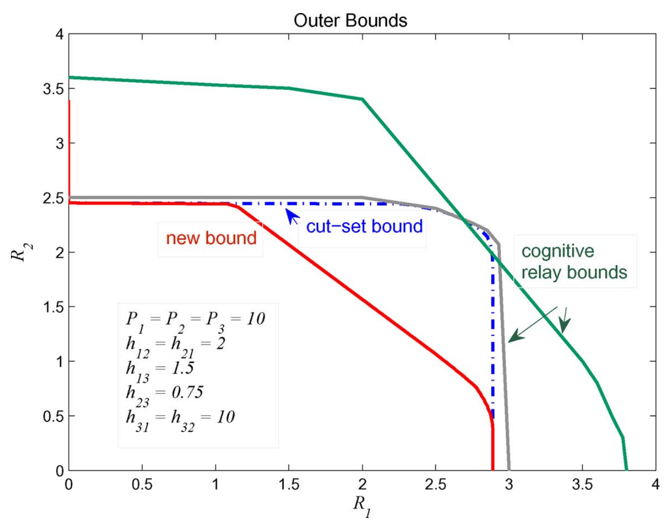

Fig. 8. Comparison of the new bound of Theorem 4 with the cut-set outer bound of Lemma 3 and the outer bounds developed to the ICR from [12] (i.e., cognitive relay bounds).

achieves capacity were presented. Our results demonstrate that forwarding interference—with a goal of enhancing an undesired signal so that it can be canceled out at the assisted receiver—can improve the performance. Therefore, in addition to forwarding desired messages to their intended destinations, interference forwarding should be considered when relaying in networks with multiple communication pairs.

The considered encoding scheme does not include rate splitting at the encoders and/or the relay. The largest performance gains are then obtained when the interference is strong, because then the interference cancellation can readily be realized via interference forwarding at the relay. This approach allowed us to easily identify the scenarios in which interference forwarding brings benefits. Rate splitting also facilitates (partial) interference cancellation. In future work, we will investigate the gains of interference forwarding when accompanied by rate splitting.

We presented a new sum-rate outer bound for the Gaussian ICR. The bound also applies to the cognitive Gaussian ICR.

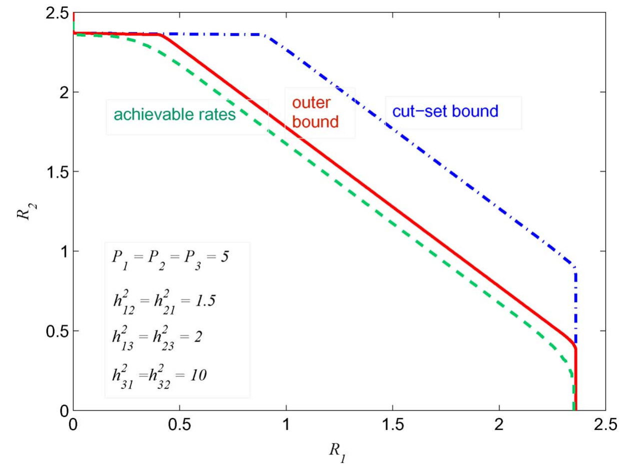

Fig. 9. Achievable rate region (24) vs. the outer bound of Theorem 4 and the cut-set outer bound (Lemma 3).

The outer bound is obtained by adapting the approach developed for the interference channels in [11]. The bound is significantly tighter than the cut-set bound in some scenarios. One limitation of the bound is that it requires a receiver to decode *both* messages. This requirement could be relaxed by using the genie technique in [30] that led to the sum capacity of the interference channel in the low interference regime [31]–[33]. The biggest difficulty of this approach when applied in our scenario is in showing the optimality of Gaussian inputs. This is one direction of our future work. The other possible extension is to apply the genie approach employed in this bound to larger networks.

Appendix A
Proof of Theorem 1

In the subsequent proofs, we shall need the following lemma:

Lemma 4: If (5) and (6) are satisfied for any distribution given by (4), then

$$I(X_1^n, X_3^n; Y_1^n | X_2^n) \le I(X_1^n, X_3^n; Y_2^n | X_2^n) \quad (37)$$
$$I(X_2^n, X_3^n; Y_2^n | X_1^n) \le I(X_2^n, X_3^n; Y_1^n | X_1^n). \quad (38)$$

Proof: Proof follows the same steps as in [34, Lemma 5]. ■

Proof of Theorem 1: We use regular encoding and backward decoding [2, Sec. 7].

