Parametric Spectrum Shaping for Downstream Spectrum Management of Digital Subscriber Lines

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***Abstract*—The performance of Digital Subscriber Line (DSL) systems is tightly dependent on network deployment. The users (lines) in the binder create mutual interference, thus decreasing the rates of all users. The Optimal Spectrum Balancing (OSB) algorithm solves the spectrum management problem to increase user rates. However, the computational complexity of the OSB algorithm is extremely high. In this paper we introduce a novel low-complexity sub-optimal algorithm, dubbed PArametric Spectrum Shaping (PASS). The complexity of the suggested algorithm is independent of the number of tones. Simulation results show that the PASS algorithm matches OSB performance in many typical DSL scenarios.**

***Index Terms*—Digital subscriber line, dynamic spectrum management, discrete multi-tone modulation, interference channel, near-far scenario.**

I. INTRODUCTION

THE Digital Subscriber Line (DSL) is a family of technologies that provide digital data transmission over unshielded twisted pairs (UTP) of copper wires that were originally used for the telephone service. The performance of DSL systems is constrained by the crosstalk between the lines. Dynamic Spectrum Management (DSM) is a family of techniques that enables increased data rates by reducing interferences between lines. The well known near-far scenario is illustrated in Fig. 1.

The first group of N_1 users is located at the Central Office (CO), and the second group of N_2 transmitters is located far away from the CO. Consequently, the downstream interference from the Remote Terminal (RT) users[1] on the CO users[2] is extremely strong whereas the interference from the CO users on the RT users is weak. A well known distributed low-complexity DSM algorithm is known as iterative water-filling (IWF) [1], [2], [3]. In this technique each user determines its power level by water-filling over the interference from other users plus the background noise. The IWF algorithm is highly suboptimal in the near-far scenario case, due to its greedy nature. The Optimal Spectrum Balancing (OSB) [4] algorithm solves the spectrum management problem by its dual problem at each tone separately and an exhaustive search over the Lagrange multiplier of each user to meet power constraints. This method suffers from exponential complexity with the number of users. Centralized sub-optimal algorithms that have been proposed recently include Successive Convex Approximation for Low-complExity (SCALE) [5] and Iterative Spectrum Balancing [6], [7]. Recent decentralized sub-optimal low-complexity algorithms include Autonomous Spectrum Balancing (ASB) [8], Band Preference Spectrum Management (BPSM) [9], [10], and Iterative Power Pricing (IPP) [11].

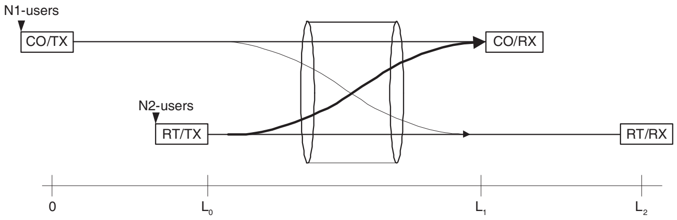

Fig. 1. Illustration of the Near-Far Scenario. The interference from the RT users on the CO users is extremely strong whereas the interference from the CO users on the RT users is very weak.

In this paper we introduce a novel centralized algorithm that considers rates in the average sense, dubbed PArametric Spectrum Shaping (PASS). The PASS algorithm shapes the Power Spectrum Density (PSD) according to a parametric model. Hence, the complexity of the algorithm depends solely on the choice of the model and not on the number of frequency bins. Simulation results show that the proposed algorithm achieves near-OSB performance in many typical DSL near-far scenarios.

