Game Theoretic Aspects of the Multi-Channel ALOHA Protocol in Cognitive Radio Networks

Kobi Cohen, Amir Leshem, and Ephraim Zehavi

***Abstract*— In this paper we consider the problem of distributed throughput maximization of cognitive radio networks with the multi-channel ALOHA medium access protocol. First, we characterize the Nash Equilibrium Points (NEPs) of the network when users solve an unconstrained rate maximization (i.e., the total transmission probability equals one). Then, we focus on constrained rate maximization, where user rates are subject to a total transmission probability constraint. We propose a simple best-response algorithm that solves the constrained rate maximization, where each user updates its strategy using its local channel state information (CSI) and by monitoring the channel utilization. We prove the convergence of the proposed algorithm using the theory of potential games. Furthermore, we show that the network approaches a unique equilibrium as the number of users increases. Then, we formulate the problem of choosing the access probability as a leader-followers Stackelberg game, where a single user is chosen to be the leader to manage the network. We show that a fully distributed setup can be applied to approximately optimize the network throughput for a large number of users. Finally, we extend the model to the case where primary and secondary users co-exist in the same frequency band.**

***Index Terms*—Cognitive radio networks, collision channels, multi-channel ALOHA, Nash equilibrium point, potential games, Stackelberg game.**

I. Introduction

THE increasing demand for wireless communication, along with spectrum utilization inefficiency, have triggered the development of dynamic spectrum access schemes for cognitive radio networks. The technology enabling different intelligent devices and networks to co-exist in the same frequency band is called cognitive radio. A good overview of the various dynamic spectrum access models for cognitive radio networks can be found in [1]. Dynamic spectrum access strategies can be categorized into three main models [1]: a hierarchical model that allows secondary (unlicensed) cognitive users to use the spectrum whenever they do not interfere with primary (licensed) users, a dynamic exclusive use model, where the spectrum bands are licensed to services for exclusive use, and finally an open sharing model among users that acts as the basis for managing a spectral region. In this paper we mainly focus on the third model, as was done in [2]–[4], so generally we will not assume that there are primary and secondary users in the networks. However, in Section V we extend the model to the case where primary and secondary users co-exist in the same frequency band.

Multi-channel systems are widely used in cognitive radio networks. In multi-channel systems, the bandwidth is divided into K orthogonal sub-bands using Orthogonal Frequency Division Multiple Access (OFDMA). Each sub-band can be a cluster of multiple carriers. A diversity of channel realizations is advantageous when users exploit local CSI to access good channels. Multi-channel systems have been widely investigated recently in cognitive radio networks. A related work on this subject can be found in [1], [2], [4]–[6].

Medium Access Control (MAC) schemes are used to manage users' access to the shared channels. The slotted ALOHA access protocol is a popular tool primarily because of its ease of implementation and its random-access [7]. In each time-slot, a user can access a shared channel according to a specific transmission probability. Transmission is successful only if a single user tries to access a shared channel in a given time-slot. If two or more users transmit in the same time slot over the same channel a collision occurs. Here, we examine the ALOHA protocol with multi-channel cognitive radio systems, dubbed multi-channel ALOHA [8]–[10]. A related work on stabilization and analysis of the multi-channel ALOHA protocol can be found in [8], [9].

We consider distributed optimization algorithms, where users make autonomous decisions based on local information and where coordination or message-passing between users are not required. In wireless networks, distributed optimization algorithms are simple to implement and generally preferred over centralized solutions. A natural framework to analyze distributed optimization algorithms in wireless networks is non-cooperative game-theory. Optimization of cognitive radio networks, random sensing and access games were examined in [11]–[14]. The problem of multi-radio multi-channel allocation non-cooperative games was investigated in [10], [15]–[17]. In [16], a distributed learning algorithm was proposed that converges in some special cases. In multi-radio multi-channel allocation, users are encouraged to spread resources over channels, since the utility of each channel decreases with the number of radios transmitting over it. This is not the case in our setup. A related work on optimization of a single-channel ALOHA using game theoretic tools can be found in [18]–[20]. In this setup, the utility of each user increases with the transmission probability. Here, we consider a similar model.

In this paper we present a game theoretic approach to the problem of distributed rate maximization of a multi-channel ALOHA in cognitive radio networks. A related work on cognitive radio networks using ALOHA-based protocols can be found in [21]–[23]. In the multi-channel ALOHA

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The authors are with the Faculty of Engineering, Bar-Ilan University, RamatGan, Israel (e-mail: kobi.cohen10@gmail.com).

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protocol, each user tries to randomly access a channel using a probability vector defining the access probability to the various channels. First, we characterize the Nash Equilibrium Points (NEPs) of the network when users solve the unconstrained rate maximization. We show that in this case, for any NEP, each user's probability vector is a standard unit vector (i.e., each user occupies a single channel with probability 1 and does not try to access other channels). In terms of the unconstrained rate maximization, we are mainly interested in the case where the number of channels is greater or equal to the number of users, to avoid collisions. Specifically, in the case where the number of users, N, is equal to the number of channels there are $N!$ NEPs.

Next, we consider the more interesting case where the number of users is much larger than the number of channels. In this case, to reduce the load (i.e., reduce the number of collisions), we focus on constrained rate maximization, where user rates are subject to a total transmission probability constraint. We characterize the NEPs when users solve the problem under a total probability constraint. We propose a simple best-response algorithm that solves the constrained rate maximization, where each cognitive user updates its strategy using its local CSI and by monitoring the channel utilization. A best-response algorithm is a common method in non-cooperative games to achieve a NEP [24]–[27]. We prove that the constrained rate maximization can be formulated as a potential game [28]. In potential games, the incentive of all players to change their strategy can be expressed in a one global function, the potential function. The existence of a bounded potential function corresponding to the constrained rate maximization problem implies that the convergence of the proposed algorithm is guaranteed. Furthermore, the convergence is in finite time, starting from any point and using any updating dynamics across users. Next, we analyze the algorithm's performance when the number of users approaches infinity. We show that the network approaches a unique equilibrium as the number of users increases.

Then, we discuss practical network management for constrained distributed rate maximization using the proposed best response algorithm. We formulate the problem as a leader-followers Stackelberg game, where a single cognitive user is chosen to be the leader to manage the network. Such leader-follower dynamic management has recently been investigated in cognitive radio networks [29]–[31]. We show that a fully distributed Stackelberg game setup can be applied that approximately optimizes the network throughput for a large number of users.

Finally, we extend the model to the hierarchical or exclusive use model (i.e., we assume that there are primary and secondary users in the network). We consider the case where the primary users have a high priority to transmit and the secondary users have a low priority to transmit, such that QoS requirements are satisfied. This approach, which limits interferences to primary users such that QoS requirements are satisfied was used in [32]–[34].

The rest of this paper is organized as follows. In Section II we present the network model and game formulation. In Sections III and IV we discuss the unconstrained and the constrained rate maximization problems, respectively. In Section V we extend the model to the case where primary and secondary users co-exist in the same frequency band. In Section VI we provide simulation results to demonstrate the algorithm's performance.