Code Construction: Choose a distribution $\mathcal{P} = p(u_1)p(x_1|u_1)p(u_2)p(x_2|u_2)p(x_3|u_1,u_2)$.

1) Generate 2^{nR_1} codewords $u_1^n(v_1)$, $v_1 = 1, \ldots, 2^{nR_1}$, by choosing $u_{1,i}(v_1)$ independently according to $P_{U_1}(\cdot)$.
2) For each v_1: generate 2^{nR_1} codewords $x_1^n(v_1, w_1)$ using $\prod_{i=1}^n P_{X_1|U_1}(\cdot|u_{1,i}(v_1))$, $w_1 = 1, \ldots, 2^{nR_1}$.
3) Generate 2^{nR_2} codewords $u_2^n(v_2)$, $v_2 = 1, \ldots, 2^{nR_2}$, by choosing $u_{2,i}(v_2)$ independently according to $P_{U_2}(\cdot)$.
4) For each v_2: generate 2^{nR_2} codewords $x_2^n(v_2, w_2)$ using $\prod_{i=1}^n P_{X_2|U_2}(\cdot|u_{2,i}(v_2))$, $w_2 = 1, \ldots, 2^{nR_2}$.
5) For each pair (v_1, v_2): Generate $x_3^n(v_1, v_2)$ independently from symbol to symbol using $\prod_{i=1}^n P_{X_3|U_1,U_2}(\cdot|u_{1,i}(v_1), u_{2,i}(v_2))$.

	block 1	block 2	block 3	block 4
source 1	x1(1, w1(1))	x1(w1(1),w1(2))	x1(w1(2),w1(3))	x1(w1(3),1)
source 2	x2(1,w2(1))	x2(w2(1),w2(2))	x2(w2(2),w2(3))	x2(w2(3),1)
relay	x3(1,1)	x3(w1(1),w2(1))	x3(w1(2),w2(2))	x3(w1(3),w2(3))

Fig. 10. Encoding at the sources and at the relay. $w_t(b), t = 1, 2$ denotes the message sent by the source t in block b, for $b = 1, \ldots B-1$. In this example, $B = 4$.

Encoders: (see Fig. 10).

As in [1], the message at each source is sent over B blocks. In block $b = 1, \ldots, B$, encoder 1 transmits $x_1^n(w_1(b-1), w_1(b))$ and encoder 2 transmits $x_2^n(w_2(b-1), w_2(b))$. We choose $w_t(0) = w_t(B) = 1$, $t = 1, 2$. At the end of block $b-1$, the relay observes $y_3^n(b-1)$, and chooses $(\tilde{w}_1(b-1), \tilde{w}_2(b-1))$ such that

$$\begin{aligned}(x_1^n(\tilde{w}_1(b-2), \tilde{w}_1(b-1)), x_2^n(\tilde{w}_2(b-2), \tilde{w}_2(b-1)),\\ u_1^n(\tilde{w}_1(b-2)), u_2^n(\tilde{w}_2(b-2)), y_3^n(b-1)) \in T_\epsilon^n(\mathcal{P})\end{aligned} \tag{39}$$

where $(\tilde{w}_1(b-2), \tilde{w}_2(b-2))$ are relay estimates made in the previous block, and $T_\epsilon^n(\mathcal{P})$ denotes the jointly ϵ-typical set with respect to $\mathcal{P}$, see [27, Sec. 8.6].

In block b, the relay then transmits $x_3^n(\tilde{w}_1(b-1), \tilde{w}_2(b-1))$.

Decoders: Assume $(1, 1)$ is sent in each block. Decoders 1 and 2 use backward decoding. In particular, they start by decoding in block B.

Decoder 1 has observed $y_1^n(B)$. It chooses $(\hat{w}_1(B-1), \hat{w}_2(B-1))$ if

$$\begin{aligned}(u_1^n(\hat{w}_1(B-1)), x_1^n(\hat{w}_1(B-1), 1), u_2^n(\hat{w}_2(B-1)),\\ x_2^n(\hat{w}_2(B-1), 1), x_3^n(\hat{w}_1(B-1),\\ \hat{w}_2(B-1)), y_1^n(B)) \in T_\epsilon(\mathcal{P}).\end{aligned}$$

If there is more than one such pair, choose one. If there is no such pair, choose $(1, 1)$.