II. SYSTEM MODEL AND PROBLEM STATEMENT

Assuming that discrete multi-tone modulation (DMT) is employed, the received signal at the modems is modeled independently on each tone k by:

$$\mathbf{x}(k) = \mathbf{H}(k)s(k) + \mathbf{w}(k), k = 1...K \tag{1}$$

where $\mathbf{s}(k) = [s_1(k), s_2(k), ..., s_N(k)]^T$ is the transmitted signal at the kth tone for each line in the binder, N, K are the number of users and the number of tones respectively. $\mathbf{x}(k), \mathbf{w}(k)$ are the received signal and the additive noise at tone k, respectively. The noise Power Spectral Density (PSD) is given by $\sigma_n(k) = E[|w_n(k)|^2]$, $H(k)$ is the channel transfer matrix on tone k. The $H(k)$ entries $h_{n,n}(k)$ and $h_{n,m}(k)$ are the direct channel of user n and the crosstalk channel from user m to user n, respectively. The transmit PSD of the nth user at tone k is given by:

$$p_n(k) = E[|s_n(k)|^2]. \tag{2}$$

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[1] RT users or strong users refer to users who are connected to the CO through the RT.

[2] CO users or weak users refer to users who are connected to the CO directly.

Assuming an Additive White Gaussian Noise (AWGN) channel, the maximum bitloading for the nth user at tone k in DMT systems is given by [12]:

$$b_n(k) = \log_2\left(1 + \frac{1}{\Gamma}\frac{|h_{n,n}(k)|^2 p_n(k)}{\sum\limits_{m\neq n}|h_{n,m}(k)|^2 p_m(k) + \sigma_n^2(k)}\right) \tag{3}$$

where Γ is the Signal to Noise Ratio (SNR) gap to capacity which includes the noise margin and coding gain The rate at line n is denoted by:

$$R_n = \Delta f \sum_k b_n(k) \tag{4}$$

where Δf is the bandwidth of each tone. The spectrum management problem, as formulated in [4], [8], is given by:

$$\begin{aligned} \max_{p_n, n\in N} R_1 \quad s.t. \quad & R_n \geq R_{n,target}, \forall n > 1, \\ & \sum_{k=1}^{K} p_n(k) \leq P_n^{tot}, \forall n = 1,..,N \end{aligned} \tag{5}$$

Here $R_{n,target}$ denotes the target rate of user n, and we can pick an arbitrary user to be user 1. The term $\mathbf{p}_n = [p_n(1), p_n(2), ..., p_n(K)]$ is a vector containing the PSD of user n.

The spectrum management problem in (5) is nonconvex due to the interference between users. However, the rate region is approximately convex in DMT systems with small tone spacings. This approximation becomes exact as the tone spacing approaches zero [4]. This problem is also high dimensional, since the problem dimension equals the number of tones of each user times the number of users. For instance, in VDSL2 systems, where a wide bandwidth is used, the number of tones can reach several thousands. In that case, solving the spectrum management problem directly by implementing the OSB algorithm results in extremely high computational cost, due to the exhaustive search over the Lagrange multipliers. Therefore, we suggest a parametric PSD shaping, which only searches the shaping parameters and approximates the OSB solution.

III. PARAMETRIC SPECTRUM SHAPING APPROACH

In this section we introduce the Parametric Spectrum Shaping (PASS) algorithm. Although the method can be applied to G groups of users, for simplicity we consider the two group case. Consider the near-far scenario case of two groups in the downstream direction as depicted in Fig. 1. Due to symmetry in the channels with equal lengths, we split the users into two groups, obtaining the same rates in each group [5],[13]. An adequate scheme for the strong users is to maximize the weak users rate under a target rate constraint, while the weak users maximize their rate regardless of the strong users[3]. We assume that the total transmit power per user equals the sum of allowed power per user per tone $\sum_{k=1}^{K} p_{mask}(k) = P_n^{tot}$ which means that every user can transmit the maximum power allowed by the mask constraint at each tone. We also assume identical PSD for all RT users as in [5],[13]. Let $\mathcal{P}_n(\boldsymbol{\theta}, k)$ be a continuous function which parameterizes the power at each tone k with respect to the parameter vector $\boldsymbol{\theta}$. We can formulate the power allocation problem for the two groups with weights α and $(1-\alpha)$ as:

$$\begin{aligned} & \max_{\boldsymbol{\theta}_w, \boldsymbol{\theta}_s} \alpha N_w R_w + (1-\alpha) N_s R_s \\ s.t & \\ & \mathcal{P}_w(\boldsymbol{\theta}_w, k) \leq p_{w,mask}(k), \forall k \\ & \mathcal{P}_s(\boldsymbol{\theta}_s, k) \leq p_{s,mask}(k), \forall k \\ & \mathcal{P}_w(\boldsymbol{\theta}_w, k), \mathcal{P}_s(\boldsymbol{\theta}_s, k) \geq 0, \forall k \end{aligned} \tag{6}$$

where

$$\begin{aligned} R_w &= \Delta f \sum_k \log_2\left(1 + \frac{1}{\Gamma}\frac{|h_{w,w}(k)|^2 \mathcal{P}_w(\boldsymbol{\theta}_w, k)}{Q_w}\right) \\ R_s &= \Delta f \sum_k \log_2\left(1 + \frac{1}{\Gamma}\frac{|h_{s,s}(k)|^2 \mathcal{P}_s(\boldsymbol{\theta}_s, k)}{Q_s}\right) \end{aligned} \tag{7}$$

where

$$\begin{aligned} Q_w &= N_s |h_{w,s}(k)|^2 \mathcal{P}_s(\boldsymbol{\theta}_s, k) \\ &\quad + (N_w - 1)|h_{w,w}(k)|^2 \mathcal{P}_w(\boldsymbol{\theta}_w, k) + \sigma_w^2(k) \\ Q_s &= N_w |h_{s,w}(k)|^2 \mathcal{P}_w(\boldsymbol{\theta}_w, k) \\ &\quad + (N_s - 1)|h_{s,s}(k)|^2 \mathcal{P}_s(\boldsymbol{\theta}_s, k) + \sigma_s^2(k) \end{aligned} \tag{8}$$

where $\mathcal{P}_w(\boldsymbol{\theta}_w, k), \mathcal{P}_s(\boldsymbol{\theta}_s, k)$ are the power allocations at tone k for weak and strong users respectively; $p_{w,mask}(k), p_{s,mask}(k)$ are the PSD constraints on the weak and strong users at tone k respectively; $|h_{w,w}(k)|^2, |h_{s,s}(k)|^2$ are the average direct channel gains of weak and strong users respectively. $|h_{s,w}(k)|^2, |h_{w,s}(k)|^2$ are the average interference channel gains from the strong users to the weak users and from weak users to the strong users. Due to the symmetry in the channel models [5],[13], the resulting rates for users having equal loop lengths end up the same. R_w, R_s are the mean rates of the weak and strong groups when all users in a group use the same spectrum shaping. Let k_c be the cutoff tone, above which the weak user does not operate. An example of a parametric model for the strong group that yields good results is given by:

$$\mathcal{P}_s(\boldsymbol{\theta}_s, k) = \begin{cases} p_{s,mask}(k), & k \geq k_c \\ p_{s,mask}(k) \cdot e^{-a_s\sqrt{k} - b_s k^2 - c_s k - d_s}, & k < k_c \end{cases}. \tag{9}$$

where k_c is computed offline, $e^{-a_s\sqrt{k} - b_s k^2 - c_s k - d_s} \leq 1 \forall k$ and $\boldsymbol{\theta}_s = [a_s, b_s, c_s, d_s]$ is the PSD shaping parameter vector. The optimization is done by applying Newton's method to determine $\boldsymbol{\theta}_w, \boldsymbol{\theta}_s$. The number of iterations is very low in Newton's method (usually fewer than 50 for a very large parameter space) and does not depend on the number of tones [14]. We initialize the PASS algorithm with a simplified solution to the PASS problem (9), where $b_s = 0, c_s = 0, d_s = 0$. Simulation results confirm that the PSD allocation in the simplified solution is sufficient for approximately fitting this model to the PSD achieved by the OSB algorithm. Note that the magnitude squared of the DSL insertion loss versus the frequency f and the length l obeys a simple parametric model [15], [16]: $|H^{IL}(f, l)| = e^{-\gamma l \sqrt{f}}$, where γ is the intrinsic line constant. This means that in the initialization stage we only need to solve a simple one dimensional optimization problem. Simulation results for PSD allocation achieved by

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[3] This approach is common to various algorithms such as ASB and IPP.