II. Network Model and Game Formulation

In this paper we consider a wireless network containing N users (or equivalently, N pairs of transmitters and receivers) who transmit over K orthogonal collision channels. The users transmit using the slotted ALOHA scheme. We make the following assumptions:

- In each time slot each user is allowed to access a single channel.
- A transmission is successful only if no other user tries to access the same channel simultaneously. If two or more users tries to access the same channel simultaneously, a collision occurs.
- Each user knows its local CSI, which can be obtained by a pilot signal in practical implementations.
- Each user perfectly estimates the load on all channels (i.e., monitors the channel utilization for a sufficient time)

In this paper we denote the collision-free achievable rate of user n at channel k by $u_n(k) \geq 0$. The collision-free achievable rate is given by $u_n(k) = W \log\left(1 + \frac{1}{\lambda}\text{SNR}\right)$, where W is the channel bandwidth and λ is the SNR gap to capacity which includes the noise margin and coding gain. Furthermore, we define a virtual zero-rate channel $u_n(0) = 0, \ ,\forall n$, i.e., accessing a channel $k = 0$ refers to no-transmission.
The collision-free rate vector of user n in all $K+1$ channels is given by:

$$\mathbf{u}_n \triangleq \begin{bmatrix} u_n(0) & u_n(1) & u_n(2) & \cdots & u_n(K) \end{bmatrix}, \qquad (1)$$

and the collision-free rate matrix of all N users in all $K+1$ channels is given by:

$$\mathbf{U} \triangleq \begin{bmatrix} u_1(0) & u_1(1) & u_1(2) & \cdots & u_1(K) \\ u_2(0) & u_2(1) & u_2(2) & \cdots & u_2(K) \\ \vdots \\ u_N(0) & u_N(1) & u_N(2) & \cdots & u_N(K) \end{bmatrix}. \qquad (2)$$

Let $p_n(k)$ be the probability that user n tries to access channel k. Let $\mathcal{P}_n$ be the set of all probability vectors of user n in all $K+1$ channels. A probability vector $\mathbf{p}_n \in \mathcal{P}_n$ of user n is given by:

$$\mathbf{p}_n \triangleq \begin{bmatrix} p_n(0) & p_n(1) & p_n(2) & \cdots & p_n(K) \end{bmatrix}. \qquad (3)$$

Let $\mathcal{P}$ be the set of all probability matrices of all N users in all $K+1$ channels. The probability matrix $\mathbf{P} \in \mathcal{P}$ is given by:

$$\mathbf{P} \triangleq \begin{bmatrix} p_1(0) & p_1(1) & p_1(2) & \cdots & p_1(K) \\ p_2(0) & p_2(1) & p_2(2) & \cdots & p_2(K) \\ \vdots \\ p_N(0) & p_N(1) & p_N(2) & \cdots & p_N(K) \end{bmatrix}, \qquad (4)$$

where $\sum_{k=0}^{K} p_n(k) = 1 \ \forall n$.
Let $\mathcal{P}_{-n}$ be the set of all probability matrices of all N users

in all $K+1$ channels, except user n. The probability matrix $\mathbf{P}_{-n} \in \mathcal{P}_{-n}$ is given by:

$$\mathbf{P}_{-n} \triangleq \begin{bmatrix} p_1(0) & p_1(1) & p_1(2) & \cdots & p_1(K) \\ \vdots & & & & \\ p_{n-1}(0) & p_{n-1}(1) & p_{n-1}(2) & \cdots & p_{n-1}(K) \\ p_{n+1}(0) & p_{n+1}(1) & p_{n+1}(2) & \cdots & p_{n+1}(K) \\ \vdots & & & & \\ p_N(0) & p_N(1) & p_N(2) & \cdots & p_N(K) \end{bmatrix} . \quad (5)$$

We focus in this paper on stationary access strategies, where each user decides whether or not to access a channel based on the current utility matrix and all other users' strategies.

Definition 1: A stationary strategy for user n is a mapping from $\{\mathbf{P}_{-n}, \mathbf{u}_n\}$ to $\mathbf{p}_n \in \mathcal{P}_n$.

Remark 1: Note that $\mathbf{u}_n$ depends on the local CSI of user n, which can be obtained by a pilot signal in practical implementations. On the other hand, in what follows we show that user n does not need the complete information on matrix $\mathbf{P}_{-n}$ to update its strategy, but only to monitor channel utilization by other users, defined by:

$$q_n(k) \triangleq 1 - \prod_{i \neq n} (1 - p_i(k)) \; . \quad (6)$$

Remark 2: The probability matrix $\mathbf{P}$ is called the multi-strategy matrix and contains all users' strategies, whereas $\mathbf{P}_{-n}$ is the multi-strategy matrix containing all users' strategies except the strategy of user n.

When user n perfectly monitors the k^{th} channel utilization, it observes:

$$v_n(k) \triangleq 1 - q_n(k) = \prod_{i \neq n} (1 - p_i(k)) \; , \quad (7)$$

which is the probability that the k^{th} channel is available.

Since a collision occurs when more than one user tries to access the same channel, the expected rate of user n in the k^{th} channel is given by:

$$r_n(k) \triangleq u_n(k) v_n(k) \; . \quad (8)$$

Hence, the expected rate of user n is given by:

$$R_n \triangleq R_n(\mathbf{p}_n, \mathbf{P}_{-n}) = \sum_{k=1}^{K} p_n(k) r_n(k) \; . \quad (9)$$

We define the non-cooperative multi-channel ALOHA game in this paper as follows:

Definition 2: The non-cooperative multi-channel ALOHA (MCA) game is given by $\Gamma_{MCA}(K, P_{max}) = (\mathcal{N}, \mathcal{P}, R)$, where $\mathcal{N} = \{1, 2, ..., N\}$ denotes the set of players (or users), $\mathcal{P}$ denotes the set of multi-strategy matrices, such that $\sum_{k=1}^{K} p_n(k) \leq P_{max}$ for all $n \in \mathcal{N}$. $R : \mathcal{P} \to \mathbb{R}^N$, given in (9), denotes the payoff (i.e., rate) function.

Note that P_{max} physically signifies the total channel access probability of each user.

In this paper, we consider a distributed rate maximization problem, where each user tries to maximize its own expected rate subject to a total transmission probability constraint:

$$\max_{\mathbf{p}_n} \quad R_n \quad \text{s.t.} \quad \sum_{k=1}^{K} p_n(k) \leq P_{max} \; . \quad (10)$$

We are interested in unconstrained (i.e., $P_{max} = 1$) and constrained (i.e., $P_{max} < 1$) solutions. A NEP for our model is the multi-strategy matrix $\mathbf{P}$, given in (4), which is stable in the sense that none of the users can increase its utility by unilaterally modifying its strategy $\mathbf{p}_n$.

Definition 3: A multi-strategy matrix $\mathbf{P} = \left[\mathbf{p}_1^T \; \mathbf{p}_2^T \; ... \; \mathbf{p}_N^T\right]^T$ is a Nash Equilibrium Point (NEP) if

$$R_n(\mathbf{p}_n, \mathbf{P}_{-n}) \geq R_n(\tilde{\mathbf{p}}_n, \mathbf{P}_{-n}) \;\; \forall n \, , \forall \tilde{\mathbf{p}}_n \; . \quad (11)$$

Next, we examine the unconstrained and constrained NEP solutions of $\Gamma_{MCA}(K, P_{max})$.