Similarly, in any block b, decoder 1 has observed $y_1^n(b)$, and has decided $(\hat{w}_1(b), \hat{w}_2(b))$. It then chooses (assuming decoding in block $b+1$ was correct) $(\hat{w}_1(b-1), \hat{w}_2(b-1))$ if

$$\begin{aligned}(u_1^n(\hat{w}_1(b-1)), x_1^n(\hat{w}_1(b-1), 1),\\ u_2^n(\hat{w}_2(b-1)), x_2^n(\hat{w}_2(b-1), 1),\\ x_3^n(\hat{w}_1(b-1), \hat{w}_2(b-1)), y_1^n(b)) \in T_\epsilon(\mathcal{P}).\end{aligned}$$

The exact same decoding is done at destination 2 by using $y_2^n(b)$ instead of $y_1^n(b)$ for all $b = 1, \ldots, B$.

Analysis: Since $(1, 1)$ was sent in each block, the error events at decoder 1 in block $b+1$ are $E_1 = \{\hat{w}_1(b) \neq 1, \hat{w}_2(b) \neq 1\}$ and $E_2 = \{\hat{w}_1(b) \neq 1, \hat{w}_2(b) = 1\}$.

Consider the probability of event E_1

$$\begin{aligned}&P[\hat{W}_1 \neq 1, \hat{W}_2 \neq 1]\\ &= \sum_{w_1=2}^{2^{nR_1}} \sum_{w_2=2}^{2^{nR_2}} P[(U_1^n(w_1), X_1^n(w_1, 1),\\ &\quad U_2^n(w_2), X_2^n(w_2, 1), X_3^n(w_1, w_2), Y_1^n) \in T_\epsilon(\mathcal{P})]\\ &\leq 2^{-n[I(U_1, U_2, X_1, X_2, X_3; Y_1) - (R_1+R_2) - \delta]}\end{aligned} \tag{40}$$

by [27, Sec.8.6.1]. From (40), achieving arbitrarily small error probability of E_1 requires

$$R_1 + R_2 < I(X_1, X_2, X_3; Y_1) \tag{41}$$

yielding (3c).

Consider the probability of event E_2

$$\begin{aligned}&P[\hat{W}_1 \neq 1, \hat{W}_2 = 1]\\ &= \sum_{w_1=2}^{2^{nR_1}} P[(U_1^n(w_1), X_1^n(w_1, 1), \qquad (42)\\ &\quad U_2^n(1), X_2^n(1, 1), X_3^n(w_1, 1), Y_1^n) \in T_\epsilon(\mathcal{P})]\\ &\leq 2^{-n[I(U_1, X_1, X_3; Y_1 | U_2, X_2) - R_1 - \delta]} \qquad (43)\end{aligned}$$

by [27, Sec.8.6.1]. From (43), achieving arbitrarily small error probability of E_2 requires

$$R_1 < I(U_1, X_1, X_3; Y_1 | U_2, X_2) \tag{44}$$

or equivalently

$$R_1 < I(X_1, X_3; Y_1 | U_2, X_2) \tag{45}$$

yielding (3a). Similar analysis holds for decoder 2 resulting in (3b) and (3d). The three error events at the relay in block b are, as in the MAC [27], $E_{r,12} = \{\tilde{w}_1(b) \neq 1, \tilde{w}_2(b) \neq 1\}$, $E_{r,1} = \{\tilde{w}_1(b) \neq 1, \tilde{w}_2(b) = 1\}$, and $E_{r,2} = \{\tilde{w}_1(b) = 1, \tilde{w}_2(b) \neq 1\}$. Then, from (39)

$$\begin{aligned}P[E_{r,12}] &= P[\tilde{W}_1 \neq 1, \tilde{W}_2 \neq 1]\\ &= \sum_{w_1=2}^{2^{nR_1}} \sum_{w_2=2}^{2^{nR_2}} P[(U_1^n(1), X_1^n(1, w_1),\\ &\quad U_2^n(1), X_2^n(1, w_2), X_3^n(1, 1), Y_3^n) \in T_\epsilon(\mathcal{P})]\\ &\leq 2^{-n[I(X_1, X_2; Y_3 | U_1, U_2, X_3) - (R_1+R_2) - \delta]}\end{aligned} \tag{46}$$

and hence

$$R_1 + R_2 \leq I(X_1, X_2; Y_3 | U_1, U_2, X_3) \tag{47}$$

yielding (39).

