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The Value of Cooperation Between Relays in the Multiple Access Channel With Multiple Relays

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ABSTRACT

We study the discrete, memoryless multiple-access channel with two independent sources, two relays and a single destination. We refer to this configuration as the multiple-access channel with multiple relays (MACMR), which is a generalization of the multiple-access relay channel (MARC) model obtained by adding a relay node. We present inner and outer bounds on the capacity region of the MACMR. The inner bound is based on a hierarchical decode-and-forward scheme, in which each relay decodes the messages of the lower hierarchy. We extend the regular encoding, sliding-window decoding and backward decoding techniques, previously applied to MARCs and multiple-relay channels, to MACMRs. The outer bounds are obtained using the cut-set bound. For Gaussian MACMRs the bounds are evaluated and compared to those obtained for the multiple-access channel with parallel relays. We conclude that a significant improvement in performance can be obtained by letting the relays interact with each other. Copyright © 0000 John Wiley & Sons, Ltd.

1. INTRODUCTION

In this work we study the multiple-access channel with multiple relays (MACMR). This model extends the multiple-access relay channel (MARC) by adding relays to assist the communication from the sources to the destination. We first recall the main results on the two component channels of the MACMR: the MARC and the multiple-relay channel.

1.1. The Multiple-Access Relay Channel

The MARC is a network in which several users communicate with a single destination in the presence of a relay [1]. A two-user MARC is depicted in Figure 1. The MARC is a suitable model for situations in which direct cooperation between the nodes is either undesirable or not possible, but an intermediate relay node is available to aid communications between the sources and the destination. This model, therefore, applies to hybrid wireless LAN/WAN networks, sensor networks, and ad hoc networks. An outer bound on the capacity of the discrete memoryless MARC (DM-MARC) using cut-sets was derived in [1] (see also [2]). An achievable rate region for the Gaussian MARC was also obtained in [1] by extending the decode-and-forward (DF) coding scheme proposed in [3]. The DF scheme of [3] combines block Markov superposition encoding, random partitioning (binning), and successive decoding. An achievable rate region was also obtained in [4] using DF based on block Markov encoding and backward decoding (see [5]). In [6], potentially tighter outer bounds on the capacity region of the DM-MARC as well as achievable rate regions based on the compress-and-forward strategy, [3, Theorem 6], were obtained, and an achievable rate region for Gaussian MARCs, based on the amplify-and-forward strategy, was derived.

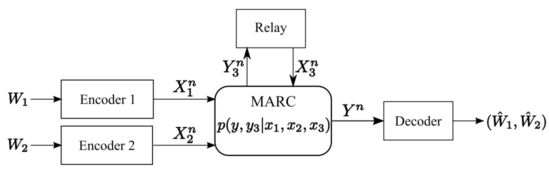

Figure 1. A two-user multiple-access relay channel.

Additionally, in [6], [7] a new code construction for MARCs using offset encoding with the DF strategy was presented. This construction facilitates the more practical window decoding (see [8]) at the destination, while achieving the same rate region as in backward decoding. At the same time, this construction avoids the excessive delay associated with backward decoding.

One simple instance of the MARC is the degraded Gaussian MARC [9]. A K-user Gaussian MARC is degraded when, given the transmitted signal at the relay, the multiple-access signal received at the destination is a noisier version of the signal received at the relay. For a K-user degraded Gaussian MARC, Sankar et al. developed an inner bound on the capacity region using Gaussian codebooks at the sources and at the relay and the DF strategy. Outer bounds on the capacity region were obtained by specializing the cut-set bound of [10, Theorem 15.10.1] to the case of independent sources and by applying the degradedness condition.

In [11] Ho et al. proposed and analyzed a decode-and-forward scheme for the MARC with generalized feedback. In their scheme, a common auxiliary random variable is used for facilitating cooperation. This requires that all nodes successfully decode the messages intended for cooperation. The coding scheme of [11] can be viewed as a combination of the schemes for the MARC and for the two-way relay channel [12]. Another type of MARC with feedback is the MARC with relay-source feedback that was studied by Hou et al. in [13].

1.2. The Multiple Relay Channel

In the classic relay channel [3], a single helper node assists the communication from a single source to a single destination. In recent years there has been increasing interest in scenarios that include multiple relays. For instance, the Gaussian parallel relay channel considered in [14] includes two relay nodes. In [15] the DF scheme was applied to the physically degraded Gaussian multiple-relay channel and its capacity was characterized. The general multiple-level relay channel, where each level of relaying consists of one or more nodes was studied in [16]. In this paper the irregular encoding/sliding-window decoding DF scheme of [3] was extended to multiple relays, and an achievable rate expression was derived.

In [17], Xie and Kumar proposed a new coding scheme for the Gaussian multiple-level relay channel by combining regular encoding with sliding-window decoding. The scheme they developed is an extension of [8] to a multistage format (i.e., sliding-window). This scheme gives the same achievable rate for the single-relay channel as in [3]. However, it is easier to extend to the multiple-level relay channel, and generally achieves higher rates than those achieved in [16]. An achievable rate using the regular encoding/sliding-window decoding scheme for the DM multiple-relay channel was derived in [18].

Kramer, Gastpar and Gupta applied in [19] the regular encoding/sliding-window decoding scheme to memoryless relay networks, and generalized the approach of [17] to additional classes of relay networks. They also generalized regular encoding/backward decoding to the multiple-relay channel. The achievable rates of the two regular encoding methods turn out to be the same. However, the *delay* of sliding-window decoding is lower than that of backward decoding. Additional related work can be found in [28], [29] and [30].

1.3. Previous Models and Results for MACMRs

We now describe two models for MACMRs that were considered recently, which are most relevant to the present work. These models serve as a baseline for our work. The first model is the multiple-access channel (MAC) with multiple parallel relays (MPR-MAC) studied in [20]–[22]. Two relays are said to be parallel if there is no direct link between them, while each relay has a direct link from the source and a direct link towards the destination. In [20], del Coso et al. derived the capacity region for the multiple-access channel assisted by N parallel relays that have only buffering and amplifying capabilities. In [21], the authors derived rate regions for the MAC assisted by N parallel relays using the DF strategy. The work [21] considered both full-duplex and half-duplex relaying. The asymptotic sum-rate for this channel under Rayleigh fading was also presented. In [22], an achievable rate region with linear relaying was derived for the MPR-MAC. In linear relaying, each relay node transmits at each symbol time a linear combination of its previously received channel outputs, see [24]. Additional results on the MPR-MAC can be found in [23].

• The scenario considered in this work generalizes the one in [20]–[22] as we consider non-parallel relays: in the present work, each relay can use the signal from the other relay to enhance coordination.

The second model is the multiple-access relay network (MARN), studied in [25]. The MARN consists of multiple transmitters, multiple relays and a single receiver. The authors obtained an achievable rate region for MARNs by considering the partial decode-and-forward (PDF) strategy at the relays. They showed that the region obtained using the PDF strategy subsumes the region obtained by for the MARC in [19]. They also define the semi-deterministic MARN, in which the output of every transmitter-relay link is a deterministic function of the input from the transmitter. The authors present inner and outer bounds on the capacity region of the semi-deterministic MARN.

• The key difference between the present contribution and the work of [25] is in the way the relays cooperate: in our network the relays cooperate with *each other* in order to improve their effectiveness in assisting the communication between sources and destination. In the scheme of [25], each relay cooperates with the sources, but the relays *do not cooperate between themselves*. Our scheme is therefore more general then the one in [25], but the one in [25] has the advantage of scalability, and it can be easily adapted to any number of relays. These differences will be elaborated later.

The rest of this paper is organized as follows: In Section 2 we introduce the notation, channel model and definitions. In Section 3 we present the coding scheme and derive an achievable rate region for the MACMR. In Section 4 we derive the cut-set outer bound for the MACMR. We also write explicit expressions for the outer and inner bounds for the Gaussian MACMR. In Section 5 the bounds are numerically evaluated and comparison with previous work is made. It is demonstrated that in some scenarios, our achievable rate region is outside the outer bound for the MACMR with parallel relays (i.e., the MPR-MAC) considered in [23]. Finally, Section 6 concludes the work.

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2. NOTATIONS AND CHANNEL MODEL

In the following we denote random variables with upper case letters, e.g. X, Y, and their realizations with lower case letters, x, y. A random variable (RV) X takes values in a set $\mathcal{X}$. We use $p_X(x)$ to denote the probability mass function (p.m.f.) of a discrete RV X on $\mathcal{X}$. For brevity we may omit the subscript X when it is the uppercase version of the realization symbol x. We use $p_{X|Y}(x|y)$ to denote the conditional p.m.f. of X given Y. We denote column vectors with boldface letters, e.g. $\mathbf{x}$, $\mathbf{y}$; the i'th element of a vector $\mathbf{x}$ is denoted with x_i and we use x_i^j where $i<j$ to denote the vector $(x_i, x_{i+1}, ..., x_{j-1}, x_j)$; x^j is a short form notation for x_1^j, and $\mathbf{x} \equiv x^n$. A vector of random variables is denoted by $\mathbf{X} \equiv X^n$. $I(\cdot;\cdot)$ denotes the mutual information between two random variables, $H(\cdot)$ and $h(\cdot)$ denote the entropy and differential entropy, respectively, as defined in [10, Chapter 2, Chapter 8]. X^* denotes the conjugate of X and $\mathbb{A}^H$ denotes the Hermitian transpose of a matrix $\mathbb{A}$. We denote with $A_\epsilon^{(n)}$ the set of weakly jointly-typical sequences as defined in [10, Chapter 3]. Finally, we denote the proper, circularly symmetric, complex Normal distribution with mean μ and variance σ^2 by $\mathcal{CN}(\mu, \sigma^2)$, and $\mathbb{E}\{X\}$ is the stochastic expectation of X.

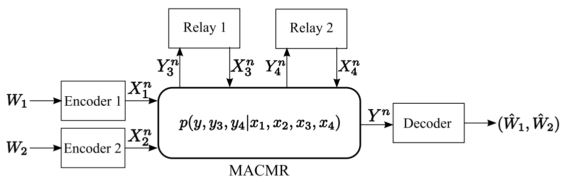

Figure 2. The multiple-access channel with multiple relays.

The multiple-access channel with multiple relays is depicted in Figure 2. In this network two users communicate with a single destination with the help of two relays. There are four channel inputs, X_1, X_2, X_3, X_4, and three channel outputs, Y, Y_3 and Y_4. The transmitters send independent messages W_1 and W_2, representing information at rates R_1 and R_2 respectively. Transmission is carried out in blocks of length n. $X_{1,i}$ and $X_{2,i}$, the channel inputs from sources $1, 2$ at time $i, i = 1, 2, ...n$, are functions of W_1, W_2 respectively. The relays' channel inputs $X_{3,i}$ and $X_{4,i}$ are causal functions of theirs received signals, $Y_{3,1}^{i-1}$ and $Y_{4,1}^{i-1}$, respectively. The relays are assisting both transmitters to communicate with the receiver. The destination uses the channel output Y^n to decode the messages (W_1, W_2). The channel is assumed to be causal, time-invariant, and memoryless as characterized by the conditional probability distribution

$$p(y_i, y_{3,i}, y_{4,i}|x_{1,1}^i, x_{2,1}^i, x_{3,1}^i, x_{4,1}^i, y_1^{i-1}, y_{3,1}^{i-1}, y_{4,1}^{i-1}, w_1, w_2) = p(y_i, y_{3,i}, y_{4,i}|x_{1,i}, x_{2,i}, x_{3,i}, x_{4,i}).$$

We define an (R_1, R_2, n) code for the MACMR to consist of

1. Two sets of integers $\mathcal{W}_1 \triangleq \{1, 2, ..., 2^{nR_1}\}$ and $\mathcal{W}_2 \triangleq \{1, 2, ..., 2^{nR_2}\}$, called message sets.
2. Two encoding functions at the sources, $e_k : \mathcal{W}_k \mapsto \mathcal{X}_k^n, k = 1, 2$.
3. Two sets of relay functions $\{f_{ji}\}_{i=1}^n, j = 3, 4$, such that $x_{j,i} = f_{ji}(y_{j,1}^{i-1}),\ \ i = 1, 2, \ldots, n,\ j = 3, 4$.
4. A decoding function at the destination, $g : \mathcal{Y}^n \mapsto \mathcal{W}_1 \times \mathcal{W}_2$

We define the average probability of error for this code as

$$\begin{aligned} P_e^{(n)} &\triangleq \Pr\{g(\mathbf{Y}) \neq (W_1, W_2)\} \\ &= \frac{1}{2^{nR_1} \cdot 2^{nR_2}} \sum_{(w_1, w_2) \in \mathcal{W}_1 \times \mathcal{W}_2} \Pr\{g(\mathbf{Y}) \neq \\ &\qquad (w_1, w_2)|W_1 = w_1, W_2 = w_2\}, \end{aligned}$$

under the assumption that the messages W_1 and W_2 are drawn according to a uniform distribution over $\mathcal{W}_1 \times \mathcal{W}_2$. The rate pair (R_1, R_2) is said to be achievable for the MACMR if there exists a sequence of (R_1, R_2, n) codes with $P_e^{(n)} \to 0$, as $n \to \infty$.

In the following $w_{k,b}$ denotes the message transmitted from node $k \in \{1, 2\}$ at message interval b.