III. Unconstrained Rate Maximization

In this section, we characterize unconstrained NEP solutions of $\Gamma_{MCA}(K, P_{max})$, i.e., we set $P_{max} = 1$. When considering unconstrained solutions, we are interested in the case where $K \geq N$ to avoid collisions. Practically, each user monitors the channel utilization $v_n(k)$ for all $k = 1, ..., K$ (i.e., the complete $\mathbf{P}_{-n}$ is not required), and tries to access only a single available channel, which is the best response to all users' strategies $\mathbf{P}_{-n}$.

Theorem 1: Assume that $N \leq K$ in $\Gamma_{MCA}(K, 1)$. Then, the following hold:

a) For any NEP, each user's probability vector is a standard unit vector with probability 1 (i.e., each user tries to access a single channel with probability 1 and does not try to access other channels).

b) The network converges to a NEP in N iterations.

Proof:

$a)$ Assume that $N-1$ users play a multi-strategy matrix $\mathbf{P}_{-n} \in \mathcal{P}_{-n}$ and user n solves (10) after estimating $r_n(k) = u_n(k) v_n(k)$. Let[1] $k^* = \arg \max_k \{r_n(k)\}$. For any strategy $\mathbf{p}_n \in \mathcal{P}_n$, we have:

$$\begin{aligned} R_n &= \sum_{k=1}^{K} p_n(k) r_n(k) \leq \sum_{k=1}^{K} p_n(k) r_n(k^*) \\ &\leq r_n(k^*) \sum_{k=1}^{K} p_n(k) \leq r_n(k^*) \end{aligned}$$

Since we consider the unconstrained rate maximization $\left(\sum_{k=1}^{K} p_n(k) \leq P_{max} = 1\right)$, the upper bound is achieved by setting $p_n(k^*) = 1$.

$b)$ To prove the theorem we show that every user selects a channel only once. In the following, the superscript (t) denotes the iteration index. Without loss of generality, assume that in the first iteration $(t = 1)$ user 1 selects a channel $k = 1$, i.e., $p_1^{(t)}(1)|_{t=1} = p_1^{(1)}(1) = 1$. In the next iteration, for any user $n \neq 1$, we have: $r_n^{(2)}(1) = 0 < r_n^{(2)}(k) \;, \forall k = 2, ..., K$. Hence, user n will select a different channel $k \neq 1$. The process continues until the network converges to a NEP in the N^{th} iteration. ∎

[1] Since we consider a continuous value utility, k^* is unique with probability 1. When considering a quantized utility, we choose the best continuous value channel gain in cases where k^* is not unique.

We infer from Theorem 1 that the unconstrained distributed rate maximization is equivalent to a channel assignment problem, where each user chooses a single channel. Once a channel is taken by some user, no other user can access the same channel, since it has a zero utility.

In the case where $N = K$ any permutation that avoids a collision is a NEP. For instance, in the case of 3 users and 4 channels (note that channel $k = 0$ is the virtual zero-rate channel), the following multi-strategy matrix is a NEP:

$$\mathbf{P} = \left[\begin{array}{c|ccc} 0 & 0 & 1 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 1 \end{array}\right],$$

since any user that unilaterally modifies its strategy gets a zero utility (due to collision or no-transmission). In this case we have $N!$ NEPs.

In the case where $K > N$ any permutation that avoids a collision and maximizes every users' rate (given other users' strategies) is a NEP. For instance, consider the case of 2 users and 3 channels and assume that $u_1(3) \le u_1(2)$ and $u_2(3) \le u_2(1)$. The following multi-strategy matrix is a NEP:

$$\mathbf{P} = \left[\begin{array}{c|ccc} 0 & 0 & 1 & 0 \\ 0 & 1 & 0 & 0 \end{array}\right],$$

since none of the users can increase its utility by unilaterally modifying its strategy $\mathbf{p}_n$. As a result, there exist at most $(K \cdot (K-1) \cdots (K-N+1))$ NEPs.

In the case where $N > K$ any permutation is a NEP if at least K users access different channels. For instance, in the case of 3 users and 2 channels, the following multi-strategy matrix is a NEP:

$$\mathbf{P} = \left[\begin{array}{c|cc} 0 & 0 & 1 \\ 0 & 1 & 0 \\ 0 & 1 & 0 \end{array}\right],$$

since any user that unilaterally modifies its strategy gets a zero utility (due to a collision or accessing the virtual channel). Note that a better NEP can be obtained if users 2 or 3 access the virtual channel (i.e., do not transmit). In any practical system, users that get a zero utility should not transmit to avoid collisions.

IV. Constrained Rate Maximization

We now discuss the more interesting case, where $N > K$. In this case, unconstrained solutions lead to collisions or to zero utilities for some users. Therefore, constrained solutions should be used. According to Theorem 1, setting $P_{max} < 1$ is necessary to avoid collisions. First, we show the following result:

Theorem 2: Assume that $N > K$ in $\Gamma_{MCA}(K, P_{max})$ and $P_{max} < 1$. Let $k^* = \arg\max_k \{r_n(k)\}$, where $r_n(k)$ is defined in (8). Then, each user n plays the strategy:

$$p_n(k) = \begin{cases} 1 - P_{max}, & \text{if } k = 0 \\ P_{max}, & \text{if } k = k^* \\ 0, & \text{otherwise} \end{cases}, \quad (12)$$

with probability 1.

TABLE I
Proposed Best Response Algorithm

```
-  Initialize:
-  for n = 1,...,N users do:
-       estimate u_n(k) for all k = 1,...,K
-       k* <- arg max_k {u_n(k)}
-       p_n(k*) <- P_max
-  end for

-  repeat:
-  for n = 1,...,N users do:
-       estimate v_n(k) for all k = 1,...,K
-       compute r_n(k) = u_n(k)v_n(k)
-                   for all k = 1,...,K
-       k* <- arg max_k {r_n(k)}
-       p_n(k*) <- P_max
-  end for
-  until convergence
```

Proof: Assume that $N - 1$ users play a multi-strategy matrix $\mathbf{P}_{-n} \in \mathcal{P}_{-n}$ and user n wants to solve (10) after estimating $r_n(k) = u_n(k)v_n(k)$. Let $r_n(k^*) = \max_k \{r_n(k)\}$. For any strategy $\mathbf{p}_n \in \mathcal{P}_n$, we have:

$$R_n = \sum_{k=1}^{K} p_n(k) r_n(k) \le \sum_{k=1}^{K} p_n(k) r_n(k^*) \le r_n(k^*) \sum_{k=1}^{K} p_n(k) \le r_n(k^*) P_{max}$$

The upper bound is achieved by setting $p_n(k^*) = P_{max}$. ■

We infer from Theorem 2 that in each iteration each user will access a single channel with probability P_{max} and will not try to access other channels. However, in contrast to the unconstrained solutions, other users can still access occupied channels, since the utility is strictly positive in all channels. We discuss the convergence in a subsequent section.

As a result of Theorem 2, we obtain a best response algorithm, given in Table I. A best-response algorithm is a common method in non-cooperative games to achieve a NEP [24]–[27]. The proposed algorithm solves the constrained rate maximization problem (10). In the initialization step, each user selects the channel with the maximal collision-free rate $u_n(k)$. This can be done by all users simultaneously in a single iteration. Then, each user occasionally monitors the channel utilization and updates its strategy by selecting the channel with the maximal achievable rate $r_n(k)$ given the utilization of the channels.