$$\begin{aligned}P[E_{r,1}] &= P[\tilde{W}_1 \neq 1, \tilde{W}_2 = 1]\\ &= \sum_{w_1=2}^{2^{nR_1}} P[(U_1^n(1), X_1^n(1, w_1),\\ &\quad U_2^n(1), X_2^n(1, 1), X_3^n(1, 1), Y_3^n) \in T_\epsilon(\mathcal{P})]\\ &\leq 2^{-n[I(X_1; Y_3 | X_2, U_1, U_2, X_3) - R_1 - \delta]}.\end{aligned} \tag{48}$$

Similar steps as in (48) show that

$$P[E_{r,2}] \leq 2^{-n[I(X_2; Y_3 | X_1, U_1, U_2, X_3) - R_2 - \delta]}. \tag{49}$$

From (48) and (49), we, respectively, obtain (3e) and (3f). ■

APPENDIX B
PROOF OF THEOREM 2

We first prove that bound (3c) is tight. Following Fano's inequality, we have

$$\begin{aligned} n(R_1+R_2) &\le I(W_1;Y_1^n)+I(W_2;Y_2^n) \\ &\le^{(a)} I(W_1;Y_1^n)+I(W_2;Y_2^n|W_1) \\ &=^{(b)} I(X_1^n;Y_1^n)+I(W_2;Y_2^n|X_1^n) \\ &\le I(X_1^n;Y_1^n)+I(W_2,X_2^n,X_3^n;Y_2^n|X_1^n) \\ &=^{(c)} I(X_1^n;Y_1^n)+I(X_2^n,X_3^n;Y_2^n|X_1^n) \\ &\le^{(d)} I(X_1^n;Y_1^n)+I(X_2^n,X_3^n;Y_1^n|X_1^n) \\ &= I(X_1^n,X_2^n,X_3^n;Y_1^n) \\ &\le \sum_{i=1}^n I(X_{1,i},X_{2,i},X_{3,i};Y_{1,i}) \end{aligned} \tag{50}$$

where (a) follows by independence of (W_1,W_2); (b) and (c) follow by the encoding, and (d) follows by (6) and Lemma 4.

Using a similar approach, it can be shown that bound (3d) is tight. Bounds (3a) and (3b) can be shown by following the same approach as in [2, Sec. 6]:

$$\begin{aligned} nR_1 &\le I(W_1;Y_1^n|W_2) \\ &= \sum_{i=1}^n I(W_1;Y_{1,i}|W_2,Y_1^{i-1}) \\ &=^{(a)} \sum_{i=1}^n I(W_1;Y_{1,i}|W_2,Y_1^{i-1},X_2^i) \\ &\le \sum_{i=1}^n I(W_1,X_{1,i},X_{3,i};Y_{1,i}|W_2,Y_1^{i-1},X_2^i) \\ &= \sum_{i=1}^n I(X_{1,i},X_{3,i};Y_{1,i}|W_2,Y_1^{i-1},X_2^i) \\ &\le^{(b)} \sum_{i=1}^n I(X_{1,i},X_{3,i};Y_{1,i}|X_{2,i},U_{2,i}) \end{aligned} \tag{51}$$

where (a) follows by the encoding; (b) follows by Markovity $(W_2,Y_1^{i-1}) \to (X_{1,i},X_{2,i},X_{3,i}) \to Y_{3,i}$ and by denoting

$$U_{2,i} = X_2^{i-1}. \tag{52}$$

Bound (7b) can be shown by repeating the same steps for R_2. The approach in [2, Sec. 6] also implies the chain (4).

Bounds (7e) and (7f) can be shown using similar steps as in [1, Lemma 4]:

$$\begin{aligned} nR_1 &\le I(W_1;Y_1^n|W_2) \\ &\le I(W_1;Y_1^n,Y_3^n|W_2) \\ &= \sum_{i=1}^n I(W_1;Y_{1,i},Y_{3,i}|W_2,Y_1^{i-1},Y_3^{i-1}) \\ &\le^{(a)} \sum_{i=1}^n I(W_1;Y_{1,i},Y_{3,i}|W_2,Y_1^{i-1},Y_3^{i-1},X_3^i,X_2^i) \\ &= \sum_{i=1}^n H(Y_{1,i},Y_{3,i}|W_2,Y_1^{i-1},Y_3^{i-1},X_3^i,X_2^i) \\ &\quad - H(Y_{1,i},Y_{3,i}|W_2,Y_1^{i-1},Y_3^{i-1},X_3^i,X_2^i,X_{1,i},W_1) \\ &\le \sum_{i=1}^n H(Y_{1,i},Y_{3,i}|X_{3,i},X_2^{i-1}) \\ &\quad - H(Y_{1,i},Y_{3,i}|X_{3,i},X_2^i,X_{1,i}) \\ &=^{(b)} \sum_{i=1}^n I(X_{1,i};Y_{1,i},Y_{3,i}|X_{3,i},X_{2,i},U_{2,i}) \end{aligned} \tag{53}$$

where (a) follows by the Markov chain $W_1 \to (W_2,Y_1^{i-1},Y_3^{i-1}) \to X_3^i,X_2^i$, and (b) by using (52).