3. A NEW ACHIEVABLE REGION

In this section we present a new achievable region for the multiple-access channel with multiple relays. The region is obtained by extending the multi-relay DF scheme to MACMRs with two sources and two relays. The achievable region is characterized in the following theorem:

Theorem 1
Any non-negative rate pair (R_1, R_2) satisfying constraints (1)–(3):

$$R_1 \leq I(X_1; Y_3|X_2, X_3, X_4, V_1^{(2)}) \tag{1a}$$
$$R_2 \leq I(X_2; Y_3|X_1, X_3, X_4, V_2^{(2)}) \tag{1b}$$
$$R_1 + R_2 \leq I(X_1, X_2; Y_3|X_3, X_4, V_1^{(2)}, V_2^{(2)}), \tag{1c}$$

$$R_1 \leq I(X_1; Y_4|X_2, X_3, X_4, V_1^{(2)}) + I(X_3, V_1^{(2)}; Y_4|X_4, V_1^{(1)}, V_2^{(2)}) \tag{2a}$$
$$R_2 \leq I(X_2; Y_4|X_1, X_3, X_4, V_2^{(2)}) + I(X_3, V_2^{(2)}; Y_4|X_4, V_1^{(2)}, V_2^{(1)}) \tag{2b}$$
$$R_1 + R_2 \leq I(X_1, X_2, X_3; Y_4|X_4, V_1^{(1)}, V_2^{(1)}), \tag{2c}$$

$$R_1 \leq I(X_1, X_3, X_4; Y|X_2, V_2^{(1)}, V_2^{(2)}) \tag{3a}$$
$$R_2 \leq I(X_2, X_3, X_4; Y|X_1, V_1^{(1)}, V_1^{(2)}) \tag{3b}$$
$$R_1 + R_2 \leq I(X_1, X_2, X_3, X_4; Y), \tag{3c}$$

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subject to input distribution

$$p(x_1, x_2, x_3, x_4, v_1^{(1)}, v_1^{(2)}, v_2^{(1)}, v_2^{(2)}) = p(v_1^{(1)})p(v_2^{(1)})p(v_1^{(2)}|v_1^{(1)})p(v_2^{(2)}|v_2^{(1)})p(x_4|v_1^{(1)}, v_2^{(1)}) p(x_1|v_1^{(2)})p(x_2|v_2^{(2)})p(x_3|v_1^{(2)}, v_2^{(2)}, x_4), \qquad (4)$$

is achievable.

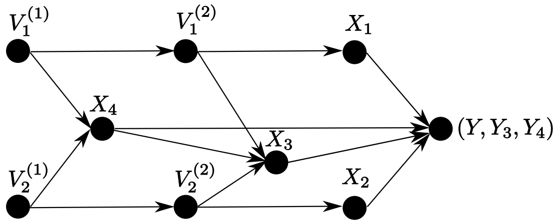

Figure 3. Schematic description of the Markov chain for the MACMR.

In the constraints above, equations (1) are decoding constraints at relay 1, equations (2) are decoding constraints at relay 2 and equations (3) are decoding constraints at the destination. The Markov chain (4) is schematically depicted in Figure 3. This chain lends itself to the following interpretation: consider transmission of $B-2$ messages. $V_1^{(1)}$, $V_1^{(2)}$, $V_2^{(1)}$ and $V_2^{(2)}$ are auxiliary random variables that enable cooperation between the relays and the two sources. $V_1^{(2)}$ and $V_2^{(2)}$ are functions of the messages W_1 and W_2 respectively, sent by the sources in the previous message interval. $V_1^{(1)}$ and $V_2^{(1)}$ are functions of the messages W_1 and W_2 respectively, sent by the sources two message intervals before. Relay 1 is cooperating with both sources and with relay 2 by sending signals which depend on $V_1^{(2)}$, $V_2^{(2)}$ and X_4. Relay 2 is cooperating with both sources by sending signals which depend on $V_1^{(1)}$ and $V_2^{(1)}$. The two sources are also transmitting signals which depend on $V_1^{(2)}$, $V_2^{(2)}$.

Note that in the present work, coordination at time b is achieved using the messages sent at the two previous blocks: source k cooperates with relay 1 through the message $w_{k,b-1}$ and with relay 2 through the message $w_{k,b-2}$, $k \in \{1,2\}$.

In the MARN [25] source-relay cooperation at time b is achieved through the messages that were sent at the previous block, i.e., only at block $b-1$: source 1 cooperates with relay 1 through the message $w_{1,b-1}^{r1}$ and with relay 2 through the message $w_{1,b-1}^{r2}$. Source 2 cooperates with relay 1 through the message $w_{2,b-1}^{r1}$ and with relay 2 through the message $w_{2,b-1}^{r2}$. The sources do not cooperate with each other. Note that each relay coordinates its transmissions with the sources, but relay 1 and relay 2 *do not coordinate their transmissions with each other*. Also note that while in our scheme source-relay coordination is achieved for the entire messages, in [25] coordination is done using only parts of the messages.

To demonstrate the difference between the schemes, consider a multihop wireless network example in which relay 1 is the only node that can reliably decode the sources' messages, as depicted in Figure 4. In the scheme proposed in [25] the relays do not cooperate with each other, and because each relay decodes only part of each message, relay 2 becomes useless and the scheme [25] uses the channel as a standard MARC. In the scheme proposed in the present paper the relays cooperate also with each other, and due to this interaction, relay 2 is useful also in multihop scenarios: First, the signals from both sources are observed at relay 1, then relay 2 observes the signal of relay 1, and lastly, the destination observes the signals from both relays.

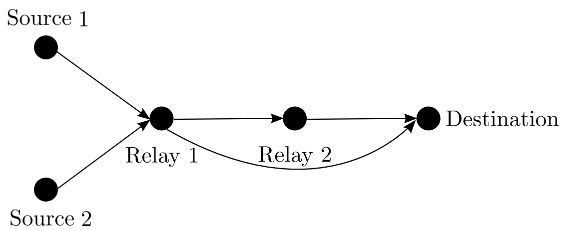

Figure 4. A multihop MACMR scenario.

The rest of this section is dedicated to the detailed proof of Theorem 1.

3.1. Proof Outline of Theorem 1

The channel model in Figure 2 combines elements of both the MARC and the multiple-relay channel. Encoding and decoding techniques employed in both models are therefore used in our code construction. Due to the observations described in Sections 1.1 and 1.2, the coding scheme developed for the MACMR in this work combines regular encoding, MAC decoding at relay 1, sliding-window decoding at relay 2, and backward decoding at the destination.

An important property of our coding scheme is that, due to the use of multiple relays, the codebooks used at any two consecutive message blocks are independent of each other. Therefore, in the decoding process, when simultaneously considering consecutive blocks, the error events arising from each block are independent, see [18].

3.2. The Coding Scheme for Theorem 1

Each source sends $B-2$ messages, each sent using a codeword of n channel symbols, in B transmission blocks. For source k, $k \in \{1,2\}$, the messages are denoted with $w_{k,b} \in \mathcal{W}_k$, $b = 1, 2, \ldots, B-2$. Note that as $B \to \infty$, for a fixed n, the bit rate $R_k(B-2)/B$ is arbitrarily close to R_k, $k \in \{1,2\}$.

Fix the input distribution (4). Codebook construction now proceeds as follows:

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1. Generate at random 2^{nR_k} i.i.d n-sequences in ${\mathcal{V}_k^{(1)}}^n$, each drawn according to
$$p(\mathbf{v}_k^{(1)}) = \prod_{i=1}^{n} p(v_{k,i}^{(1)}),$$
$k \in \{1,2\}$. Index them as $\mathbf{v}_k^{(1)}(s_k^{(1)})$, $s_k^{(1)} \in \mathcal{W}_k$, $k \in \{1,2\}$.
2. For each $\mathbf{v}_k^{(1)}(s_k^{(1)})$, generate 2^{nR_k} conditionally independent n-sequences $\mathbf{v}_k^{(2)}(s_k^{(2)}|s_k^{(1)}), s_k^{(2)} \in \mathcal{W}_k$, drawn according to
$$p\big(\mathbf{v}_k^{(2)}\big|\mathbf{v}_k^{(1)}(s_k^{(1)})\big) = \prod_{i=1}^{n} p\big(v_{k,i}^{(2)}\big|v_{k,i}^{(1)}(s_k^{(1)})\big),$$
$k \in \{1,2\}$.
3. For each $\mathbf{v}_k^{(2)}\big(s_k^{(2)}\big|s_k^{(1)}\big)$ generate 2^{nR_k} conditionally independent n-sequences $\mathbf{x}_k\big(w_k|s_k^{(2)}, s_k^{(1)}\big)$, $w_k \in \mathcal{W}_k$, drawn according to
$$p\big(\mathbf{x}_k\big|\mathbf{v}_k^{(2)}(s_k^{(2)}|s_k^{(1)})\big) = \prod_{i=1}^{n} p\big(x_{k,i}\big|v_{k,i}^{(2)}(s_k^{(2)}|s_k^{(1)})\big),$$
$k \in \{1,2\}$.
4. For each $s_1^{(1)}, s_2^{(1)}$ generate a conditionally independent n-sequence $\mathbf{x}_4(s_1^{(1)}, s_2^{(1)})$ drawn according to
$$p\big(\mathbf{x}_4|\mathbf{v}_1^{(1)}(s_1^{(1)}), \mathbf{v}_2^{(1)}(s_2^{(1)})\big) = \prod_{i=1}^{n} p\big(x_{4,i}|v_{1,i}^{(1)}(s_1^{(1)}), v_{2,i}^{(1)}(s_2^{(1)})\big).$$
5. For each $\big(\mathbf{x}_4(s_1^{(1)}, s_2^{(1)}), \mathbf{v}_1^{(2)}(s_1^{(2)}|s_1^{(1)}), \mathbf{v}_2^{(2)}(s_2^{(2)}| s_2^{(1)})\big)$ generate a conditionally independent n-sequence $\mathbf{x}_3(s_1^{(2)}, s_2^{(2)}, s_1^{(1)}, s_2^{(1)})$ drawn according to

$$p\Big(\mathbf{x}_3\Big|\mathbf{v}_1^{(2)}(s_1^{(2)}|s_1^{(1)}), \mathbf{v}_2^{(2)}(s_2^{(2)}|s_2^{(1)}), \mathbf{x}_4(s_1^{(1)}, s_2^{(1)})\Big) = \prod_{i=1}^{n} p\big(x_{3,i}|v_{1,i}^{(2)}(s_1^{(2)}|s_1^{(1)}), v_{2,i}^{(2)}(s_2^{(2)}|s_2^{(1)}), x_{4,i}(s_1^{(1)}, s_2^{(1)})\big).$$

Let $\mathcal{C}_0$ denote the joint codebook for the two sources and the two relays:
$$\mathcal{C}_0 \triangleq \big\{\mathbf{x}_1(w_1|s_1^{(2)}, s_1^{(1)}), \mathbf{x}_2(w_2|s_2^{(2)}, s_2^{(1)}), \mathbf{x}_3(s_1^{(2)}, s_2^{(2)}, s_1^{(1)}, s_2^{(1)}), \mathbf{x}_4(s_1^{(1)}, s_2^{(1)}), \mathbf{v}_1^{(1)}(s_1^{(1)}), \mathbf{v}_1^{(2)}(s_1^{(2)}|s_1^{(1)}), \mathbf{v}_2^{(1)}(s_2^{(1)}), \mathbf{v}_2^{(2)}(s_2^{(2)}|s_2^{(1)})\big\},$$
for all $w_k \in \mathcal{W}_k$, $s_k^{(1)} \in \mathcal{S}_k^{(1)}$, $s_k^{(2)} \in \mathcal{S}_k^{(2)}$, $k \in \{1,2\}$. Repeating the above steps 1–5 independently one more time, we generate an additional random codebook $\mathcal{C}_1$. These two codebooks are used alternately as follows: In block $b = 1, 2, ..., B$, the codebook $\mathcal{C}_{b \ mod \ 2}$ is used. Hence, for any two consecutive blocks, codewords from different blocks are independent. This property is used in the analysis of the probability of error. The joint codebooks are known at all network nodes. To establish cooperation $s_k^{(1)}$ and $s_k^{(2)}$ are chosen to be
$$s_k^{(2)} = w_{k,b-1},$$
$$s_k^{(1)} = w_{k,b-2}, \quad k = 1, 2.$$

Encoding at time b: Let $\hat{w}_{k,b}$ denote the estimate at relay 1 of the message sent at block b by sender k, $k \in \{1,2\}$, $\hat{\hat{w}}_{k,b}$ denote this estimate at relay 2, and $\hat{\hat{\hat{w}}}_{k,b}$ denote this estimate at the destination receiver. At the beginning of each block $b \in \{3, 4, ..., B\}$, relay 1 is assumed to have message estimates $\hat{w}_{1,b-1}, \hat{w}_{2,b-1}$ of $w_{1,b-1}, w_{2,b-1}$ as well as estimates $\hat{w}_{1,b-2}, \hat{w}_{2,b-2}$ of $w_{1,b-2}, w_{2,b-2}$. Relay 2 is assumed to have message estimates $\hat{\hat{w}}_{1,b-2}, \hat{\hat{w}}_{2,b-2}$, of $w_{1,b-2}, w_{2,b-2}$. Let $w_{1,b} \in \mathcal{W}_1$, $w_{2,b} \in \mathcal{W}_2$ be the new messages to be sent at block b.