Next, we examine the convergence of the proposed algorithm. Unlike the unconstrained solutions, convergence of the algorithm is not guaranteed in N iterations. However, in the following we use the theory of potential games to show that the distributed constrained rate maximization (10) indeed converges in finite time.

Remark 3: The convergence of sequential updating across users of the proposed algorithm will be proved using the theory of potential games and is based on knowledge of channel utilization $q_n(k) = 1 - v_n(k)$, defined in (6). To simplify the presentation, similar to [19] for a single-channel ALOHA, we assume that each cognitive user estimates the channel utilization perfectly (i.e., monitors the channel utilization for a sufficient time) before it updates its strategy. However, due to estimation errors in practical systems, users may switch their strategy, although their actual utilities may be reduced. Therefore, to stabilize the convergence of the algorithm, users should look for an $\epsilon-$NEP, as defined in Definition 7. Hence, in a practical system, users should update their strategies only if they improve their utility by more than ϵ. ϵ is set, such that estimation errors will not cause users to switch strategies. Users or the controller can increase ϵ dynamically to accelerate the convergence. Increasing ϵ accelerates the convergence, but may cause loss in the utility gain.

First, we formulate the distributed rate maximization problem (10) in a more convenient form.

Proposition 1 :
Let

$$\mathbf{1}_n(k) = \begin{cases} 1 & , \quad \text{if } p_n(k) = P_{max} \\ 0 & , \quad \text{otherwise} \end{cases} , \tag{13}$$

be the indicator function, which indicates whether user n tries to access a channel k.
Let

$$N(k) = \sum_{n=1}^{N} \mathbf{1}_n(k) \tag{14}$$

be the number of users that access channel k.
For every user n consider the following optimization problem:

$$\begin{aligned} \arg \max_{k} \quad & \tilde{R}_n \triangleq \log\left(u_n(k)\right) - N(k)\log\left(\frac{1}{1-P_{max}}\right) \\ s.t. \quad & p_n(k) = P_{max} \ , \end{aligned} \tag{15}$$

Then, the solution to (15) solves the optimization problem in (10).

Proof: The proof follows from the proof of Theorem 7. ■

Next, we define a modified multi-channel ALOHA game $\widetilde{\Gamma}_{MCA}(K, P_{max})$ as follows:

Definition 4: The modified non-cooperative multi-channel ALOHA (MCA) game is given by
$\widetilde{\Gamma}_{MCA}(K, P_{max}) = \left(\mathcal{N}, \mathcal{P}, \tilde{R}\right)$, where $\mathcal{N} = \{1, 2, ..., N\}$ denotes the set of players (or users), $\mathcal{P}$ denotes the set of multi-strategy matrices, such that $\sum_{k=1}^{K} p_n(k) \leq P_{max}$ for all $n \in \mathcal{N}$. $\tilde{R} : \mathcal{P} \rightarrow \mathbb{R}^N$, given in (15), denotes the payoff (i.e., modified rate) function.
When considering the modified game $\widetilde{\Gamma}_{MCA}(K, P_{max})$, each user solves the modified optimization problem (15). Since the solution to (15) solves (10) according to Proposition 1, then any NEP of $\widetilde{\Gamma}_{MCA}(K, P_{max})$ is a NEP of $\Gamma_{MCA}(K, P_{max})$.

In the following we show that $\Gamma_{MCA}(K, P_{max})$ and $\widetilde{\Gamma}_{MCA}(K, P_{max})$ are ordinal and exact potential games, respectively. In potential games, the incentive of all players to change their strategy can be expressed as a single global function, the potential function. In exact potential games, the improvement that each player can achieve by unilaterally changing its strategy equals the improvement in the potential function. In ordinal potential games, the utility of a player increases by unilaterally changing its strategy, if and only if the potential function increases. Hence, in both cases any local maximum of the potential function is a NEP. The existence of a bounded potential function corresponding to the constrained rate maximization problem (10) implies that the convergence of the proposed algorithm is guaranteed. Furthermore, the convergence is in finite time, starting from any point and using any updating dynamics across users[2].

Definition 5 [28]: A game $\Gamma = \left(\mathcal{N}, \mathcal{P}, \tilde{R}\right)$ is an exact potential game if there is an exact potential function $\phi : \mathcal{P} \rightarrow \mathbb{R}$ such that for every user $n \in \mathcal{N}$ and for every $\mathbf{P}_{-n} \in \mathcal{P}_{-n}$ the following holds:

$$\begin{aligned} &\tilde{R}_n(\mathbf{p}_n^{(2)}, \mathbf{P}_{-n}) - \tilde{R}_n(\mathbf{p}_n^{(1)}, \mathbf{P}_{-n}) \\ &\qquad = \phi(\mathbf{p}_n^{(2)}, \mathbf{P}_{-n}) - \phi(\mathbf{p}_n^{(1)}, \mathbf{P}_{-n}) \ , \\ &\qquad\qquad \forall \mathbf{p}_n^{(1)}, \mathbf{p}_n^{(2)} \in \mathcal{P}_n \ . \end{aligned} \tag{16}$$

Definition 6 [28]: A game $\Gamma = \left(\mathcal{N}, \mathcal{P}, \tilde{R}\right)$ is an ordinal potential game if there is an ordinal potential function $\phi : \mathcal{P} \rightarrow \mathbb{R}$ such that for every user $n \in \mathcal{N}$ and for every $\mathbf{P}_{-n} \in \mathcal{P}_{-n}$ the following holds:

$$\begin{aligned} &\tilde{R}_n(\mathbf{p}_n^{(2)}, \mathbf{P}_{-n}) - \tilde{R}_n(\mathbf{p}_n^{(1)}, \mathbf{P}_{-n}) > 0 \iff \\ &\qquad \phi(\mathbf{p}_n^{(2)}, \mathbf{P}_{-n}) - \phi(\mathbf{p}_n^{(1)}, \mathbf{P}_{-n}) > 0 \ , \\ &\qquad\qquad \forall \mathbf{p}_n^{(1)}, \mathbf{p}_n^{(2)} \in \mathcal{P}_n \ . \end{aligned} \tag{17}$$

Theorem 3: The non-cooperative multi-channel ALOHA (MCA) game $\Gamma_{MCA}(K, P_{max})$ and the modified non-cooperative MCA game $\widetilde{\Gamma}_{MCA}(K, P_{max})$ are ordinal and exact potential games, respectively, with the following bounded potential function:

$$\begin{aligned} \phi(\mathbf{P}) = &\sum_{n=1}^{N} \sum_{k=1}^{K} \log\left(u_n(k)\right) \mathbf{1}_n(k) \\ &- \log\left(\frac{1}{1-P_{max}}\right) \sum_{k=1}^{K} \frac{N(k)\left(N(k)+1\right)}{2} \ . \end{aligned} \tag{18}$$

Proof: The proof follows from the proof of Theorem 7. ■

Corollary 1: Any sequential update dynamics of the multi-channel ALOHA game $\Gamma_{MCA}(K, P_{max})$ converges to a NEP in finite time, starting from any point. Specifically, the proposed sequential best response algorithm, given in Table I, converges to a NEP in finite time.