Similar steps yield (3f). Finally, (3g) can be proven in the similar way

$$\begin{aligned} n(R_1+R_2) &\le I(W_1,W_2;Y_1^n,Y_2^n,Y_3^n) \\ &= \sum_{i=1}^n I(W_1,W_2;Y_{1,i},Y_{2,i},Y_{3,i}|Y_1^{i-1},Y_2^{i-1},Y_3^{i-1}) \\ &= \sum_{i=1}^n H(Y_{1,i},Y_{2,i},Y_{3,i}|Y_1^{i-1},Y_2^{i-1},Y_3^{i-1}) \\ &\quad - H(Y_{1,i},Y_{2,i},Y_{3,i}|Y_1^{i-1},Y_2^{i-1},Y_3^{i-1},W_1,W_2) \\ &= \sum_{i=1}^n H(Y_{1,i},Y_{2,i},Y_{3,i}|Y_1^{i-1},Y_2^{i-1},Y_3^{i-1},X_{3,i}) \\ &\quad - H(Y_{1,i},Y_{2,i},Y_{3,i}|Y_1^{i-1},Y_2^{i-1},Y_3^{i-1}, \\ &\qquad W_1,W_2,X_1^i,X_{2,i},X_{3,i}) \\ &\le \sum_{i=1}^n H(Y_{1,i},Y_{2,i},Y_{3,i}|X_{3,i}) \\ &\quad - H(Y_{1,i},Y_{2,i},Y_{3,i}|X_{1,i},X_2^i,X_3^i) \\ &= \sum_{i=1}^n I(X_{1,i},X_{2,i};Y_{1,i},Y_{2,i},Y_{3,i}|X_{3,i}). \end{aligned} \tag{54}$$

APPENDIX C
PROOF OF PROPOSITION 1

Under assumptions (13) and (14), the relay signal does not depend on W_1 and we have $X_1 = U_1$, $X_3 = f(U_2)$. The achievable rate region (3a)–(3g) reduces to

$$R_1 \le I(X_1;Y_1|X_3,X_2) \tag{55}$$
$$R_2 \le I(X_2;Y_2|X_1) \tag{56}$$
$$R_1+R_2 \le I(X_1,X_2,X_3;Y_1) \tag{57}$$
$$R_1+R_2 \le I(X_1,X_2;Y_2) \tag{58}$$
$$R_2 \le I(X_2;Y_3|X_3) \tag{59}$$

evaluated for input distributions that factor as $p(x_1)p(u_2,x_2)f(x_3|u_2)$. Comparing (55)–(59) with rate-splitting rates (15a)–(15e), we see that (56) and (58), respectively, are equal to (15b) and (15d) and are evaluated for the same distribution $p(x_1)p(x_2)$. This is expected as the relay does not impact signal Y_2 when (X_1,X_2) are known. We next show that the bound in (15a) is tighter than (55)

$$\begin{aligned} I(X_1;Y_1|U_2,X_3) &= H(X_1|U_2,X_3) - H(X_1|Y_1,U_2,X_3) \\ &=^{(a)} H(X_1|X_2,X_3) - H(X_1|Y_1,U_2,X_3) \end{aligned}$$

$$\begin{aligned}&\le H(X_1|X_2,X_3)-H(X_1|Y_1,X_2,U_2,X_3)\\&=^{(b)} I(X_1;Y_1|X_2,X_3)\end{aligned}\qquad(60)$$

where (a) follows since X_1 is independent of (U_2,X_2,X_3) under $p(x_1)p(u_2,x_2)p(x_3)$; and (b) follows since $p(x_1|y_1,x_2,u_2,x_3) = p(x_1|y_1,x_2,x_3)$. Comparing the sum-rate constraints (57) and (15c), we obtain

$$\begin{aligned}I(X_1,U_2;Y_1|X_3)&+I(X_2;Y_2|U_2,X_1)\\&\le^{(a)} I(X_1,X_2;Y_1|X_3)+I(X_2;Y_2|X_1)\\&\le^{(b)} I(X_1,X_2;Y_1|X_3)+I(X_3;Y_1)\\&=I(X_1,X_2,X_3;Y_1)\end{aligned}\qquad(61)$$

where (a) follows by Markovity and (b) follows by (16). Condition (17) assures that bound (59) is looser compared to (56).