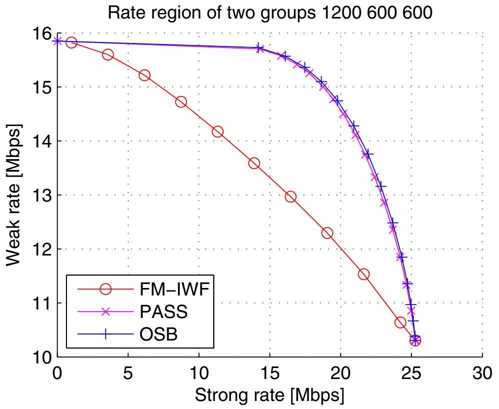

Fig. 2. Comparison of the algorithms for $1800m$ and $1200m$ lines with $600m$ overlap.

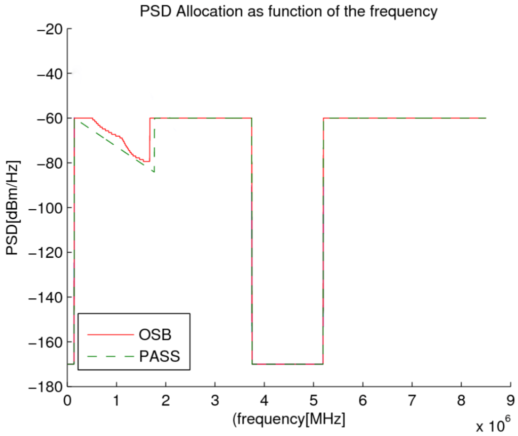

Fig. 3. Comparison of PASS and OSB power allocations for the strong group.

the OSB algorithm confirm the choice of $\mathcal{P}_s(\boldsymbol{\theta}_s, k)$ [4]. The PSD allocation in (9) implies that in the frequency range above k_c the strong users maximize their power, since they do not interfere with the weak users. However, in the frequency range below k_c the strong users reduce their transmitted power. k_c can be easily found offline by applying a bisection search on the signal to interference ratio.

IV. SIMULATION RESULTS

In this section we compare the following algorithms: 1) Fixed Margin Iterative Water Filling (FM-IWF); 2) PASS algorithm ; 3) OSB algorithm. We used a measured binder provided by the France Telecom research lab[4]. We used an ADSL2 mask for the weak group and a VDSL mask for the strong group. The PSD of the background noise was set to -140dBm/Hz. We simulated a typical downstream near-far scenario as depicted in Fig. 1, where $N_1 = 8$, $N_2 = 8$, $L_0 = 1200m$, $L_1 = 1800m$, $L_2 = 2400m$. The CO and RT used ADSL2 and VDSL respectively. The RT user changed its target rate from 0Mbps (maximal consideration) to 26Mbps (no consideration). The rate region of the CO users and RT users is presented in Fig. 2. It can be seen that the PASS algorithm outperforms IWF performance substantially and almost approaches OSB performance.

Fig 3 depicts the comparison between the power allocation made by OSB and by the PASS algorithm. In this example the strong users implement the PASS algorithm. The weak users are set to $p_{w,mask}$ due to practical considerations with typical DSL systems. However, the algorithm can be implemented for both groups.

V. CONCLUSION

In this paper we proposed a novel algorithm, dubbed PArametric Spectrum Shaping (PASS) for solving the spectrum management problem in DSL systems. The proposed algorithm makes use of a simulation-based low-dimension parametric model which approximates the optimal solution, with very low computational complexity. Simulation results show that the PASS algorithm approximately matches OSB performance in many typical DSL scenarios.

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