A. Asymptotic Analysis

In this section we characterize the algorithm's behavior in the asymptotic regime (i.e., $N \rightarrow \infty$ and K is fixed). In the

[2]Note that the same technique is used in the case of $K \geq N$ and $P_{max} < 1$, since some of the users may choose the same channel. Hence, in this case convergence is guaranteed using the theory of potential games, as well.

asymptotic regime we consider convergence to an ϵ-NEP. An ϵ-NEP for our model is a multi-strategy matrix $\mathbf{P}$, given in (4), which is stable in the sense that none of the users can increase its utility by more than $\epsilon \cdot P_{max}$ by unilaterally modifying its strategy $\mathbf{p}_n$.

Definition 7: A multi-strategy matrix $\mathbf{P}$ is an ϵ-Nash Equilibrium Point (ϵ-NEP) if:

$$R_n(\mathbf{p}_n, \mathbf{P}_{-n}) \geq R_n(\tilde{\mathbf{p}}_n, \mathbf{P}_{-n}) - \epsilon \cdot P_{max} \;\; \forall n \;, \forall \tilde{\mathbf{p}}_n \;. \tag{19}$$

If $\epsilon = 0$, ϵ-NEP is equivalent to a NEP.

In this section we assume some very general conditions on the distribution of matrix U that are common in cognitive radio networks: due to path loss attenuation, the rows (i.e., users) in the collision-free rate matrix U, defined in (2), are assumed to be independent but not-necessarily identically distributed. Due to the frequency selective fading effect, the columns (i.e., channels) for each row in collision-free rate matrix U are assumed to be identically distributed but not-necessarily independent. In this common scenario, we show in the following theorem that in the asymptotic regime (i.e. $N \to \infty$) the proposed best response algorithm converges to a unique ϵ-NEP. Each user selects the channel with the maximal collision-free rate $u_n(k)$.

Theorem 4: Assume that the proposed best response algorithm is implemented with $P_{max} = \alpha/N$. Assume that the rows in matrix U, as defined in (2), are independent but not-necessarily identically distributed, and the columns for each row in matrix U are identically distributed but not-necessarily independent. Fix the number of channels K and let $k_n^* = \arg\,\max_k \{u_n(k)\}$.
Then, in the asymptotic regime ($N \to \infty$) the network converges to a unique ϵ-NEP for any $\epsilon > 0$. Specifically, each user n plays the strategy:

$$p_n(k) = \begin{cases} 1 - P_{max} \;, & \text{if } k = 0 \\ P_{max} \;, & \text{if } k = k_n^* \\ 0 \;, & \text{otherwise} \end{cases} \;, \tag{20}$$

with probability 1.

Proof:
Let

$$\tilde{\mathbf{1}}_n(k) = \begin{cases} 1 & , \text{ if } k = k_n^* \\ 0 & , \text{ otherwise} \end{cases} \;, \tag{21}$$

be the indicator function, which indicates whether user n tries to access a channel k at the initialization step.
Let

$$\tilde{N}(k) = \sum_{n=1}^{N} \tilde{\mathbf{1}}_n(k) \tag{22}$$

be the number of users that access a channel k at the initialization step.
Since $\tilde{\mathbf{1}}_n(k)$ are identically distributed across channels, we have: $Pr\left(u_n(k) = u_n^*\right) = 1/K \; \forall k$, where K is fixed. Hence, $\mathbf{E}\left\{\tilde{\mathbf{1}}_n(k)\right\} = 1/K$. Since the average estimate of $\tilde{N}(k)$ tends to its mean according the strong law of large numbers (SLLN), we obtain:

$$\tilde{N}(k) \xrightarrow{a.s} N/K \quad \forall k \quad \text{as} \quad N \to \infty \;\;.$$

In the next iteration, user n monitors the k^{th} channel utilization after the initialization step is completed and observes $v_n(k) = (1 - P_{max})^{\tilde{N}(k)}$. By the continuous mapping theorem [35], we have:

$$v_n(k) = (1 - P_{max})^{\tilde{N}(k)} = (1 - \alpha/N)^{\tilde{N}(k)} \xrightarrow{a.s} e^{-\alpha/K}$$
$$\forall n, k \quad \text{as} \quad N \to \infty \;\;.$$

Therefore, in the next iteration each user n obtains:

$$u_n(k) v_n(k) \xrightarrow{a.s} u_n(k) e^{-\alpha/K} \quad \forall k \quad \text{as} \quad N \to \infty \;\;.$$

Since $k_n^* = \arg\,\max_k \{u_n(k)\}$ and K is fixed, we have:

$$u_n(k_n^*) v_n(k_n^*) > u_n(k) v_n(k) - \epsilon \quad \forall k \quad \text{as} \quad N \to \infty \quad \text{with probability } 1,$$

which yields a unique ϵ-NEP (20). ■

B. Determining P_{max}

We now discuss the selection of P_{max} to maximize the network throughput in the asymptotic regime (i.e., $N \to \infty$ and K is fixed), as is generally assumed in analysis of ALOHA networks. However, the simulation results show very good performance for small N, as well. Throughout the analysis in this section we again assume the general conditions on matrix U discussed in Section IV-A.

In a single-channel ALOHA systems (i.e., $K = 1$), it is well-known that $P_{max} = 1/N$ maximizes the network throughput. In the following theorem we show that $P_{max} = K/N$ maximizes the network throughput (i.e., rate) in the multi-channel ALOHA game $\Gamma_{MCA}(K, P_{max})$ using the proposed best-response algorithm presented in Table I. Furthermore, the throughput increases linearly with the number of channels K.

Theorem 5: Assume that the proposed best response algorithm is implemented. Assume that matrix U, defined in (2), satisfies the conditions specified in Theorem 4. Fix the number of channels K and let $u_n(k_n^*) = \max_k \{u_n(k)\}$.
Then, in the asymptotic regime ($N \to \infty$) setting $P_{max} = K/N$ maximizes the user sum rate, $\sum_{n=1}^{N} R_n$. Specifically, for $P_{max} = K/N$ the user sum rate increases linearly with the number of channels K:

$$\sum_{n=1}^{N} R_n = K e^{-1} \cdot \frac{1}{N} \sum_{n=1}^{N} u_n(k_n^*). \tag{23}$$

Proof: From the proof of Theorem 4, the achievable rate of user n is given by:

$$R_n = P_{max} u_n(k_n^*) v_n(k_n^*) = u_n(k_n^*) \frac{\alpha}{N} e^{-\alpha/K} \text{ as } N \to \infty.$$

Therefore, the sum rate is given by:

$$\sum_{n=1}^{N} R_n = \alpha e^{-\alpha/K} \cdot \frac{1}{N} \sum_{n=1}^{N} u_n(k_n^*).$$

Differentiating with respect to α and equating to zero yields: $\alpha = K$. ■

For $K = 1$ we obtain the well-known maximal throughput e^{-1} of the single-channel ALOHA systems. Using the best-response dynamics, each user's strategy converges to selecting the best channel. Theorem 5 states that the proposed algorithm with $P_{max} = K/N$ converges to this limit at each channel as N increases. In Section V-A we show that $P_{max} = K/N$ maximizes the user expected rates for a finite N, as well.