- Sender 1 sends $\mathbf{x}_1\big(w_{1,b}\big|\mathbf{v}_1^{(2)}(w_{1,b-1}|w_{1,b-2})\big)$.
- Sender 2 sends $\mathbf{x}_2\big(w_{2,b}\big|\mathbf{v}_2^{(2)}(w_{2,b-1}|w_{2,b-2})\big)$.
- Relay 1 sends $\mathbf{x}_3\big(\mathbf{v}_1^{(2)}(\hat{w}_{1,b-1}\big|\hat{w}_{1,b-2}), \mathbf{v}_2^{(2)}(\hat{w}_{2,b-1}|\hat{w}_{2,b-2}), \mathbf{x}_4(\hat{w}_{1,b-2}, \hat{w}_{2,b-2})\big)$.
- Relay 2 sends $\mathbf{x}_4\big(\mathbf{v}_1^{(1)}(\hat{\hat{w}}_{1,b-2}), \mathbf{v}_2^{(1)}(\hat{\hat{w}}_{2,b-2})\big)$.

The transmitted signals at time b are summarized in Table I.

Remark 1
In the first block, $b = 1$, relay 1 has no information for cooperation. To start the cooperation $\hat{w}_{k,0}$, $\hat{w}_{k,-1}$, $k = 1, 2$, are set to constant 1. To start cooperation at relay 2, $\hat{\hat{w}}_{k,-1}$ and $\hat{\hat{w}}_{k,0}$, $k = 1, 2$, are set to constant 1. In the last two blocks, the sources do not transmit any new information, thus $w_{k,B-1}$ and $w_{k,B}$, $k = 1, 2$ are set to constant 1.

In the following, let $\mathbf{x}$ denote the codewords from codebook $\mathcal{C}_0$, and $\mathbf{x}'$ denote the codewords from codebook $\mathcal{C}_1$. The same convention is used for all codewords.
Decoding at time b: At the end of each block $b \in \{1, 2, ..., B\}$, decoding at relays 1 and 2 is done simultaneously, but independently.

In the error analysis it is assumed that at the end of each block $b \in \{1, 2, ..., B\}$, relay 1 has $\hat{w}_{1,1}^{b-1} = w_{1,1}^{b-1}$ and $\hat{w}_{2,1}^{b-1} = w_{2,1}^{b-1}$. It is also assumed that at the end of each block $b \in \{1, 2, ..., B\}$, relay 2 has $\hat{\hat{w}}_{1,1}^{b-2} = w_{1,1}^{b-2}$ and $\hat{\hat{w}}_{2,1}^{b-2} = w_{2,1}^{b-2}$.
Relay 1 declares that $(\hat{w}_{1,b}, \hat{w}_{2,b})$ are decoded if it is the

Table I. Regular encoding for the MACMR with two sources and two relays.

block b
$\mathbf{v}_1^{(1)}(w_{1,b-2})$
$\mathbf{v}_2^{(1)}(w_{2,b-2})$
$\mathbf{v}_1^{(2)}\big(w_{1,b-1}\big\vert\mathbf{v}_1^{(1)}(w_{1,b-2})\big)$
$\mathbf{v}_2^{(2)}\big(w_{2,b-1}\big\vert\mathbf{v}_2^{(1)}(w_{2,b-2})\big)$
$\mathbf{x}_1\big(w_{1,b}\big\vert\mathbf{v}_1^{(2)}(w_{1,b-1}\vert w_{1,b-2})\big)$
$\mathbf{x}_2\big(w_{2,b}\big\vert\mathbf{v}_2^{(2)}(w_{2,b-1}\vert w_{2,b-2})\big)$
$\mathbf{x}_3\big(\mathbf{v}_1^{(2)}(\hat{w}_{1,b-1}\vert\hat{w}_{1,b-2}),\mathbf{v}_2^{(2)}(\hat{w}_{2,b-1}\vert\hat{w}_{2,b-2}),\mathbf{x}_4(\hat{w}_{1,b-2},\hat{w}_{2,b-2})\big)$
$\mathbf{x}_4\big(\mathbf{v}_1^{(1)}(\hat{\hat{w}}_{1,b-2}),\mathbf{v}_2^{(1)}(\hat{\hat{w}}_{2,b-2})\big)$

unique pair in $\mathcal{W}_1\times\mathcal{W}_2$ such that at block b

$$\Big\{\mathbf{x}_1\big(\hat{w}_{1,b}|\mathbf{v}_1^{(2)}(w_{1,b-1}|w_{1,b-2})\big),\ \mathbf{x}_2\big(\hat{w}_{2,b}|\mathbf{v}_2^{(2)}(w_{2,b-1}|w_{2,b-2})\big),\ \mathbf{x}_3\big(\mathbf{v}_1^{(2)}(w_{1,b-1}|w_{1,b-2}),\mathbf{v}_2^{(2)}(w_{2,b-1}|w_{2,b-2}),\mathbf{x}_4(w_{1,b-2},w_{2,b-2})\big),\ \mathbf{x}_4\big(\mathbf{v}_1^{(1)}(w_{1,b-2}),\mathbf{v}_2^{(1)}(w_{2,b-2})\big),\mathbf{v}_1^{(1)}\big(w_{1,b-2}\big),\ \mathbf{v}_1^{(2)}\big(w_{1,b-1}|\mathbf{v}_1^{(1)}(w_{1,b-2})\big),\mathbf{v}_2^{(1)}\big(w_{2,b-2}\big),\ \mathbf{v}_2^{(2)}\big(w_{2,b-1}|\mathbf{v}_2^{(1)}(w_{2,b-2})\big),\mathbf{y}_{3,b}\Big\}\in A_\epsilon^{(n)}.$$

If no unique $(\hat{w}_{1,b},\hat{w}_{2,b})$ as above exists, an error is declared.

Remark 2
Note that relay 1 knows the previous messages of the sources, $w_{k,b-1}$ and $w_{k,b-2}$, $k=1,2$. Therefore, this decoding rule represents MAC decoding.

Relay 2 declares that $(\hat{\hat{w}}_{1,b-1},\hat{\hat{w}}_{2,b-1})$ are decoded if they are the unique values in $\mathcal{W}_1\times\mathcal{W}_2$ such that in blocks b and $b-1$ (assume now that b is even, but the all arguments and the results hold also when b is odd)

$$\Big\{\mathbf{x}_3\big(\mathbf{v}_1^{(2)}(\hat{\hat{w}}_{1,b-1}|w_{1,b-2}),\mathbf{v}_2^{(2)}(\hat{\hat{w}}_{2,b-1}|w_{2,b-2}),\ \mathbf{x}_4(w_{1,b-2},w_{2,b-2})\big),\ \mathbf{x}_4\big(\mathbf{v}_1^{(1)}(w_{1,b-2}),\mathbf{v}_2^{(1)}(w_{2,b-2})\big),\ \mathbf{v}_1^{(1)}\big(w_{1,b-2}\big),\mathbf{v}_1^{(2)}\big(\hat{\hat{w}}_{1,b-1}|\mathbf{v}_1^{(1)}(w_{1,b-2})\big),\ \mathbf{v}_2^{(1)}\big(w_{2,b-2}\big),\mathbf{v}_2^{(2)}\big(\hat{\hat{w}}_{2,b-1}|\mathbf{v}_2^{(1)}(w_{2,b-2})\big),\mathbf{y}_{4,b}\Big\}\in A_\epsilon^{(n)}$$

and

$$\Big\{\mathbf{x}'_1\big(\hat{\hat{w}}_{1,b-1}|\mathbf{v}_1'^{(2)}(w_{1,b-2}|w_{1,b-3})\big),\ \mathbf{x}'_2\big(\hat{\hat{w}}_{2,b-1}|\mathbf{v}_2'^{(2)}(w_{2,b-2}|w_{2,b-3})\big),\ \mathbf{x}'_3\big(\mathbf{v}_1'^{(2)}(w_{1,b-2}|w_{1,b-3}),\mathbf{v}_2'^{(2)}(w_{2,b-2}|w_{2,b-3}),\ \mathbf{x}'_4(w_{1,b-3},w_{2,b-3})\big),\ \mathbf{x}'_4\big(\mathbf{v}_1'^{(1)}(w_{1,b-3}),\mathbf{v}_2'^{(1)}(w_{2,b-3})\big),\mathbf{v}_1'^{(1)}\big(w_{1,b-3}\big),\ \mathbf{v}_1'^{(2)}\big(w_{1,b-2}|\mathbf{v}_1'^{(1)}(w_{1,b-3})\big),\mathbf{v}_2'^{(1)}\big(w_{2,b-3}\big),\ \mathbf{v}_2'^{(2)}\big(w_{2,b-2}|\mathbf{v}_2'^{(1)}(w_{2,b-3})\big),\mathbf{y}_{4,b-1}\Big\}\in A_\epsilon^{(n)}.$$

If no unique $(\hat{\hat{w}}_{1,b-1},\hat{\hat{w}}_{2,b-1})$ as above exists, an error is declared.

Remark 3
Note the relay 2 knows $w_{k,b-2}$, $k=1,2$, and decodes $w_{k,b-1}$, $k=1,2$, over two consecutive blocks. This represents sliding window decoding.

Decoding at the destination receiver is done using the backward decoding technique. The destination collects all of its B output blocks. Starting from the last block, the destination decodes $(w_{1,b-2},w_{2,b-2}), b=B,B-1,...,3$ by using $\mathbf{y}_b$ and by assuming that its previously decoded message estimates are correct. Namely, the destination decodes the message pair at time $b-2$ by finding a unique $(\hat{\hat{\hat{w}}}_{1,b-2},\hat{\hat{\hat{w}}}_{2,b-2})\in\mathcal{W}_1\times\mathcal{W}_2$ such that

$$\Big\{\mathbf{x}_1\big(w_{1,b}|\mathbf{v}_1^{(2)}(w_{1,b-1}|\hat{\hat{\hat{w}}}_{1,b-2})\big),\ \mathbf{x}_2\big(w_{2,b}|\mathbf{v}_2^{(2)}(w_{2,b-1}|\hat{\hat{\hat{w}}}_{2,b-2})\big),\ \mathbf{x}_3\big(\mathbf{v}_1^{(2)}(w_{1,b-1}|\hat{\hat{\hat{w}}}_{1,b-2}),\mathbf{v}_2^{(2)}(w_{2,b-1}|\hat{\hat{\hat{w}}}_{2,b-2}),\ \mathbf{x}_4(\hat{\hat{\hat{w}}}_{1,b-2},\hat{\hat{\hat{w}}}_{2,b-2})\big),\ \mathbf{x}_4\big(\mathbf{v}_1^{(1)}(\hat{\hat{\hat{w}}}_{1,b-2}),\mathbf{v}_2^{(1)}(\hat{\hat{\hat{w}}}_{2,b-2})\big),\mathbf{v}_1^{(1)}\big(\hat{\hat{\hat{w}}}_{1,b-2}\big),\ \mathbf{v}_1^{(2)}\big(w_{1,b-1}|\mathbf{v}_1^{(1)}(\hat{\hat{\hat{w}}}_{1,b-2})\big),\mathbf{v}_2^{(1)}\big(\hat{\hat{\hat{w}}}_{2,b-2}\big),\ \mathbf{v}_2^{(2)}\big(w_{2,b-1}|\mathbf{v}_2^{(1)}(\hat{\hat{\hat{w}}}_{2,b-2})\big),\mathbf{y}_b\Big\}\in A_\epsilon^{(n)}.$$

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3.3. Analysis of the Probability of Error

We analyze the probability of error averaged over all codebooks. First, note that by the symmetry of the random code construction, the averaged probability of error does not depend on the pair of messages sent. Hence, without loss of generality, we may assume that $(w_{1,b}, w_{2,b}) = (1,1)$ was sent, see e.g. [10, Chapter 7.7]. We analyze the probability of error at the relays at block b assuming that at the previous blocks there were no decoding errors [7].

At relay 1: An error occurs if either the transmitted codewords are not jointly typical with the received sequence, or there is a pair of incorrect codewords that are jointly typical with the received sequence. We define the following event for decoding $(w_{1,b}, w_{2,b})$ at time b:

$$\begin{aligned}E_{jk}=\Big\{&\mathbf{x}_1\big(j|\mathbf{v}_1^{(2)}(w_{1,b-1}|w_{1,b-2})\big),\\ &\mathbf{x}_2\big(k|\mathbf{v}_2^{(2)}(w_{2,b-1}|w_{2,b-2})\big),\\ &\mathbf{x}_3\big(\mathbf{v}_1^{(2)}(w_{1,b-1}|w_{1,b-2}),\\ &\mathbf{v}_2^{(2)}(w_{2,b-1}|w_{2,b-2}),\mathbf{x}_4(w_{1,b-2},w_{2,b-2})\big),\\ &\mathbf{x}_4\big(\mathbf{v}_1^{(1)}(w_{1,b-2}),\mathbf{v}_2^{(1)}(w_{2,b-2})\big),\mathbf{v}_1^{(1)}\big(w_{1,b-2}\big),\\ &\mathbf{v}_1^{(2)}\big(w_{1,b-1}|\mathbf{v}_1^{(1)}(w_{1,b-2})\big),\mathbf{v}_2^{(1)}\big(w_{2,b-2}\big),\\ &\mathbf{v}_2^{(2)}\big(w_{2,b-1}|\mathbf{v}_2^{(1)}(w_{2,b-2})\big),\mathbf{y}_{3,b}\Big\}\in A_\epsilon^{(n)}.\end{aligned}$$

