LETTERS TO THE EDITOR

Censor's acoustical Doppler effect analysis—Is it a valid method?[a]

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In the article, "Acoustical Doppler effect analysis—Is it a valid method?" [J. Acoust. Soc. Am. 83, 1223–1230 (1988)], Censor considers the problem of scattering in the presence of moving objects under conditions of space- and time-dependent moving media. He considers this situation in the context of a generalized *linear* wave equation. This equation predicts the generation of Doppler-type spectral components at frequencies that are equal to (i) the sum of the frequencies of the primary waves, (ii) the difference of the frequencies of the primary waves, and (iii) harmonics of these sum-and-difference frequencies. However, since the *nonlinear* wave equation also predicts scattered spectral components at these same frequencies, and since those predicted by the nonlinear theory are usually much stronger than those predicted by Censor's theory, the generalized linear wave equation used by Censor is generally inadequate for accurately predicting the amplitudes of the spectral components of interest. However, a limited regime is identified in which the spectral components predicted by Censor's theory can dominate those predicted by nonlinear theory.

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Controversy concerning the proper theoretical treatment of the scattering of sound in the presence of moving boundaries dates back to an article published by Censor[1] in 1972. Since then, there has been an exchange of several articles.[2–7] In an attempt to resolve the issues raised by our earlier objections, Censor has published Ref. 8.

In Ref. 8, Censor sets the stage for his theoretical treatment by first offering an interpretation of the objections we have raised regarding his theory. He states,[8] "Essentially the objections were concerned with the facts that a linear model has been assumed, *and that the motion imparted to the medium by the moving scatterers has been ignored*." [Emphasis added.] However, a careful reading of our articles will show that our concerns only regard the failure of Censor's theory to properly account for the nonlinearities involved in the problems he has solved. Thus, the arguments used in Ref. 8, which address only the question of medium motion, fail to address the objections we have raised.

Censor's article does not address the questions we have raised concerning the inherent nonlinearities involved in the problems he has solved. It is well known that the linear wave equation is not exact in acoustics and there is a vast body of literature[9] in existence regarding the theoretical foundation, and the experimental confirmation, of the subject of nonlinear acoustics. We cannot understand the use of the *linear* wave equation (although in a generalized form) as the basis of the theoretical arguments in Ref. 8 when it is the very use of the linear wave equation for the problems considered in Refs. 1 and 4 that we have called into question. The starting point for demonstrating linearity *must* be the nonlinear wave equation.

Perhaps an analogy from a different, though familiar, area of physics would be helpful in clarifying the point we have made in the preceding paragraph. Consider the problem of an electron beam interacting with an impenetrable screen that contains an aperture having dimensions that are comparable to the de Broglie wavelength of the electrons in question. Imagine obtaining a solution to this problem by applying Newtonian mechanics to the motion of the electrons. Of course, Newtonian mechanics is a fundamentally incorrect approach (since this problem is inherently quantum mechanical in nature). A modification of the solution to account for a possible motion imparted to the screen by the electron beam, but which is still based on Newtonian mechanics, would imply an unawareness of the vast literature regarding the theoretical foundation, and the experimental confirmation, of the subject of quantum mechanics.

It may be disquieting to consider that "Newton's theory is wrong," since one could certainly point to a long history of confirming experimental evidence in support of Newtonian

[a] Please also see article in this issue: D. Censor, "Acoustical Doppler effect analysis—Is it a valid method?," J. Acoust. Soc. Am. 83, 1223–1230 (1988).

mechanics. Nevertheless, Newtonian mechanics is *not* exact, and one needs to carefully consider whether it is correct to use it in any given application, particularly in problems involving very small (or very fast-moving) objects.

Similarly, one could certainly point out the long history of experimental confirmation of linear acoustics, and could cite successful measurements of the Doppler effect. Nevertheless, since the linear wave equation is not exact, one needs to carefully consider whether this equation is appropriate to use in solving acoustical problems of interest, particularly in problems that involve the presence of two primary waves which can interact nonlinearly to produce sum-and-difference frequency waves.