Next, we account for the diversity gain from the best-response dynamics as N increases. Assume that the users experience Rayleigh fading channels, $h_n(k)$, i.i.d across users and across channels (i.e., $|h_n(k)|^2$ is exponentially distributed). It is shown in [36] that the SNR diversity gain is given by $\sum_{k=1}^{K} 1/k$, and approaches $\log(K)$ as K increases. In [3], two bounds on the users' rates were obtained, where each user gets its best channel without collisions.
For low SNR, for some γ, we have:

$$E\left\{u_n(k_n^*)\right\} \leq \text{SNR} \cdot \gamma \cdot \log(K) \ ,$$

and for high SNR, where $\eta = \gamma + \frac{1}{SNR\log(2)}$, we have:

$$E\left\{u_n(k_n^*)\right\} \leq \log\log(K) + \log(\text{SNR} \cdot \eta) \ .$$

It was shown in [37] that these bounds are tight in the high SNR regime and also in the low SNR regime.
Applying these bounds to $\Gamma_{MCA}(K, P_{max})$, yields that the sum rate achieved by the proposed best response algorithm with $P_{max} = K/N$ for $N \to \infty$ are bounded by:

$$\sum_{n=1}^{N} R_n \leq Ke^{-1} \cdot \text{SNR} \cdot \gamma \cdot \log(K) \ ,$$

for low SNR, and

$$\sum_{n=1}^{N} R_n \leq Ke^{-1} \cdot (\log\log(K) + \log(\text{SNR} \cdot \eta)) \ ,$$

for high SNR.
We infer from these bounds that the diversity gain from the cognitive best response dynamics is largest when users have low SNR, e.g., when the users are limited by uncoordinated interference.

C. Network Management: A Stackelberg Game Formulation

In this section we discuss network management in practical implementations. We consider the case where the network is designed such that some user is chosen to be the leader and the rest of the users are the followers that react to the leader's strategy. Such a leader-followers scheme is called a Stackelberg game, and has recently been used in cognitive radio networks [29]–[31].

In a Stackelberg game, the leader can predict the followers' best response to its strategy and can control the network by playing a strategy that optimizes some criterion. In our setup the control parameter is the transmission probability constraint P_{max} that significantly affects the load in the network and hence the achievable user rates. Below we show that a single leader can approximately maximize the user sum rate in the network for a large number of users. Therefore, there is no need for multiple leaders in our model. A leader can be the service provider or the spectrum owner, as suggested in [30], [31], or can be a user, chosen dynamically from time to time using a leader-selection mechanism [38], [39]. In our setup, a simple leader-selection mechanism can be used from time to time. For instance, during the leader-selection step, each user can wait a random backoff-time before broadcasting a pilot signal to the all other users. The first user that broadcasts its pilot signal is selected as a leader. Below we show that the leader-followers model can be implemented in a distributed setting. Complete information on the multi-strategy matrix is not required, but only monitoring the channel loads.
Let $p \in [0, 1]$ be a control parameter and $\mathcal{L}$ be the set of all control parameters determined by the leader. Once the leader chooses $p \in \mathcal{L}$, all users (including the leader itself) set $P_{max} = p$.
A natural criterion for the leader is to play a strategy that maximizes the user sum rate in the network. Hence, the leader solves the following optimization problem:

$$\begin{aligned} \max_{p \in \mathcal{L}} \quad & \sum_{n=1}^{N} R_n^* \\ s.t. \quad & R_n^* = \max_{\mathbf{p}_n} \ R_n(\mathbf{p}_n, \mathbf{P}_{-n}, P_{max}) \ \ \forall n \\ & \sum_{k=1}^{K} p_n(k) \leq P_{max} \ \ \forall n \\ & P_{max} = p \ . \end{aligned} \tag{24}$$

The constraints in (24) denote a sub-game $\Gamma_{MCA}(K, P_{max})$, given that $P_{max} = p$ was determined by the leader. The NEPs and ϵ-NEPs for the sub-game are defined in Definitions 3 and 7, respectively. A Stackelberg equilibrium is defined as follows:
Definition 8: A pair $(p^*, \mathbf{P}^*) \in \mathcal{L} \times \mathcal{P}$ is a Stackelberg equilibrium if $(p^*, \mathbf{P}^*)$ solves (24).
The solution to a Stackelberg game is obtained via backward induction. First, the leader solves the sub-game (i.e., the constraints) of (24) as a function of p. Then, it maximizes the user sum rate. Solving (24) exactly is not practical and is computationally expensive. First, any NEP depends on the update dynamics and the objective function may have multiple local maxima points. Second, the solution requires a centralized setup. However, in the following theorem we show that for sufficiently large N, the solution can be obtained simply:

Theorem 6: Assume that the proposed best response algorithm is implemented. Assume that the matrix U, defined in (2), satisfies the conditions specified in Theorem 4. Fix the number of channels K.
Then, in the asymptotic regime ($N \to \infty$) the pair $(p^*, \mathbf{P}^*)$ is a Stackelberg equilibrium, where $p^* = K/N$ and the entries of $\mathbf{P}^*$ are given in (20) with probability 1.

Proof: From Theorem 4, the entries of $\mathbf{P}^*$ is given in (20) as $N \to \infty$ with probability 1. From the proof of Theorem 5, the sum rate is given by: $\sum_{n=1}^{N} R_n = \alpha e^{-\alpha/K} \cdot \frac{1}{N}\sum_{n=1}^{N} u_n(k_n^*)$ for all α. This is the solution to the sub-game. Maximizing $\sum_{n=1}^{N} R_n$ with respect to α yields $\alpha = K$, by Theorem 5. ■

From Theorem 6, we infer that for sufficiently large N, a distributed approximate solution can be obtained. The number of users can be obtained by the leader in a distributed fashion by monitoring the channel utilization. Assume that the leader initializes the network by determining a non-optimal $P_{max} =$

p_0. When the leader monitors the k^{th} channel perfectly it observes $v(k) = (1-p_0)^{N(k)}$, where $N(k)$ is the number of users that transmit over channel k. Hence, it computes the asymptotically optimal P_{max} by:

$$P_{max} = p^* = \frac{K}{N} = \frac{K}{\sum_{k=1}^{K} \frac{\log(v(k))}{\log(1-p_0)}} .$$

Thus, a fully distributed setup can be implemented. The simulation results show very good performance for small N as well.

V. A Hierarchical MCA for Primary and Secondary Users

In the previous sections we focused on the open sharing model. Hence, we have not discussed the case of primary (i.e., licensed) and secondary (i.e., unlicensed) users in the networks. In this section we extend our model to the case of a hierarchical or exclusive use model, where primary and secondary users co-exist in the same frequency band. We consider the case where the primary users have a transmission probability constraint $P_{max,1}$ (high priority) and the secondary users have a transmission probability constraint $P_{max,2}$ (low priority). Typically, $P_{max,2}$ can be sufficiently small as compared to $P_{max,1}$ to limit interference to primary users caused by secondary users, such that QoS requirements are satisfied. The approach of limiting interferences such that QoS requirements are satisfied was used in [32]–[34]. We discuss the choice of $P_{max,1}$ and $P_{max,2}$ in Section V-A. Again, we focus on the more interesting case, where $N > K$ and $P_{max,1} < 1$, $P_{max,2} < 1$ to avoid collisions.