By the union bound,

$$\begin{aligned}P_e^{(n)}(\text{relay }1) &= \Pr\left(E_{11}^c\bigcup\left\{\bigcup_{(j,k)\neq(1,1)}E_{jk}\right\}\right)\\ &\le p(E_{11}^c)+\sum_{j=2}^{2^{nR_1}}p(E_{j1})+\sum_{k=2}^{2^{nR_2}}p(E_{1k})\\ &\quad+\sum_{j=2}^{2^{nR_1}}\sum_{k=2}^{2^{nR_2}}p(E_{jk}),\end{aligned}$$

where $p(\cdot)$ is the conditional probability given that $(1,1)$ was sent and $(w_{k,b-1}, w_{k,b-2})$, $k=1,2$, were correctly decoded at relay 1. By the joint AEP [10, Theorem 15.2.1] $p(E_{11}^c)\to 0$ as $n\to\infty$. For $j\neq 1$, we show in Appendix A that

$$p(E_{j1})\le 2^{-n\left(I(X_1;Y_3|X_2,X_3,X_4,V_1^{(1)},V_1^{(2)},V_2^{(1)},V_2^{(2)})-10\epsilon\right)}.$$

Thus

$$\begin{aligned}&\sum_{j=2}^{2^{nR_1}}p(E_{j1})\\ &\le 2^{nR_1}2^{-n\left(I(X_1;Y_3|X_2,X_3,X_4,V_1^{(1)},V_1^{(2)},V_2^{(1)},V_2^{(2)})-10\epsilon\right)},\end{aligned}$$

and we conclude that $\sum_{j=2}^{2^{nR_1}}p(E_{j1})\to 0$, for $n\to\infty$, as long as

$$\begin{aligned}R_1 &< I(X_1;Y_3|X_2,X_3,X_4,V_1^{(1)},V_1^{(2)},V_2^{(1)},V_2^{(2)})\\ &= I(X_1;Y_3|X_2,X_3,X_4,V_1^{(2)}).\end{aligned}\tag{7}$$

In a similar way one can obtain that $\sum_{k=2}^{2^{nR_2}}p(E_{1k})\to 0$, for $n\to\infty$, as long as

$$R_2 < I(X_2;Y_3|X_1,X_3,X_4,V_2^{(2)}),\tag{8}$$

and for $j\neq 1, k\neq 1$, $\sum_{j=2}^{2^{nR_1}}\sum_{k=2}^{2^{nR_2}}p(E_{jk})\to 0$, for $n\to\infty$, as long as

$$R_1+R_2 < I(X_1,X_2;Y_3|X_3,X_4,V_1^{(2)},V_2^{(2)}).\tag{9}$$

At relay 2: An error occurs if either the transmitted codewords are not jointly typical with the received sequence at block b or at block $b-1$, or there is a pair of incorrect messages whose corresponding codewords are jointly typical with the received sequence at block b and at block $b-1$. We define the following events for decoding $(w_{1,b-1}, w_{2,b-1})$ at time b:

$$\begin{aligned}E_{jk}=\Big\{&\mathbf{x}_1'\big(j|\mathbf{v}_1'^{(2)}(w_{1,b-2}|w_{1,b-3})\big),\\ &\mathbf{x}_2'\big(k|\mathbf{v}_2'^{(2)}(w_{2,b-2}|w_{2,b-3})\big),\\ &\mathbf{x}_3'\big(\mathbf{v}_1'^{(2)}(w_{1,b-2}|w_{1,b-3}),\mathbf{v}_2'^{(2)}(w_{2,b-2}|w_{2,b-3}),\\ &\qquad\mathbf{x}_4'(w_{1,b-3},w_{2,b-3})\big),\\ &\mathbf{x}_4'\big(\mathbf{v}_1'^{(1)}(w_{1,b-3}),\mathbf{v}_2'^{(1)}(w_{2,b-3})\big),\mathbf{v}_1'^{(1)}\big(w_{1,b-3}\big),\\ &\mathbf{v}_1'^{(2)}\big(w_{1,b-2}|\mathbf{v}_1'^{(1)}(w_{1,b-3})\big),\mathbf{v}_2'^{(1)}\big(w_{2,b-3}\big),\\ &\mathbf{v}_2'^{(2)}\big(w_{2,b-2}|\mathbf{v}_2'^{(1)}(w_{2,b-3})\big),\mathbf{y}_{4,b-1}\Big\}\in A_\epsilon^{(n)}\end{aligned}$$

and

$$\begin{aligned}E_{jk}^*=\Big\{&\mathbf{x}_3\big(\mathbf{v}_1^{(2)}(j|w_{1,b-2}),\mathbf{v}_2^{(2)}(k|w_{2,b-2}),\\ &\qquad\mathbf{x}_4(w_{1,b-2},w_{2,b-2})\big),\\ &\mathbf{x}_4\big(\mathbf{v}_1^{(1)}(w_{1,b-2}),\mathbf{v}_2^{(1)}(w_{2,b-2})\big),\mathbf{v}_1^{(1)}\big(w_{1,b-2}\big),\\ &\mathbf{v}_1^{(2)}\big(j|\mathbf{v}_1^{(1)}(w_{1,b-2})\big),\mathbf{v}_2^{(1)}\big(w_{2,b-2}\big),\\ &\mathbf{v}_2^{(2)}\big(k|\mathbf{v}_2^{(1)}(w_{2,b-2})\big),\mathbf{y}_{4,b}\Big\}\in A_\epsilon^{(n)}.\end{aligned}$$

The average probability of error at relay 2 can now be bounded via

$$\begin{aligned}
P_e^{(n)}(\text{relay } 2) &= P\Bigg(\Big\{E_{11}^c \cup (E_{11}^*)^c\Big\} \\
&\qquad \bigcup \Big\{ \bigcup_{(j,k)\neq(1,1)} (E_{jk} \cap E_{jk}^*)\Big\}\Bigg) \\
&\overset{(a)}{\le} p(E_{11}^c) + p\big((E_{11}^*)^c\big) + \sum_{(j,k)\neq(1,1)} p(E_{jk} \cap E_{jk}^*) \\
&\overset{(b)}{=} p(E_{11}^c) + p\big((E_{11}^*)^c\big) \\
&\qquad + \sum_{(j,k)\neq(1,1)} p(E_{jk}) \cdot p(E_{jk}^*) \\
&\overset{(c)}{\le} 2\epsilon + \sum_{j=2}^{2^{nR_1}} p(E_{j1})p(E_{j1}^*) + \sum_{k=2}^{2^{nR_2}} p(E_{1k})p(E_{1k}^*) \\
&\qquad + \sum_{j=2}^{2^{nR_1}} \sum_{k=2}^{2^{nR_2}} p(E_{jk})p(E_{jk}^*),
\end{aligned}$$

where $p(\cdot)$ is the conditional probability given that $(1,1)$ was sent and $(w_{k,b-2}, w_{k,b-3})$, $k=1,2$, were correctly decoded at relay 2. In (a) we applied the union bound, and in (b) we used the fact that codewords from two consecutive blocks are generated independently. In (c) we used the joint AEP for bounding $p(E_{11}^c) \le \epsilon$ and $p\big((E_{11}^*)^c\big) \le \epsilon$.

Similarly to the derivation in Appendix A it follows that $\sum_{j=2}^{2^{nR_1}} p(E_{j1})p(E_{j1}^*) \to 0$, for $n \to \infty$, as long as

$$\begin{aligned}
R_1 <& I(X_1; Y_4 | X_2, X_3, X_4, V_1^{(2)}) \\
&+ I(X_3, V_1^{(2)}; Y_4 | X_4, V_1^{(1)}, V_2^{(2)}), \qquad (11)
\end{aligned}$$

$\sum_{k=2}^{2^{nR_2}} p(E_{1k})p(E_{1k}^*) \to 0$, for $n \to \infty$, as long as

$$\begin{aligned}
R_2 <& I(X_2; Y_4 | X_1, X_3, X_4, V_2^{(2)}) \\
&+ I(X_3, V_2^{(2)}; Y_4 | X_4, V_2^{(1)}, V_1^{(2)}), \qquad (12)
\end{aligned}$$

and $\sum_{j=2}^{2^{nR_1}} \sum_{k=2}^{2^{nR_2}} p(E_{jk})p(E_{jk}^*) \to 0$, for $n \to \infty$, as long as

$$R_1 + R_2 < I(X_1, X_2, X_3; Y_4 | X_4, V_1^{(1)}, V_2^{(1)}). \qquad (13)$$

At the destination receiver: An error occurs if either the transmitted codewords are not jointly typical with the received sequence, or there is a pair of incorrect codewords that are jointly typical with the received sequence. We define the following event for decoding $(w_{1,b-2}, w_{2,b-2})$:

$$\begin{aligned}
E_{jk} = \Big\{ & \mathbf{x}_1\big(w_{1,b} | \mathbf{v}_1^{(2)}(w_{1,b-1}|j)\big), \\
& \mathbf{x}_2\big(w_{2,b} | \mathbf{v}_2^{(2)}(w_{2,b-1}|k)\big), \\
& \mathbf{x}_3\big(\mathbf{v}_1^{(2)}(w_{1,b-1}|j), \mathbf{v}_2^{(2)}(w_{2,b-1}|k), \mathbf{x}_4(j,k)\big), \\
& \mathbf{x}_4\big(\mathbf{v}_1^{(1)}(j), \mathbf{v}_2^{(1)}(k)\big), \mathbf{v}_1^{(1)}(j), \\
& \mathbf{v}_1^{(2)}\big(w_{1,b-1} | \mathbf{v}_1^{(1)}(j)\big), \mathbf{v}_2^{(1)}(k), \\
& \mathbf{v}_2^{(2)}\big(w_{2,b-1} | \mathbf{v}_2^{(1)}(k)\big), \mathbf{y}_b \Big\} \in A_\epsilon^{(n)}.
\end{aligned}$$

Then, by the union bound,

$$\begin{aligned}
P_e^{(n)}(\text{destination}) &= \Pr\left(E_{11}^c \bigcup \Big\{ \bigcup_{(j,k)\neq(1,1)} E_{jk} \Big\} \right) \\
&\le p(E_{11}^c) + \sum_{j=2}^{2^{nR_1}} p(E_{j1}) + \sum_{k=2}^{2^{nR_2}} p(E_{1k}) \\
&\qquad + \sum_{j=2}^{2^{nR_1}} \sum_{k=2}^{2^{nR_2}} p(E_{jk}),
\end{aligned}$$

where $p(\cdot)$ is the conditional probability given that $(1,1)$ was sent and $(w_{k,b}, w_{k,b-1})$, $k=1,2$, were correctly decoded at the destination. From the joint AEP $p(E_{11}^c) \to 0$, as $n \to \infty$. Similarly to the derivation in Appendix A we obtain that $\sum_{j=2}^{2^{nR_1}} p(E_{j1}) \to 0$, for $n \to \infty$, as long as

$$R_1 < I(X_1, X_3, X_4; Y | X_2, V_2^{(1)}, V_2^{(2)}), \qquad (14)$$

$\sum_{k=2}^{2^{nR_2}} p(E_{1k}) \to 0$, for $n \to \infty$, as long as

$$R_2 < I(X_2, X_3, X_4; Y | X_1, V_1^{(1)}, V_1^{(2)}), \qquad (15)$$

and $\sum_{j=2}^{2^{nR_1}} \sum_{k=2}^{2^{nR_2}} p(E_{jk}) \to 0$, for $n \to \infty$, as long as

$$R_1 + R_2 < I(X_1, X_2, X_3, X_4; Y). \qquad (16)$$

Collecting constraints (7)–(9), (11)–(13) and (14)–(16), we obtain the rate constraints of the theorem.

4. GAUSSIAN MACMR

In this section we focus on the Gaussian channel. We obtain an outer bound on the capacity region, and an achievable rate region based on Theorem 1. The Gaussian MACMR is depicted in Figure 5.

The relationship between the channel inputs and channel outputs is:

$$Y_3 = h_{3,1}X_1 + h_{3,2}X_2 + h_{3,4}X_4 + Z_3 \qquad (17a)$$

$$Y_4 = h_{4,1}X_1 + h_{4,2}X_2 + h_{4,3}X_3 + Z_4 \qquad (17b)$$

$$Y = h_1X_1 + h_2X_2 + h_3X_3 + h_4X_4 + Z, \qquad (17c)$$

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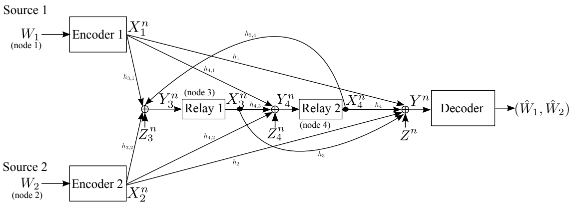

Figure 5. Gaussian MACMR.

where Z_3, Z_4, Z are complex Normal RVs, $\mathcal{CN}(0,1)$, independent of each other. The channel input signals are subject to per-symbol average power constraints: $\mathbb{E}\{|X_k|^2\} \leq P_k$, $k = \{1,2,3,4\}$. We consider the time-invariant channel, therefore, $h_{i,j}$, $i = \{3,4\}$, $j = \{1,2\}$, $h_{4,3}$, $h_{3,4}$, and $\{h_k\}_{k=1}^4$ are constant complex scalars, and are known at all nodes.