APPENDIX D
PROOF OF THEOREM 4

We consider signaling at achievable rates (R_1,R_2). Each receiver t, $t=1,2$ can then reliably decode its desired message W_t. A genie gives receiver 1 the signal Y_{1g} given by (30).

Receiver 1 processes its channel output and the information obtained from the genie in the same manner as in [11]: after decoding W_1, it forms

$$\hat{Y}_1=\alpha Y_1+\beta Y_{1g}+(h_{21}-\alpha-\beta d_1)X_1\qquad(62)$$

for some real numbers α and $\beta\neq 0$. This yields

$$\begin{aligned}\hat{Y}_1=h_{21}X_1+(\alpha h_{12}+\beta d_2)X_2+(\alpha h_{13}+\beta d_5)X_3\\+(\alpha+\beta d_3)Z_1+\beta d_4\tilde{Z}_1.\end{aligned}\qquad(63)$$

We choose

$$\alpha h_{12}+\beta d_2=1\qquad \alpha h_{13}+\beta d_5=h_{23}\qquad(64)$$

which yield conditions (33) and (34). Then, (63) becomes

$$\hat{Y}_1=h_{21}X_1+X_2+h_{23}X_3+(\alpha+\beta d_3)Z_1+\beta d_4\tilde{Z}_1.\qquad(65)$$

Comparing (65) to the channel output at receiver 2 given by (2), we conclude that when the equivalent noise variance in (65) is smaller than the noise variance at receiver 2, i.e., when

$$(\alpha+\beta d_3)^2+(\beta d_4)^2\le 1\qquad(66)$$

then, since receiver 2 can decode W_2, receiver 1 can decode W_2 as well. This conclusion holds *regardless* of what the relay channel input is. By substituting the expression for α from (34) into (66), we obtain (32), which is an equivalent condition to the condition for the interference channel bound in [11].

Because receiver 1 can reliably decode both messages, we can now bound the sum rate using Fano's inequality as

$$\begin{aligned}&n(R_1+R_2)\\&\le^{(a)} I(W_1,W_2;Y_1^n,Y_{1g}^n)\\&=\sum_{i=1}^n\Big[H(Y_{1,i},Y_{1g,i}|Y_1^{i-1},Y_{1g}^{i-1})\\&\qquad -H(Y_{1,i},Y_{1g,i}|Y_1^{i-1},Y_{1g}^{i-1},W_1,W_2)\Big]\\&\le\sum_{i=1}^n\Big[H(Y_{1,i},Y_{1g,i})\\&\qquad -H(Y_{1,i},Y_{1g,i}|Y_1^{i-1},Y_{1g}^{i-1},W_1,W_2)\Big]\\&\le\sum_{i=1}^n\Big[H(Y_{1,i},Y_{1g,i})\\&\qquad -H(Y_{1,i},Y_{1g,i}|Y_1^{i-1},Y_{1g}^{i-1},X_1^i,X_2^i,X_3^i,W_1,W_2)\Big]\\&=^{(b)}\sum_{i=1}^n\Big[H(Y_{1,i},Y_{1g,i})\\&\qquad -H(Y_{1,i},Y_{1g,i}|Y_1^{i-1},Y_{1g}^{i-1},X_1^i,X_2^i,X_3^i)\Big]\\&=^{(c)}\sum_{i=1}^n\Big[H(Y_{1,i},Y_{1g,i})\\&\qquad -H(Y_{1,i},Y_{1g,i}|X_{1,i},X_{2,i},X_{3,i})\Big]\\&=\sum_{i=1}^n I(X_{1,i},X_{2,i},X_{3,i};Y_{1,i},Y_{1g,i})\end{aligned}\qquad(67)$$

where (a) follows because receiver 1 can decode both messages, (b) follows by causality, and (c) follows by the memoryless property of the channel.

By introducing a time-sharing random variable in (67) as in [27, Th. 14.10.1], we obtain the sum-rate bound as

$$R_1+R_2\le I(X_1,X_2,X_3;Y_1,Y_{1g}).\qquad(68)$$

It follows from the maximum entropy theorem [27, Th. 9.6.5] that Gaussian inputs maximize the mutual information expression in (68). As the final step, we optimize this bound over parameters $d_i, i=1,\ldots,5$ subject to (64) and (66).

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