Definition 9: The non-cooperative hierarchical multi-channel ALOHA (HMCA) game is given by $\Gamma_{HMCA}(K, P_{max,1}, P_{max,2}) = (\mathcal{N}_p, \mathcal{N}_s, \mathcal{P}, R)$, where $\mathcal{N} = \mathcal{N}_p \bigcup \mathcal{N}_s = \{1, 2, ..., N\}$ denotes the set of players (or users), where $\mathcal{N}_p$ is the set of primary users, $\mathcal{N}_s$ is the set of secondary users, and $\mathcal{N}_p \bigcap \mathcal{N}_s = \emptyset$. $\mathcal{P}$ denotes the set of multi-strategy matrices, such that $\sum_{k=1}^{K} p_n(k) \le P_{max,1}$ for all $n \in \mathcal{N}_p$ and $\sum_{k=1}^{K} p_n(k) \le P_{max,2}$ for all $n \in \mathcal{N}_s$. $R : \mathcal{P} \rightarrow \mathbb{R}^N$, given in (9), denotes the payoff (i.e., rate) function.

Again, each user tries to maximize its own expected rate subject to a total transmission probability constraint:

$$\max_{\mathbf{p}_n} \quad R_n \quad \text{s.t.} \quad \sum_{k=1}^{K} p_n(k) \le P_n \ , \tag{25}$$

where $P_n = P_{max,1}$ for all $n \in \mathcal{N}_p$, and $P_n = P_{max,2}$ for all $n \in \mathcal{N}_s$.

Similar to Theorem 2, it can be shown that each user n plays the following strategy:

$$p_n(k) = \begin{cases} 1 - P_n \ , & \text{if } k = 0 \\ P_n \ , & \text{if } k = k^* \\ 0 \ , & \text{otherwise} \end{cases} , \tag{26}$$

where $P_n = P_{max,1}$ for all $n \in \mathcal{N}_p$, and $P_n = P_{max,2}$ for all $n \in \mathcal{N}_s$.

As in Table I, we obtain a best response algorithm, where each user occasionally monitors the channel utilization and updates its strategy by selecting the channel with the maximal achievable rate $r_n(k)$ given the utilization of the channels. A primary user transmits with probability $P_{max,1}$ and the secondary user transmits with probability $P_{max,2}$.

Next, we examine the convergence of the proposed best-response algorithm. In the following we show that $\Gamma_{HMCA}(K, P_{max,1}, P_{max,2})$ is an ordinal potential game, which implies the convergence of the algorithm.

Theorem 7: The non-cooperative hierarchical multi-channel ALOHA (HMCA) game $\Gamma_{HMCA}(K, P_{max,1}, P_{max,2})$ is an ordinal potential game, with the following bounded ordinal potential function:

$$\phi(\mathbf{P}) = \sum_{n=1}^{N} \sum_{k=1}^{K} \log\left(\frac{1}{1-P_n}\right) \times \left(\log(u_n(k)) - \frac{L(k) + \log\left(\frac{1}{1-P_n}\right)}{2} \right) \mathbf{1}_n(k), \tag{27}$$

where $L(k) = \sum_{n=1}^{N} \log\left(\frac{1}{1-P_n}\right) \mathbf{1}_n(k)$ denotes the load on channel k. $P_n = P_{max,1}$ for all $n \in \mathcal{N}_p$, and $P_n = P_{max,2}$ for all $n \in \mathcal{N}_s$.

Proof: To show that $\phi(\mathbf{P})$ is an ordinal potential function of $\Gamma_{HMCA}(K, P_{max,1}, P_{max,2})$ we need to show that (17) holds. First, we formulate the distributed rate maximization problem (25) in a more convenient form. Note that every user n selects a channel k_n^* such that setting $p_n(k_n^*) = P_n$, where $P_n = P_{max,1}$ for all $n \in \mathcal{N}_p$, and $P_n = P_{max,2}$ for all $n \in \mathcal{N}_s$, maximizes R_n in (25). Using the monotonicity of the logarithm function we have:

$$\begin{aligned} k_n^* &= \arg \max_k \ R_n \\ &= \arg \max_k \ u_n(k) P_n \prod_{i \in \mathcal{N}_k \ , \ i \neq n} (1 - P_i) \\ &= \arg \max_k \ \log(u_n(k)) - L(k) \\ &= \arg \max_k \ \tilde{R}_n \ , \end{aligned} \tag{28}$$

where $\mathcal{N}_k$ is the set of all users that select channel k and

$$\tilde{R}_n \triangleq \log(u_n(k)) - L(k) \ . \tag{29}$$

For each user n consider the following optimization problem:

$$\arg \max_k \ \tilde{R}_n \quad s.t. \quad p_n(k) = P_n \ , \tag{30}$$

where $P_n = P_{max,1}$ for all $n \in \mathcal{N}_p$, and $P_n = P_{max,2}$ for all $n \in \mathcal{N}_s$.

The solution to (30) solves the optimization problem in (25). Note that in the case of equal transmission probability $P_n = P_{max}$ for all n, we have: $L(k) = \log\left(\frac{1}{1-P_{max}}\right) N(k)$. Thus, the modified optimization problem (15) in Proposition 1 is a special case of (30).

We apply the ordinal potential function that was introduced in [40] to $\Gamma_{HMCA}(K, P_{max,1}, P_{max,2})$. Assume that user n_0 selects channel k_1 according to strategy $\mathbf{p}_{n_0}^{(1)}$ and changes its strategy by selecting channel k_2 according to strategy $\mathbf{p}_{n_0}^{(2)}$. As a result, we have:

$$\begin{aligned}
&\mathbf{1}_{n_0}^{(1)}(k_1) = 1 \quad , \quad \mathbf{1}_{n_0}^{(1)}(k_2) = 0 \ , \\
&\mathbf{1}_{n_0}^{(2)}(k_1) = 0 \quad , \quad \mathbf{1}_{n_0}^{(2)}(k_2) = 1 \ , \\
&\mathbf{1}_{n_0}^{(2)}(k) = \mathbf{1}_{n_0}^{(1)}(k) = 0 \ , \ \forall k \neq k_1, k_2 \ , \\
&\mathbf{1}_{n}^{(2)}(k) = \mathbf{1}_{n}^{(1)}(k) \ , \ \forall k \ \ \forall n \neq n_0 \ ,
\end{aligned}$$

and

$$\begin{aligned}
&L^{(2)}(k_2) = L^{(1)}(k_2) + \log\left(\tfrac{1}{1-P_{n_0}}\right) \ , \\
&L^{(2)}(k_1) = L^{(1)}(k_1) - \log\left(\tfrac{1}{1-P_{n_0}}\right) \ , \\
&L^{(2)}(k) = L^{(1)}(k) \ , \ \forall k \neq k_1, k_2 \ .
\end{aligned}$$