4.1. An Outer Bound

We next provide an outer bound on the capacity region of the MACMR:

Proposition 1
The capacity region of the Gaussian MACMR is outer bounded by the region obtained as the union of all non-negative rate pairs (R_1, R_2) that satisfy

$$\begin{aligned} R_1 \leq \min\big\{ & I(X_1; Y_3, Y_4, Y | X_2, X_3, X_4), \\ & I(X_1, X_3; Y_4, Y | X_2, X_4), \\ & I(X_1, X_4; Y_3, Y | X_2, X_3), \\ & I(X_1, X_3, X_4; Y | X_2)\big\} \qquad (18) \\ R_2 \leq \min\big\{ & I(X_2; Y_3, Y_4, Y | X_1, X_3, X_4), \\ & I(X_2, X_3; Y_4, Y | X_1, X_4), \\ & I(X_2, X_4; Y_3, Y | X_1, X_3), \\ & I(X_2, X_3, X_4; Y | X_1)\big\} \qquad (19) \\ R_1 + R_2 \leq \min\big\{ & I(X_1, X_2; Y_3, Y_4, Y | X_3, X_4), \\ & I(X_1, X_2, X_3; Y_4, Y | X_4), \\ & I(X_1, X_2, X_4; Y_3, Y | X_3), \\ & I(X_1, X_2, X_3, X_4; Y)\big\}, \qquad (20) \end{aligned}$$

for some X_1, X_2, X_3 and X_4 zero mean, jointly complex Normal RVs with an arbitrary correlation matrix.

Proof
The mutual information expressions in (18)–(20) follow directly from the cut-set bound [10, Theorem 15.10.1]. Following similar steps as in [19, Proposition 2, Theorem 8], we write each mutual information expression in (18)–(20) as

$$I(\mathbf{X}_{\mathcal{S}}; \mathbf{Y}_{\mathcal{U}} | \mathbf{X}_{\mathcal{S}^c}) = h(\mathbf{Y}_{\mathcal{U}} | \mathbf{X}_{\mathcal{S}^c}) - h(\mathbf{Z}_{\mathcal{U}}), \qquad (21)$$

where $\mathcal{S}$ is any subset of $\mathcal{T} \triangleq \{1,2,3,4\}$, and $\mathcal{S}^c$ is the complement of $\mathcal{S}$ in $\mathcal{T}$. $\mathcal{U} \triangleq \big\{\text{Destination}, \mathcal{S}^c\big\}$, such that $Y_1 = Y_2 = \varnothing$, where $\varnothing$ denotes the empty set. Observe that $h(\mathbf{Z}_{\mathcal{U}})$ is independent of $\mathbf{X}_{\mathcal{T}}$, hence, maximizing (21) becomes an entropy maximization problem. The best $\mathbf{X}_{\mathcal{T}}$ has zero mean because every node t, $t \in \mathcal{T}$ uses less power by sending $\mathbf{X}_t - \mathbb{E}\{\mathbf{X}_t\}$ rather than $\mathbf{X}_t$, and this change does not affect the mutual information expressions (21). Suppose the vector $\mathbf{X}_{\mathcal{T}}$ that maximizes (18)–(20) has a covariance matrix $\mathbb{Q}_{\mathbf{X}_{\mathcal{T}}}$. This $\mathbb{Q}_{\mathbf{X}_{\mathcal{T}}}$ fixes $\mathbb{Q}_{\mathbf{Y}_{\mathcal{U}}, \mathbf{X}_{\mathcal{S}^c}}$ for all $\mathcal{U}, \mathcal{S}$, where $\mathbb{Q}_{\mathbf{A},\mathbf{B}}$ is the covariance matrix of the vector $[\mathbf{A}^T \mathbf{B}^T]^T$. But once $\mathbb{Q}_{\mathbf{Y}_{\mathcal{U}}, \mathbf{X}_{\mathcal{S}^c}}$ is fixed, then $h(\mathbf{Y}_{\mathcal{U}} | \mathbf{X}_{\mathcal{S}^c})$ is maximized by making $\mathbf{Y}_{\mathcal{U}}$ and $\mathbf{X}_{\mathcal{S}^c}$ jointly Gaussian [26, Lemma 1]. Hence, choosing $\mathbf{X}_{\mathcal{T}}$ to be jointly Gaussian with covariance matrix $\mathbb{Q}_{\mathbf{X}_{\mathcal{T}}}$ maximizes $h(\mathbf{Y}_{\mathcal{U}} | \mathbf{X}_{\mathcal{S}^c})$ for every mutual information expression in (21).

We conclude that the maximizing distribution for (18)–(20) is zero-mean jointly Gaussian. □

We now evaluate the expressions in (18)–(20). Choosing Gaussian $\mathbf{X}_{\mathcal{T}}$, we observe that the mutual information expressions in (18)–(20) can be simplified as follows. Let

$$\mathbf{h} \triangleq \begin{bmatrix} h_{3,1} \\ h_{4,1} \\ h_1 \end{bmatrix}, \mathbf{Z} \triangleq \begin{bmatrix} Z_3 \\ Z_4 \\ Z \end{bmatrix},$$

$$\mathbb{C}_1 = \begin{bmatrix} P_1 & \alpha_{12} & \alpha_{13} & \alpha_{14} \\ \alpha_{21} & P_2 & \alpha_{23} & \alpha_{24} \\ \alpha_{31} & \alpha_{32} & P_3 & \alpha_{34} \\ \alpha_{41} & \alpha_{42} & \alpha_{43} & P_4 \end{bmatrix} \triangleq \begin{bmatrix} t_{11} & \mathbf{t}_{21}^H \\ \mathbf{t}_{21} & \mathbb{T}_{22} \end{bmatrix}$$

where $\alpha_{ij} \triangleq \mathbb{E}\{X_i X_j^*\}$, $\mathbb{T}_{22}$ is a 3×3 matrix, and all transmitters use maximum power. Let $\mathbb{I}_k$ denote the $k \times k$ identity matrix. Then, we obtain from (18)

$$\begin{aligned} R_1 \leq & I(X_1; Y_3, Y_4, Y | X_2, X_3, X_4) \\ \stackrel{(a)}{=} & h(Y_3, Y_4, Y | X_2, X_3, X_4) \\ & \qquad - h(Y_3, Y_4, Y | X_1, X_2, X_3, X_4) \\ = & h(\overbrace{h_{3,1}X_1 + Z_3}^{\hat{Y}_3}, \overbrace{h_{4,1}X_1 + Z_4}^{\hat{Y}_4}, \overbrace{h_1 X_1 + Z}^{\hat{Y}} | \\ & \qquad X_2, X_3, X_4) - h(Z_3, Z_4, Z) \\ \stackrel{(b)}{=} & \log_2\Big(\det\big(\text{cov}(\hat{Y}_3, \hat{Y}_4, \hat{Y} | X_2, X_3, X_4)\big)\Big) \\ = & \log_2\Big(\det\big(\text{cov}(\mathbf{h} \cdot X_1 + \mathbf{Z} | X_2, X_3, X_4)\big)\Big) \\ \stackrel{(c)}{=} & \log_2\Big(\det\big(\text{cov}(\mathbf{h} \cdot X_1 | X_2, X_3, X_4) + \text{cov}(\mathbf{Z})\big)\Big) \\ = & \log_2\Big(\det\big(\mathbb{I}_3 + \mathbf{h} \cdot \text{cov}(X_1 | X_2, X_3, X_4) \cdot \mathbf{h}^H\big)\Big) \\ \stackrel{(d)}{=} & \log_2\Big(\det\big(\mathbb{I}_3 + \mathbf{h} \cdot (t_{11} - \mathbf{t}_{21}^H \mathbb{T}_{22}^{-1} \mathbf{t}_{21}) \cdot \mathbf{h}^H\big)\Big), \end{aligned} \qquad (22)$$

where (a) follows from the definition of mutual information [10, Chapter 2], (b) follows as $\mathbf{X}_{\mathcal{T}}$ is a jointly Gaussian vector, (c) follows from the independence of $\mathbf{X}_{\mathcal{T}}$

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and **Z**, and (d) follows from [27, Section VI]. Similarly,

$$\begin{aligned} R_1 \leq & I(X_1,X_3,X_4;Y|X_2) \\ = & h(Y|X_2) - h(Y|X_1,X_2,X_3,X_4) \\ = & h(\overbrace{h_1X_1+h_3X_3+h_4X_4+Z}^{\hat{Y}}|X_2) - h(Z) \\ \leq & \log_2\Big((\pi e)\det\big(\text{cov}(\hat{Y}|X_2)\big)\Big) - \log_2(\pi e) \\ = & \log_2\Big(\det\big(\text{cov}(\hat{Y}|X_2)\big)\Big) \\ = & \log_2\Big(\det\big(t_{11} - t_{12}t_{22}^{-1}t_{12}^*)\big)\Big), \end{aligned} \tag{23}$$

where

$$t_{11} = \text{Trace}\left\{ \begin{bmatrix} h_1^* \\ h_3^* \\ h_4^* \end{bmatrix} \cdot \begin{bmatrix} h_1 & h_3 & h_4 \end{bmatrix} \cdot \begin{bmatrix} P_1 & \alpha_{13} & \alpha_{14} \\ \alpha_{31} & P_3 & \alpha_{34} \\ \alpha_{41} & \alpha_{43} & P_4 \end{bmatrix} \right\} + 1,$$

$$t_{22}^{-1} = P_2^{-1}, \quad t_{12} = h_1\alpha_{12} + h_3\alpha_{32} + h_4\alpha_{42}.$$

Following similar steps we obtain

$$\begin{aligned} R_1 \leq & I(X_1,X_3;Y_4,Y|X_2,X_4) \\ \leq & \log_2\Big(\det\big(\mathbb{I}_2 + \mathbb{H}\cdot(\mathbb{T}_{11} - \mathbb{T}_{12}\mathbb{T}_{22}^{-1}\mathbb{T}_{12}^H)\cdot\mathbb{H}^H\big)\Big), \end{aligned} \tag{24}$$

where

$$\mathbb{H} \triangleq \begin{bmatrix} h_{4,1} & h_{4,3} \\ h_1 & h_3 \end{bmatrix},$$

$$\mathbb{C}_3 = \begin{bmatrix} P_1 & \alpha_{13} & \alpha_{12} & \alpha_{14} \\ \alpha_{31} & P_3 & \alpha_{32} & \alpha_{34} \\ \alpha_{21} & \alpha_{23} & P_2 & \alpha_{24} \\ \alpha_{41} & \alpha_{43} & \alpha_{42} & P_4 \end{bmatrix} \triangleq \begin{bmatrix} \mathbb{T}_{11} & \mathbb{T}_{12} \\ \mathbb{T}_{12}^H & \mathbb{T}_{22} \end{bmatrix},$$

such that each sub-matrix $\mathbb{T}_{ij}$ is a two dimensional matrix. Lastly, using steps as in the previous bounds we obtain

$$\begin{aligned} R_1 \leq & I(X_1,X_4;Y_3,Y|X_2,X_3) \\ \leq & \log_2\Big(\det\big(\mathbb{I}_2 + \mathbb{H}\cdot(\mathbb{T}_{11} - \mathbb{T}_{12}\mathbb{T}_{22}^{-1}\mathbb{T}_{12}^H)\cdot\mathbb{H}^H\big)\Big), \end{aligned} \tag{25}$$

where

$$\mathbb{H} \triangleq \begin{bmatrix} h_{3,1} & h_{3,4} \\ h_1 & h_4 \end{bmatrix},$$

$$\mathbb{C}_4 = \begin{bmatrix} P_1 & \alpha_{14} & \alpha_{12} & \alpha_{13} \\ \alpha_{41} & P_4 & \alpha_{42} & \alpha_{43} \\ \alpha_{21} & \alpha_{24} & P_2 & \alpha_{23} \\ \alpha_{31} & \alpha_{34} & \alpha_{32} & P_3 \end{bmatrix} \triangleq \begin{bmatrix} \mathbb{T}_{11} & \mathbb{T}_{12} \\ \mathbb{T}_{12}^H & \mathbb{T}_{22} \end{bmatrix},$$

such that each sub-matrix $\mathbb{T}_{ij}$ is a two dimensional matrix.

The expressions for the bounds on R_2 in (19) are obtained similarly to the bound on R_1. The expressions for the bounds on $R_1 + R_2$ in (20) are derived in Appendix B.

All the inequalities (22)-(25) are achieved with equality when the Gaussian vector (X_1, X_2, X_3, X_4) has a zero mean. The outer bound is now obtained by finding the maximizing covariance matrix such that for each possible value of R_1, the largest R_2 is obtained.