We have solved scattering problems for several different geometries using both Censor's theory and the nonlinear theory.[3,7] In the cases we considered, the contributions to the sum-and-difference frequency components predicted by the nonlinear theory overwhelm those predicted by Censor's theory within a fraction of a wavelength of the scatterer's surface. However, after further consideration of the matter, we would like to modify somewhat our position regarding the validity of Censor's theory. We have previously stated[2,3,5,7] that there is no meaningful regime of experimental parameters in which the effects caused by the Doppler mechanism used in Censor's theory can dominate those caused by nonlinearities. However, the Doppler effects predicted by Censor's theory *can* be dominant in one case: This is the case in which the scatterer is driven to produce a wavelength that is very much larger than the characteristic geometrical dimensions of the scatterer.[10] The Doppler mechanism can dominate here because in this case the amplitude of the radiated signal associated with the oscillation of the scatterer's surface is very small and therefore the situation is very unfavorable to the nonlinear parametric generation of sum-and-difference frequency waves.[11] On the other hand, if the experiment reported in Ref. 10 were repeated using an angular frequency Ω for the oscillation of the scatterer's surface that had an associated wavelength comparable to or less than the dimensions of the scatterer, nonlinear theory would be required to interpret the experimental measurements so obtained.

Returning to our electron-beam analogy, one *can* neglect quantum mechanical effects, even for the problem considered in the analogy, provided that the electrons' de Broglie wavelength is much smaller than the aperture dimensions. In the scattering situations considered by Censor, one can neglect nonlinear effects provided that the characteristic length l of the geometry of the scatterer is much smaller than $2\pi c/\Omega$, where c is the sound speed. Only in this limited regime can the Doppler effects predicted by Censor's theory be made to dominate those produced by nonlinear mechanisms.

[1]D. Censor, "Scattering by time-varying obstacles," J. Sound Vib. **25**, 101–110 (1972).

[2]P. H. Rogers, "Comments on 'Scattering by time varying obstacles'," J. Sound Vib. **28**, 764–768 (1973).

[3]J. C. Piquette and A. L. Van Buren, "Nonlinear scattering of acoustic waves by vibrating surfaces," J. Acoust. Soc. Am. **76**, 880–889 (1984).

[4]D. Censor, "Harmonic and transient scattering from time varying obstacles," J. Acoust. Soc. Am. **76**, 1527–1534 (1984).

[5]J. C. Piquette and A. L. Van Buren, "Comments on 'Harmonic and transient scattering by time varying obstacles [J. Acoust. Soc. Am. **76**, 1527–1534 (1984)],' " J. Acoust. Soc. Am. **79**, 179–180 (1986).

[6]D. Censor, "Reply to 'Comments on Harmonic and transient scattering from time varying obstacles [J. Acoust. Soc. Am. **76**, 1527–1534 (1984)],' " J. Acoust. Soc. Am. **79**, 181–182 (1986).

[7]J. C. Piquette and A. L. Van Buren, "Some further remarks regarding scattering of an acoustic wave by a vibrating surface," J. Acoust. Soc. Am. **80**, 1533–1536 (1986).

[8]D. Censor, "Acoustical Doppler effect analysis—Is it a valid method?," J. Acoust. Soc. Am. **83**, 1223–1230 (1988).

[9]Readers unfamiliar with nonlinear acoustics can get a good introduction to the subject in, e.g., R. T. Beyer, *Physical Acoustics*, edited by W. P. Mason (Academic, New York, 1965), Vol. II, part B.

[10]M. Cox and P. H. Rogers, "Automated noninvasive motion measurement of auditory organs in fish using ultrasound," J. Vib., Acoust. Stress Reliability Design **109**, 55–59 (1987).

[11]Our previous articles only considered the case in which the scatterer's surface is caused to oscillate by actively driving the scatterer as a source. The approach taken in Ref. 10 to enhance the low-frequency amplitude is to immerse the scatterer in a low-frequency sound field generated by a source external to the scatterer. This approach can significantly enhance the low-frequency amplitude of oscillation as compared with the amplitude obtainable by directly driving the scatterer as a source.