4.2. Evaluation of the Achievable Region of Theorem 1 with Gaussian Inputs

Let $V_1^{(1)}, V_2^{(1)}, V_{1,0}, V_{2,0}, X_{1,0}, X_{2,0}, X_{3,0}, X_{4,0}$ be complex Normal RVs, $\mathcal{CN}(0,1)$, mutually independent. Let

$$V_1^{(2)} = \sqrt{\alpha_1}V_1^{(1)} + \sqrt{\beta_1}V_{1,0} \tag{26a}$$

$$V_2^{(2)} = \sqrt{\alpha_2}V_2^{(1)} + \sqrt{\beta_2}V_{2,0} \tag{26b}$$

$$X_4 = \sqrt{\alpha_{4,1}}V_1^{(1)} + \sqrt{\alpha_{4,2}}V_2^{(1)} + \sqrt{\alpha_{4,3}}X_{4,0} \tag{26c}$$

$$X_3 = \sqrt{\alpha_3}X_4 + \sqrt{\beta_3}V_1^{(2)} + \sqrt{\gamma_3}V_2^{(2)} + \sqrt{\delta}X_{3,0} \tag{26d}$$

$$X_1 = \sqrt{\phi_1}V_1^{(2)} + \sqrt{\theta_1}X_{1,0} \tag{26e}$$

$$X_2 = \sqrt{\phi_2}V_2^{(2)} + \sqrt{\theta_2}X_{2,0}. \tag{26f}$$

From the average power constraints on the channel input signals, $\mathbb{E}\{|X_k|^2\} \leq P_k$, $k = 1,2,3,4$, we have

$$\begin{aligned} \mathbb{E}\{|X_1|^2\} &= \phi_1(\alpha_1+\beta_1) + \theta_1 \leq P_1 \\ \mathbb{E}\{|X_2|^2\} &= \phi_2(\alpha_2+\beta_2) + \theta_2 \leq P_2 \\ \mathbb{E}\{|X_3|^2\} &= |\sqrt{\alpha_3\alpha_{4,1}} + \sqrt{\beta_3\alpha_1}|^2 \\ &\quad + |\sqrt{\alpha_3\alpha_{4,2}} + \sqrt{\gamma_3\alpha_2}|^2 + \alpha_3\alpha_{4,3} \\ &\quad + \beta_3\beta_1 + \gamma_3\beta_2 + \delta \leq P_3 \\ \mathbb{E}\{|X_4|^2\} &= \alpha_{4,1} + \alpha_{4,2} + \alpha_{4,3} \leq P_4, \end{aligned}$$

where $\alpha_1, \beta_1, \alpha_2, \beta_2, \alpha_{4,1}, \alpha_{4,2}, \alpha_{4,3}, \alpha_3, \beta_3, \gamma_3, \delta, \phi_1, \theta_1, \phi_2, \theta_2$ are all non-negative real numbers. Then, the mutual information expressions (1)-(3) can be written as

$$\begin{aligned} R_1 \leq & I(X_1;Y_3|X_2,X_3,X_4,V_1^{(2)}) \\ \stackrel{(a)}{=} & h(Y_3|X_2,X_3,X_4,V_1^{(2)}) \\ & - h(Y_3|X_1,X_2,X_3,X_4,V_1^{(2)}) \\ = & h(h_{3,1}X_1 + Z_3|X_2,X_3,X_4,V_1^{(2)}) - h(Z_3) \\ \stackrel{(b)}{=} & h(h_{3,1}\sqrt{\phi_1}V_1^{(2)} + h_{3,1}\sqrt{\theta_1}X_{1,0} \\ & + Z_3|X_2,X_3,X_4,V_1^{(2)}) - h(Z_3) \\ = & h(h_{3,1}\sqrt{\theta_1}X_{1,0} + Z_3|X_2,X_3,X_4,V_1^{(2)}) - h(Z_3) \\ \stackrel{(c)}{=} & \log_2\big((\pi e)\text{cov}(h_{3,1}\sqrt{\theta_1}X_{1,0} + Z_3|X_2,X_3, \\ & X_4,V_1^{(2)})\big) - \log_2\big((\pi e)\text{cov}(Z_3)\big) \\ = & \log_2\big(|h_{3,1}|^2\theta_1 + 1\big), \end{aligned}$$

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where (a) follows from the definition of mutual information [10, Chapter 2], (b) follows from the assignments (26), and (c) follows as all variables are jointly Gaussian. Similarly,

$$R_2 \le I(X_2; Y_3 | X_1, X_3, X_4, V_2^{(2)}) = \log_2\left(|h_{3,2}|^2\theta_2 + 1\right),$$

$$R_1 + R_2 \le I(X_1, X_2; Y_3 | X_3, X_4, V_1^{(2)}, V_2^{(2)}) = \log_2\left(|h_{3,1}|^2\theta_1 + |h_{3,2}|^2\theta_2 + 1\right),$$

$$\begin{aligned} R_1 \le& I(X_1; Y_4 | X_2, X_3, X_4, V_1^{(2)}) \\ &+ I(X_3, V_1^{(2)}; Y_4 | X_4, V_1^{(1)}, V_2^{(2)}) \\ =& \log_2(|h_{4,1}|^2\theta_1 + 1) \\ &+ \log_2\Big(1 + \frac{|h_{4,1}\sqrt{\phi_1\beta_1} + h_{4,3}\sqrt{\beta_3\beta_1}|^2 + |h_{4,3}|^2\delta}{|h_{4,1}|^2\theta_1 + |h_{4,2}|^2\theta_2 + 1}\Big), \end{aligned}$$

$$\begin{aligned} R_2 \le& I(X_2; Y_4 | X_1, X_3, X_4, V_2^{(2)}) \\ &+ I(X_3, V_2^{(2)}; Y_4 | X_4, V_1^{(2)}, V_2^{(1)}) \\ =& \log_2(|h_{4,2}|^2\theta_2 + 1) \\ &+ \log_2\Big(1 + \frac{|h_{4,2}\sqrt{\phi_2\beta_2} + h_{4,3}\sqrt{\gamma_3\beta_2}|^2 + |h_{4,3}|^2\delta}{|h_{4,1}|^2\theta_1 + |h_{4,2}|^2\theta_2 + 1}\Big), \end{aligned}$$

$$\begin{aligned} R_1 + R_2 \le& I(X_1, X_2, X_3; Y_4 | X_4, V_1^{(1)}, V_2^{(1)}) \\ =& \log_2\big(|h_{4,1}\sqrt{\phi_1\beta_1} + h_{4,3}\sqrt{\beta_3\beta_1}|^2 \\ &+ |h_{4,2}\sqrt{\phi_2\beta_2} + h_{4,3}\sqrt{\gamma_3\beta_2}|^2 \\ &+ |h_{4,1}|^2\theta_1 + |h_{4,2}|^2\theta_2 + |h_{4,3}|^2\delta + 1\big), \end{aligned}$$

$$\begin{aligned} R_1 \le& I(X_1, X_3, X_4; Y | X_2, V_2^{(1)}, V_2^{(2)}) \\ =& \log_2\big(|h_1\sqrt{\phi_1\alpha_1} + h_3\sqrt{\alpha_3\alpha_{4,1}} + h_3\sqrt{\beta_3\alpha_1} \\ &+ h_4\sqrt{\alpha_{4,1}}|^2 + |h_1\sqrt{\phi_1\beta_1} + h_3\sqrt{\beta_3\beta_1}|^2 \\ &+ |h_3\sqrt{\alpha_3\alpha_{4,3}} + h_4\sqrt{\alpha_{4,3}}|^2 \\ &+ |h_1|^2\theta_1 + |h_3|^2\delta + 1\big), \end{aligned}$$

$$\begin{aligned} R_2 \le& I(X_2, X_3, X_4; Y | X_1, V_1^{(1)}, V_1^{(2)}) \\ =& \log_2\big(|h_2\sqrt{\phi_2\alpha_2} + h_3\sqrt{\alpha_3\alpha_{4,2}} + h_3\sqrt{\gamma_3\alpha_2} \\ &+ h_4\sqrt{\alpha_{4,2}}|^2 + |h_2\sqrt{\phi_2\beta_2} + h_3\sqrt{\gamma_3\beta_2}|^2 \\ &+ |h_3\sqrt{\alpha_3\alpha_{4,3}} + h_4\sqrt{\alpha_{4,3}}|^2 \\ &+ |h_2|^2\theta_2 + |h_3|^2\delta + 1\big), \end{aligned}$$

and

$$\begin{aligned} R_1 + R_2 \le& I(X_1, X_2, X_3, X_4; Y) \\ =& \log_2\big(|h_1\sqrt{\phi_1\alpha_1} + (h_3\sqrt{\alpha_3} + h_4)\sqrt{\alpha_{4,1}} \\ &+ h_3\sqrt{\beta_3\alpha_1}|^2 + |h_1\sqrt{\phi_1\beta_1} + h_3\sqrt{\beta_3\beta_1}|^2 \\ &+ |h_2\sqrt{\phi_2\alpha_2} + (h_3\sqrt{\alpha_3} + h_4)\sqrt{\alpha_{4,2}} \\ &+ h_3\sqrt{\gamma_3\alpha_2}|^2 + |h_2\sqrt{\phi_2\beta_2} \\ &+ h_3\sqrt{\gamma_3\beta_2}|^2 + |\sqrt{\alpha_{4,3}}(h_4 + h_3\sqrt{\alpha_3})|^2 \\ &+ |h_1|^2\theta_1 + |h_2|^2\theta_2 + |h_3|^2\delta + 1\big). \end{aligned}$$

5. NUMERICAL RESULTS AND COMPARISON WITH PREVIOUS WORK

To demonstrate the benefit of coordination between relays we compare our results to the MPR-MAC with two parallel relays, depicted in Figure 6. In the MPR-MAC model, there is no wireless connectivity between the relays, e.g., each relay uses a directional transmit antenna to the destination. Thus, the received signals at the destination

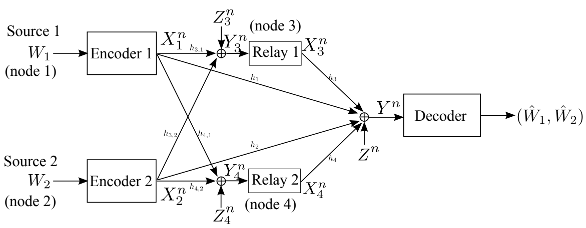

Figure 6. Gaussian MAC with two parallel relays.

and at the relays are given by (see [23])

$$Y_3 = h_{3,1}X_1 + h_{3,2}X_2 + Z_3 \quad (27a)$$
$$Y_4 = h_{4,1}X_1 + h_{4,2}X_2 + Z_4 \quad (27b)$$
$$Y = h_1X_1 + h_2X_2 + h_3X_3 + h_4X_4 + Z, \quad (27c)$$

where Z_3, Z_4, Z are additive white Gaussian noises. The average transmit power of node k is constrained to P_k. Note that the MPR-MAC channel outputs are similar to the MACMR channel outputs with the exception that each relay cannot receive signals transmitted by the other relay. More precisely, equations (27a), (27b) can be obtained by setting $h_{3,4}$ and $h_{4,3}$ to be zero in equations (17a) and (17b). Hence, the MPR-MAC is a special case of our model. It is interesting to evaluate the impact of the parallel relays restriction, that is, how much do we benefit by letting the relays communicate with each other. To this aim we compare the region of Theorem 1 with the achievable region obtained in [23] for the MPR-MAC. The outer bound of Proposition 1 was evaluated for the MPR-MAC and for the MACMR. Different configurations of the MPR-MAC and the MACMR were considered.

The numerical evaluations were carried out through the interior-point method, and each evaluation was repeated with random initial conditions to guarantee convergence.

5.1. Linear MACMR vs. Linear MPR-MAC

The first configuration is a scenario in which the relays and the destination are located on a straight line, see Figure 7, with $d_A < d_B < d_C$. For both the MACMR

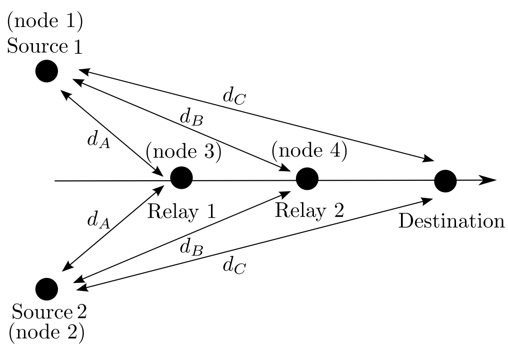

Figure 7. Line MACMR network.

and the MPR-MAC, we used $h_1 = h_2 = 0.1$, $h_3 = 0.2$, $h_4 = 0.4$, $h_{3,1} = h_{3,2} = 0.5$, $h_{4,1} = h_{4,2} = 0.2$. For the MACMR we set $h_{4,3} = h_{3,4} = 1$, while for the MPR-MAC we set $h_{4,3} = h_{3,4} = 0$. In Figure 8 we observe that

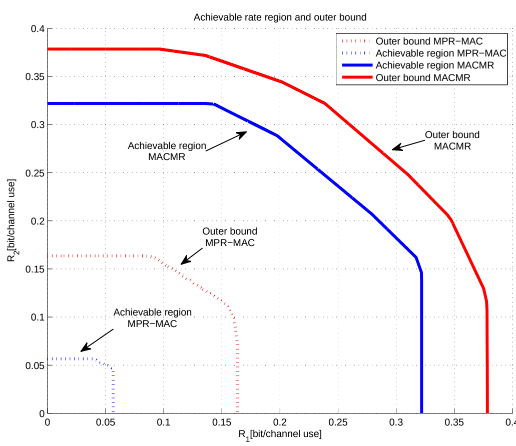

Figure 8. Achievable rate regions and outer bounds for the MACMR and the MPR-MAC for the line network of Figure 7.

the achievable rate region for the MACMR contains the outer bound for the MPR-MAC model.

We next change the locations of the relays and check the effect of the relative location of the sources, the relays and the destination on the performance.

5.2. MACMR vs. MPR-MAC when the Relays Are Close to the Sources

The second configuration is a scenario in which the relays are located closer to the sources than to the destination. This configuration is depicted in Figure 9, with $d_A < d_B < d_D < d_C$. The values of the channel coefficients

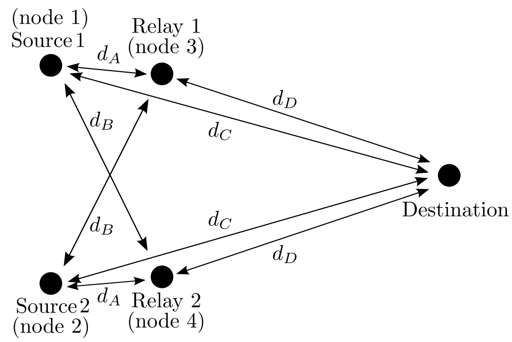

Figure 9. MACMR with relays closer to the sources.

are $h_1 = h_2 = 0.1$, $h_3 = h_4 = 0.2$, $h_{3,1} = h_{4,2} = 0.5$, $h_{3,2} = h_{4,1} = 0.3$, and $h_{4,3} = h_{3,4} = h_{rr}$, where for the MACMR $h_{rr} = 1$ and for the MPR-MAC $h_{rr} = 0$. In

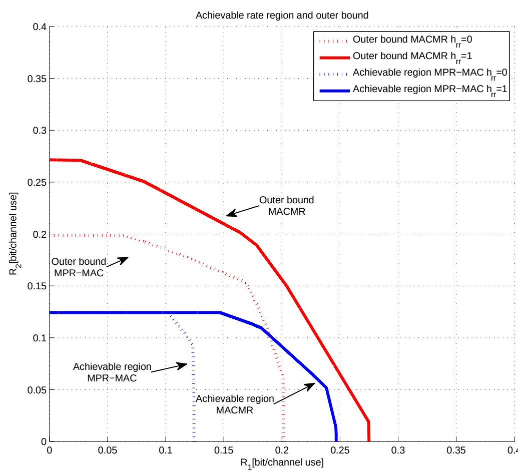

Figure 10. Achievable rate regions and outer bounds for the MACMR and the MPR-MAC for relays close to the sources network, see Figure 9.

Figure 10 we observe that the achievable rate region for the MACMR is greater than the achievable rate region obtained for the MPR-MAC model. Source 1 can transmit at higher rates when $h_{rr} = 1$. This is because letting the relays coordinate their transmissions allows relay 1 to assist also relay 2, thereby improving the decoding performance at relay 2. Although this change also allows relay 1 to observe signals from relay 2, it does not affect the decoding performance at relay 1, as relay 1 knows a-priori the signal from relay 2 due to the DF scheme. Therefore,

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when $h_{rr} = 1$ decoding at relay 1 becomes the bottleneck of the network. In the configuration depicted in Figure 9, the distance between source 1 and relay 1 is shorter than that between source 2 and relay 1. Therefore, source 1 can transmit at higher rates than source 2. Observe that part of the achievable region of the MACMR is outside the outer bound for the MPR-MAC.

5.3. MACMR vs. MPR-MAC when the Relays Are Close to the Destination

The third configuration is a scenario in which the relays are located closer to the destination than to the sources. This configuration is depicted in Figure 11, with $d_D < d_A < d_B < d_C$. The values of the channel coefficients

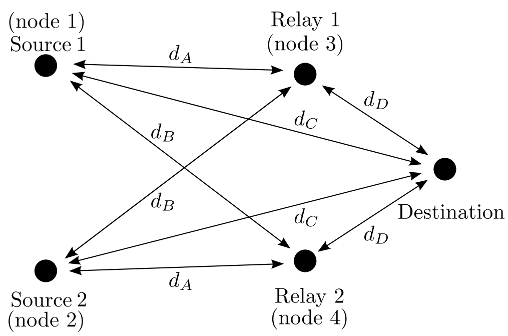

Figure 11. MACMR with relays closer to the destination.

are $h_1 = h_2 = 0.1$, $h_3 = h_4 = 0.6$, $h_{3,1} = h_{4,2} = 0.3$, $h_{3,2} = h_{4,1} = 0.2$, and $h_{4,3} = h_{3,4} = h_{rr}$, where for the MACMR $h_{rr} = 1$ and for the MPR-MAC $h_{rr} = 0$. In Figure 12 we observe that the achievable rate region for the MACMR is greater than the achievable rate region obtained for the MPR-MAC model. Source 1 can transmit at higher rates when $h_{rr} = 1$. As in previous configuration, the distance between source 1 and relay 1 is less than the distance between source 2 and relay 1. Recalling that the bottleneck is decoding at relay 1 leads again to the situation that source 1 can transmit at higher rates than source 2.

In order to evaluate the significance of the ability of the relays to receive each other's signals, and to understand how the channel coefficients $h_{4,3} = h_{3,4}$ affect the achievable region, we carried out a numerical evaluation which compared the achievable region of the MACMR for different values of $h_{4,3} = h_{3,4} = h_{rr}$, to the outer bound for the MPR-MAC. The value of $\max\{R_1 + R_2\}$ was used as a figure-of-merit for the different regions. Figure 13 shows the MACMR achievable regions for $h_{rr} = 0, 0.15, 0.3, 1$, and the MPR-MAC outer bound (i.e., setting $h_{rr} = 0$ in the MACMR). Lastly, Figure 14 depicts the difference between the maximal sum-rates of the achievable regions for the MACMR and the outer bound for the MPR-MAC (the fluctuations at high values of h_{rr} are due to numerical accuracy. The figure in monotonically increasing with h_{rr}). Observe that for $h_{rr} \geq 0.2$ the MACMR achievable sum-rate is greater than the MPR-MAC maximal sum-rate. This can be explained as follows: as h_{rr} increases, the quality of the link between the relays improves, and the relays receive better (i.e., less noisy) signals from each other. Therefore, the relays in the MACMR improve their coordination with each other and improve their assistance to the communication between sources and destination. As a result, the achievable region for the MACMR is increasing.

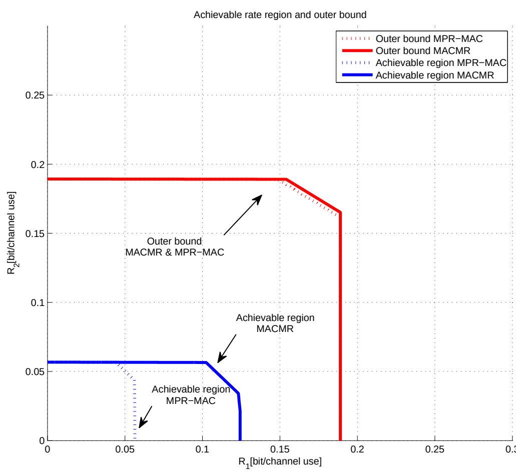

Figure 12. Achievable rate regions and outer bounds for the MACMR and the MPR-MAC for relays close to destination, see Figure 11.

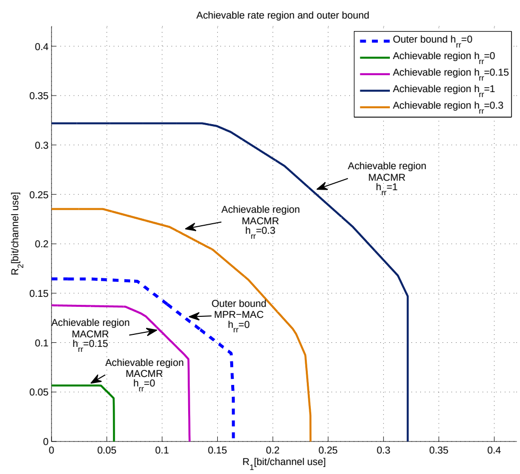

Figure 13. MACMR achievable rate regions for different values of h_{rr} and the MPR-MAC outer bound, for the line network depicted in Figure 7.

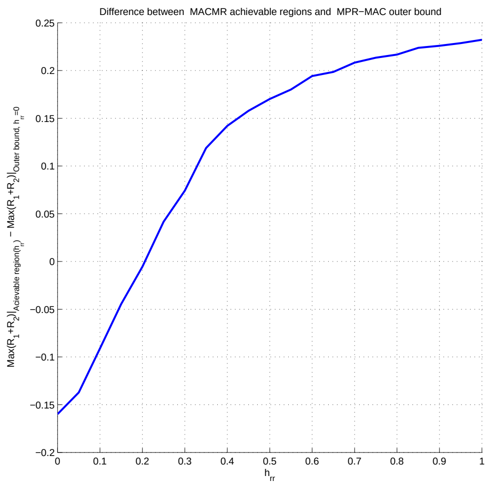

Figure 14. Difference between the maximum sum-rates for the achievable region of the MACMR and for the outer bound of the MPR-MAC, for the line network depicted in Figure 7, as a function of h_{rr}.

6. CONCLUSIONS

In this work we derived a new achievable rate region for the MACMR with two sources and two relays. We proposed a coding scheme in which the two relays assist the communication between the sources and destination. Each relay decodes the messages based on its channel output and forwards them to both the receiver and the other relay. The proposed coding scheme combines several techniques: regular encoding, sliding-window decoding at the relays, and backward decoding at the destination. We also derived the cut-set outer bound on the capacity. The achievable rate region and the outer bound were numerically evaluated for the Gaussian MACMR. To understand the benefit of our scheme over the previously proposed MPR-MAC model we carried out a numerical evaluation for different scenarios. For the case of two relays, we showed that because MACMR enables better cooperation in the network, the MACMR achievable region contains that of the MPR-MAC and in some scenarios it even contains the MPR-MAC outer bound. Our scheme also outperforms the scheme presented in [25] for the MACMR. We explained that this is because in [25] the relays do not cooperate and can only decode part of the sources' messages. This limitation is more pronounced in multihop wireless networks.

Future work includes error exponent analysis for the MACMR, in order to evaluate the effect of multiple relays on the relationship between the probability of error and the blocklength. Note that the present work can also be extended to more than two relays. To that aim, observe that cooperation is done in a hierarchical manner, namely, relay 2, which does not know the codewords to be sent by relay 1, is at the first (lower) level, and relay 1, which knows the codewords to be transmitted by relay 2 is at the second (top) level. In general, the assignment of the relays into levels affects the performance of this scheme. Thus, when there are multiple relays, we first need to decide how many levels to use, then assign the relays into levels, and finally, introduce additional auxiliary RVs to facilitate coordination (i.e., statistical dependence) between the codewords sent by the nodes at all levels, at each time block.

In conclusion, allowing the relays to utilize each other's transmissions can substantially increase the achievable region. Our numerical evaluations showed that this improvement depends on the quality of the links between the relays. This conclusion has important practical implication and should be taken into account when designing communication systems which employ cooperative strategies using multiple relays.

APPENDIX A: BOUNDING $p(E_{j1})$ AT RELAY 1

We now provide a detailed analysis of the error probability for relay 1.

Let $\mathbf{s} \triangleq (\mathbf{x}_1, \mathbf{x}_2, \mathbf{x}_3, \mathbf{x}_4, \mathbf{v}_1^{(1)}, \mathbf{v}_1^{(2)}, \mathbf{v}_2^{(1)}, \mathbf{v}_2^{(2)})$. Then,

$$\begin{aligned}
p(E_{j1}) &= p\Big(\Big\{\mathbf{x}_1\big(j|\mathbf{v}_1^{(2)}(w_{1,b-1}|w_{1,b-2})\big),\\
&\quad \mathbf{x}_2\big(1|\mathbf{v}_2^{(2)}(w_{2,b-1}|w_{2,b-2})\big), \mathbf{x}_3\big(\mathbf{v}_1^{(2)}(w_{1,b-1}|w_{1,b-2}),\\
&\quad \mathbf{v}_2^{(2)}(w_{2,b-1}|w_{2,b-2}), \mathbf{x}_4(w_{1,b-2}, w_{2,b-2})\big),\\
&\quad \mathbf{x}_4\big(\mathbf{v}_1^{(1)}(w_{1,b-2}), \mathbf{v}_2^{(1)}(w_{2,b-2})\big), \mathbf{v}_1^{(1)}(w_{1,b-2}),\\
&\quad \mathbf{v}_1^{(2)}\big(w_{1,b-1}|\mathbf{v}_1^{(1)}(w_{1,b-2})\big), \mathbf{v}_2^{(1)}(w_{2,b-2}),\\
&\quad \mathbf{v}_2^{(2)}\big(w_{2,b-1}|\mathbf{v}_2^{(1)}(w_{2,b-2})\big), \mathbf{y}_{3,b}\Big\} \in A_\epsilon^{(n)}\Big)\\
&\overset{(a)}{=} \sum_{(\mathbf{s},\mathbf{y}_3)\in A_\epsilon^{(n)}} p\Big(\mathbf{v}_1^{(1)}(w_{1,b-2})\Big) p\Big(\mathbf{v}_2^{(1)}(w_{2,b-2})\Big)\\
&\quad p\Big(\mathbf{v}_1^{(2)}\big(w_{1,b-1}|\mathbf{v}_1^{(1)}(w_{1,b-2})\big)\Big) p\Big(\mathbf{v}_2^{(2)}\big(w_{2,b-1}|\\
&\quad \mathbf{v}_2^{(1)}(w_{2,b-2})\big)\Big) p\Big(\mathbf{x}_4\Big|\mathbf{v}_1^{(1)}(w_{1,b-2}), \mathbf{v}_2^{(1)}(w_{2,b-2})\Big)\\
&\quad p\Big(\mathbf{x}_1\big(j|\mathbf{v}_1^{(2)}(w_{1,b-1}|w_{1,b-2})\big)\Big) p\Big(\mathbf{x}_2\big(1|\\
&\quad \mathbf{v}_2^{(2)}(w_{2,b-1}|w_{2,b-2})\big)\Big) p\Big(\mathbf{x}_3\Big|\mathbf{v}_1^{(2)}(w_{1,b-1}|w_{1,b-2}),\\
&\quad \mathbf{v}_2^{(2)}(w_{2,b-1}|w_{2,b-2}), \mathbf{x}_4(w_{1,b-2}, w_{2,b-2})\Big)\\
&\quad p\Big(\mathbf{y}_3\Big|\mathbf{x}_2\big(1|\mathbf{v}_2^{(2)}(w_{2,b-1}|w_{2,b-2})\big),\\
&\quad \mathbf{x}_3\big(\mathbf{v}_1^{(2)}(w_{1,b-1}|w_{1,b-2}), \mathbf{v}_2^{(2)}(w_{2,b-1}|w_{2,b-2}),\\
&\quad \mathbf{x}_4(w_{1,b-2}, w_{2,b-2})\big), \mathbf{x}_4\big(\mathbf{v}_1^{(1)}(w_{1,b-2}),\\
&\quad \mathbf{v}_2^{(1)}(w_{2,b-2})\big), \mathbf{v}_1^{(1)}(w_{1,b-2}), \mathbf{v}_1^{(2)}\big(w_{1,b-1}|\\
&\quad \mathbf{v}_1^{(1)}(w_{1,b-2})\big), \mathbf{v}_2^{(1)}(w_{2,b-2}), \mathbf{v}_2^{(2)}\big(w_{2,b-1}|\\
&\quad \mathbf{v}_2^{(1)}(w_{2,b-2})\big)\Big)
\end{aligned}$$

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$$
\begin{aligned}
&\overset{(b)}{\leq} 2^{n\left(H(\mathbf{S},Y_3)+\epsilon\right)} 2^{-nH(V_1^{(1)})} 2^{-nH(V_2^{(1)})} \\
&\quad 2^{-nH(V_1^{(2)}|V_1^{(1)})} 2^{-nH(V_2^{(2)}|V_2^{(1)})} 2^{-nH(X_4|V_1^{(1)},V_2^{(1)})} \\
&\quad 2^{-nH(X_1|V_1^{(2)})} 2^{-nH(X_2|V_2^{(2)})} 2^{-nH(X_3|V_1^{(2)},V_2^{(2)},X_4)} \\
&\quad 2^{-nH(Y_3|X_2,X_3,X_4,V_1^{(1)},V_1^{(2)},V_2^{(1)},V_2^{(2)})} 2^{9\epsilon n} \\
&\overset{(c)}{=} 2^{-n\left(H(Y_3|X_2,X_3,X_4,V_1^{(1)},V_1^{(2)},V_2^{(1)},V_2^{(2)})-H(Y_3|\mathbf{S})-10\epsilon\right)} \\
&\overset{(d)}{=} 2^{-n\left(I(X_1;Y_3|X_2,X_3,X_4,V_1^{(1)},V_1^{(2)},V_2^{(1)},V_2^{(2)})-10\epsilon\right)},
\end{aligned}
$$

where (a) follows from the codebook construction of Section 3.2 and the fact that $j \neq 1$, (b) follows from the properties of conditionally i.i.d. sequences [10, Chapter 15.2], (c) follows from the distribution chain (4) and the chain rule for mutual information [10, Theorem 2.5.2], and (d) follows from the definition of mutual information [10, Chapter 2.4].

APPENDIX B: SUM-RATE CUT-SET BOUNDS FOR THE GAUSSIAN MACMR

$$
\begin{aligned}
R_1 + R_2 &\leq I(X_1, X_2; Y_3, Y_4, Y | X_3, X_4) \\
&\leq \log_2 \Big(\det \big(\mathbb{I}_3 \\
&\qquad + \mathbb{H} \cdot (\mathbb{T}_{11} - \mathbb{T}_{12}\mathbb{T}_{22}^{-1}\mathbb{T}_{12}^H) \cdot \mathbb{H}^H\big)\Big)
\end{aligned}
$$

where

$$
\mathbb{H} \triangleq \begin{bmatrix} h_{3,1} & h_{3,2} \\ h_{4,1} & h_{4,2} \\ h_1 & h_2 \end{bmatrix}, \mathbb{C}_1 \triangleq \begin{bmatrix} \mathbb{T}_{11} & \mathbb{T}_{12} \\ \mathbb{T}_{12}^H & \mathbb{T}_{22} \end{bmatrix},
$$

as defined in Section 4.1, such that each sub-matrix $\mathbb{T}_{ij}$ is a two-dimensional matrix. Next,

$$
\begin{aligned}
R_1 + R_2 &\leq I(X_1, X_2, X_3, X_4; Y) \\
&\leq \log_2 \Big(\mathrm{cov}(Y)\Big),
\end{aligned}
$$

where

$$
\begin{aligned}
\mathrm{cov}(Y) = \mathrm{Trace} \left\{ \begin{bmatrix} h_1^* \\ h_2^* \\ h_3^* \\ h_4^* \end{bmatrix} \cdot \begin{bmatrix} h_1 & h_2 & h_3 & h_4 \end{bmatrix} \right. \\
\left. \cdot \begin{bmatrix} P_1 & \alpha_{12} & \alpha_{13} & \alpha_{14} \\ \alpha_{21} & P_2 & \alpha_{23} & \alpha_{24} \\ \alpha_{31} & \alpha_{32} & P_3 & \alpha_{34} \\ \alpha_{41} & \alpha_{42} & \alpha_{43} & P_4 \end{bmatrix} \right\} + 1.
\end{aligned}
$$

We next bound

$$
\begin{aligned}
R_1 + R_2 &\leq I(X_1, X_2, X_3; Y_4, Y | X_4) \\
&= \log_2 \Big(\det \big(\mathbb{I}_2 \\
&\qquad + \mathbb{H} \cdot (\mathbb{T}_{11} - \mathbf{t}_{12} t_{22}^{-1} \mathbf{t}_{12}^H) \cdot \mathbb{H}^H\big)\Big),
\end{aligned}
$$

where

$$
\mathbb{H} \triangleq \begin{bmatrix} h_{4,1} & h_{4,2} & h_{4,3} \\ h_1 & h_2 & h_3 \end{bmatrix}, \mathbb{C}_1 \triangleq \begin{bmatrix} \mathbb{T}_{11} & \mathbf{t}_{12} \\ \mathbf{t}_{12}^H & t_{22} \end{bmatrix},
$$

and $\mathbb{T}_{11}$ is a 3×3 matrix. Finally,

$$
\begin{aligned}
R_1 + R_2 &\leq I(X_1, X_2, X_4; Y_3, Y | X_3) \\
&\leq \log_2 \Big(\det \big(\mathbb{I}_2 \\
&\qquad + \mathbb{H} \cdot (\mathbb{T}_{11} - \mathbf{t}_{12} t_{22}^{-1} \mathbf{t}_{12}^H) \cdot \mathbb{H}^H\big)\Big)
\end{aligned}
$$

where

$$
\begin{aligned}
\mathbb{H} &\triangleq \begin{bmatrix} h_{3,1} & h_{3,2} & h_{3,4} \\ h_1 & h_2 & h_4 \end{bmatrix}, \\
\mathbb{C}_8 &\triangleq \begin{bmatrix} P_2 & \alpha_{24} & \alpha_{21} & \alpha_{23} \\ \alpha_{42} & P_4 & \alpha_{41} & \alpha_{43} \\ \alpha_{12} & \alpha_{14} & P_1 & \alpha_{13} \\ \alpha_{32} & \alpha_{34} & \alpha_{31} & P_3 \end{bmatrix} = \begin{bmatrix} \mathbb{T}_{11} & \mathbf{t}_{12} \\ \mathbf{t}_{12}^H & t_{22} \end{bmatrix},
\end{aligned}
$$

and $\mathbb{T}_{11}$ is a 3×3 matrix.

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REFERENCES

1. G. Kramer and A. J. van Wijngaarden, "On the white Gaussian multiple-access relay channel," *Proc. IEEE Int. Symp. Inform. Theory,* Sorrento, Italy, Jun. 2000, p. 40.
2. L. Sankaranarayanan, G. Kramer and N. B. Mandayam, "Hierarchical sensor networks: Capacity theorems and cooperative strategies using the multiple-access relay channel model," *Proc. First IEEE Conference on Sensor and Ad Hoc Communications and Networks,* Santa Clara, CA, Oct. 2004.
3. T. M. Cover and A. A. El Gamal, "Capacity theorems for the relay channel," *IEEE Trans. Inform. Theory,* vol. 25, no. 5, pp. 572–584, Oct. 1979.
4. G. Kramer, M. Gastpar and P. Gupta, "Information-theoretic multi-hopping for relay networks," *Proc. Int. Zurich Seminar on Communications,* ETH Zurich, Switzerland, Feb. 2004, pp. 192–195.
5. F. M. J. Willems. Informationtheoretical Results for the Discrete Memoryless Multiple-Access Channel.

Doctor in de Wetenschappen Proefschrift dissertation, Katholieke Univ. Leuven, Leuven, Belgium, Oct. 1982.

6. L. Sankaranarayanan, G. Kramer and N. B. Mandayam, "Capacity theorems for the multiple-access relay channel," *Proc. 42nd Annual Allerton Conf. on Commun., Control, and Computing,* Monticello, IL, Sept. 2004.
7. L. Sankar, G. Kramer and N. B. Mandayam, "Offset encoding for multiaccess relay channels," *IEEE Trans. Inform. Theory,* vol. 53, no. 10, pp. 3814–3821, Oct. 2007.
8. A. B. Carleial, "Multiple-access channels with different generalized feedback signals," *IEEE Trans. Inform. Theory*, vol. 28, no. 6, pp. 841–850, Nov. 1982.
9. L. Sankar, M. B. Narayan and H. V. Poor, "On the sum-capacity of degraded Gaussian multiple-access relay channels," *IEEE Trans. Inform. Theory,* vol 55, no. 12, pp. 5394–5411, Dec. 2009.
10. T. M. Cover and J. A. Thomas. *Elements of Information Theory*. John Wiley and Sons Inc., 2nd edition, 2006.
11. C. K. Ho, K. T. Gowda and S. Sun, "Achievable rates for multiple access relay channel with generalized feedback," *Proc. Int. Symp. Inform. Theory and its Applications,* Auckland, New Zealand, Dec. 2008.
12. B. Rankov and A. Wittneben, "Achievable rate regions for the two-way relay channel," *Proc. IEEE Int. Symp. Inform. Theory,* Seattle, WA, Jul. 2006, pp. 1668–1672.
13. J. Hou, R. Koetter and G. Kramer, "Rate regions for multiple access relay channels with relay-source feedback," *Proc. IEEE Inform. Theory Workshop,* Taormina, Italy, Oct. 2009, pp. 288–292.
14. B. Schein and R. G. Gallager, "The Gaussian parallel relay network," *Proc. IEEE Int. Symp. Inform. Theory,* Sorrento, Italy, Jun. 2000, p. 22.
15. A. Reznik, S. Kulkarni and S. Verdu, "Capacity and optimal resource allocation in the degraded Gaussian relay channel with multiple relays," *Proc. 40th Annu. Allerton Conf. Communication, Control, and Computing,* Monticello, IL, Oct. 2002.
16. P. Gupta and P. R. Kumar, "Towards an information theory of large networks: An achievable rate region," *IEEE Trans. Inform. Theory,* vol. 49, no. 8, pp. 1877–1894, Aug. 2003.
17. L. L. Xie and P. R. Kumar, "A network information theory for wireless communication: Scaling laws and optimal operation," *IEEE Trans. Inform. Theory,* vol. 50, no. 5, pp. 748–767, May 2004.
18. L. L. Xie and P. R. Kumar, "An achievable rate for the multiple-level relay channel," *IEEE Trans. Inform. Theory*, vol. 51, no. 4, pp. 1348–1358, Apr. 2005.
19. G. Kramer, M. Gastpar and P. Gupta, "Cooperative strategies and capacity theorems for relay networks," *IEEE Trans. Inform. Theory,* vol. 51, no. 9, pp. 3037–3063, Sep. 2005.
20. A. del Coso and C. Ibars, "The amplify-based multiple-relay multiple-access channel: Capacity region and MAC-BC duality," *Proc. IEEE Information Theory Workshop,* Bergen, Norway, Jul. 2007.
21. A. del Coso and C. Ibars, "Multi-access channels with multiple decode-and-forward relays: Rate region and asymptotic sum-rate," *Proc. IEEE 9th Workshop on Signal Processing Advances in Wireless Communications,* Jul. 2008, pp. 361–365.
22. A. del Coso and C. Ibars, "Linear relaying for the Gaussian multiple-access and broadcast channels," *IEEE Trans. Wireless. Comm.*, vol. 8, no. 4, pp. 2024–2035, Apr. 2009.
23. A. del Coso, Achievable Rates for Gaussian Channels with Multiple Relays. Ph.D. dissertation, Universitat Politcnica de Catalunya, Barcelona, Spain, 2008.
24. A. El-Gamal, M. Mohseni and S. Zahedi, "Bounds on capacity and minimum energy-per-bit for AWGN relay channels," *IEEE Trans. Inform. Theory*, vol. 52, no. 4, pp. 1545–1561, Apr. 2006.
25. S. Salehkalaibar, L. Ghabeli and M. R. Aref, "Achievable rate region for multiple-access-relay-networks," *IET Comm.*, vol. 4, no. 15, pp. 1792–1798, Oct. 2010.
26. J. A. Thomas, "Feedback can at most double Gaussian multiple access channel capacity," *IEEE Trans. Inform. Theory,* vol. 33, no. 5, pp. 711–716, Sep. 1987.
27. D. Ouellette, "Schur complements and statistics," *Linear Algebra and its Applications,* vol. 36, pp. 187–295, Mar. 1981.
28. A. S. Avestimehr, A. Sezgin and D. N. C. Tse, "Capacity of the two-way relay channel within a constant gap," *European Transactions on Telecommunications,* vol. 21, issue 4, pp. 363-374, , Jun. 2010.
29. T. Renk, H. Jaekel, C. Kloeck and F. K. Jondral, "General expression of outage probability in cooperative networks," *European Transactions on Telecommunications,* vol. 20, issue 6, pp. 537-548, Oct. 2009.
30. J. C. F. Li and S. Dey, "Outage minimisation in wireless relay networks with delay constraints and causal channel feedback," *European Transactions on Telecommunications,* vol. 21, issue 3, pp. 251-265, Apr. 2010.

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