Let

$$\tilde{p}_n \triangleq \log\left(\frac{1}{1-P_n}\right),$$

and let

$$\tilde{u}_n(k) \triangleq \log\left(u_n(k)\right).$$

The difference in the payoff function $\Delta\tilde{R}_{n_0}$ is given by:

$$\begin{aligned}
\Delta\tilde{R}_{n_0} &= \tilde{R}_{n_0}(\mathbf{p}_{n_0}^{(2)}, \mathbf{P}_{-n_0}) - \tilde{R}_{n_0}(\mathbf{p}_{n_0}^{(1)}, \mathbf{P}_{-n_0}) \\
&= \left[\tilde{u}_{n_0}(k_2) - L^{(2)}(k_2)\right] - \left[\tilde{u}_{n_0}(k_1) - L^{(1)}(k_1)\right].
\end{aligned}$$

The difference in the proposed function (27) $\Delta\phi$ is given by:

$$\begin{aligned}
\Delta\phi &= \phi(\mathbf{p}_{n_0}^{(2)}, \mathbf{P}_{-n_0}) - \phi(\mathbf{p}_{n_0}^{(1)}, \mathbf{P}_{-n_0}) \\
&= \sum_{n=1}^{N}\sum_{k=1}^{K} \tilde{p}_n \left(\tilde{u}_n(k) - \frac{L^{(2)}(k) + \tilde{p}_n}{2}\right)\mathbf{1}_n^{(2)}(k) \\
&\quad - \sum_{n=1}^{N}\sum_{k=1}^{K} \tilde{p}_n \left(\tilde{u}_n(k) - \frac{L^{(1)}(k) + \tilde{p}_n}{2}\right)\mathbf{1}_n^{(1)}(k) \\
&= \sum_{n=1}^{N}\sum_{k=k_1,k_2} \tilde{p}_n \left(\tilde{u}_n(k) - \frac{L^{(2)}(k) + \tilde{p}_n}{2}\right)\mathbf{1}_n^{(2)}(k) \\
&\quad - \sum_{n=1}^{N}\sum_{k=k_1,k_2} \tilde{p}_n \left(\tilde{u}_n(k) - \frac{L^{(1)}(k) + \tilde{p}_n}{2}\right)\mathbf{1}_n^{(1)}(k) \\
&= \Bigg[\tilde{p}_{n_0}\tilde{u}_{n_0}(k_2) - \sum_{n=1}^{N} \tilde{p}_n \frac{L^{(1)}(k_1) - \tilde{p}_{n_0} + \tilde{p}_n}{2}\mathbf{1}_n^{(2)}(k_1) \\
&\qquad - \sum_{n=1}^{N} \tilde{p}_n \frac{L^{(2)}(k_2) + \tilde{p}_n}{2}\mathbf{1}_n^{(2)}(k_2)\Bigg] \\
&\quad - \Bigg[\tilde{p}_{n_0}\tilde{u}_{n_0}(k_1) - \sum_{n=1}^{N} \tilde{p}_n \frac{L^{(1)}(k_1) + \tilde{p}_n}{2}\mathbf{1}_n^{(1)}(k_1) \\
&\qquad - \sum_{n=1}^{N} \tilde{p}_n \frac{L^{(2)}(k_2) - \tilde{p}_{n_0} + \tilde{p}_n}{2}\mathbf{1}_n^{(1)}(k_2)\Bigg] \\
&= \Bigg[\tilde{p}_{n_0}\tilde{u}_{n_0}(k_2) - \frac{L^{(1)}(k_1)L^{(2)}(k_1)}{2} + \frac{\tilde{p}_{n_0}L^{(2)}(k_1)}{2} \\
&\quad - \sum_{n=1}^{N}\frac{\tilde{p}_n^2}{2}\mathbf{1}_n^{(2)}(k_1) - \frac{\left(L^{(2)}(k_2)\right)^2}{2} - \sum_{n=1}^{N}\frac{\tilde{p}_n^2}{2}\mathbf{1}_n^{(2)}(k_2)\Bigg] \\
&\quad - \Bigg[\tilde{p}_{n_0}\tilde{u}_{n_0}(k_1) - \frac{\left(L^{(1)}(k_1)\right)^2}{2} - \sum_{n=1}^{N}\frac{\tilde{p}_n^2}{2}\mathbf{1}_n^{(1)}(k_1) \\
&\quad - \frac{L^{(2)}(k_2)L^{(1)}(k_2)}{2} + \frac{\tilde{p}_{n_0}L^{(1)}(k_2)}{2} - \sum_{n=1}^{N}\frac{\tilde{p}_n^2}{2}\mathbf{1}_n^{(1)}(k_2)\Bigg]
\end{aligned}$$

Note that:

$$\begin{aligned}
&\sum_{n=1}^{N}\frac{\tilde{p}_n^2}{2}\mathbf{1}_n^{(1)}(k_1) - \sum_{n=1}^{N}\frac{\tilde{p}_n^2}{2}\mathbf{1}_n^{(2)}(k_1) = \frac{\tilde{p}_{n_0}^2}{2} \ , \\
&\sum_{n=1}^{N}\frac{\tilde{p}_n^2}{2}\mathbf{1}_n^{(1)}(k_2) - \sum_{n=1}^{N}\frac{\tilde{p}_n^2}{2}\mathbf{1}_n^{(2)}(k_2) = -\frac{\tilde{p}_{n_0}^2}{2} \ .
\end{aligned}$$

and $L^{(2)}(k_1) = L^{(1)}(k_1) - \tilde{p}_{n_0}$, $L^{(1)}(k_2) = L^{(2)}(k_2) - \tilde{p}_{n_0}$. As a result, we have:

$$\begin{aligned}
\Delta\phi &= \tilde{p}_{n_0}\Big(\left[\tilde{u}_{n_0}(k_2) - L^{(2)}(k_2)\right] \\
&\qquad - \left[\tilde{u}_{n_0}(k_1) - L^{(1)}(k_1)\right]\Big) \\
&= \log\left(\frac{1}{1-P_{n_0}}\right)\Delta\tilde{R}_{n_0}.
\end{aligned}$$

Hence,

$$\begin{aligned}
&\Delta R_{n_0} = R_{n_0}(\mathbf{p}_{n_0}^{(2)}, \mathbf{P}_{-n_0}) - R_{n_0}(\mathbf{p}_{n_0}^{(1)}, \mathbf{P}_{-n_0}) > 0 \\
&\iff \Delta\tilde{R}_{n_0} = \tilde{R}_{n_0}(\mathbf{p}_{n_0}^{(2)}, \mathbf{P}_{-n_0}) - \tilde{R}_{n_0}(\mathbf{p}_{n_0}^{(1)}, \mathbf{P}_{-n_0}) > 0 \\
&\iff \Delta\phi = \phi(\mathbf{p}_{n_0}^{(2)}, \mathbf{P}_{-n_0}) - \phi(\mathbf{p}_{n_0}^{(1)}, \mathbf{P}_{-n_0}) > 0 \ .
\end{aligned}$$

Furthermore, $\phi(\mathbf{P})$ is upper bounded by $\phi(\mathbf{P}) < \sum_{n=1}^{N}\max_k \log\left(\frac{1}{1-P_n}\right)\log\left(u_n(k)\right)$.
Hence, the proposed function $\phi(\mathbf{P})$ (27) is a bounded ordinal potential function of $\Gamma_{HMCA}(K, P_{max,1}, P_{max,2})$ and the theorem follows. Note that in the case of equal transmission probability $P_n = P_{max}$ for all n, Theorem 3 follows by dividing $\phi(\mathbf{P})$ by $\log\left(\frac{1}{1-P_{max}}\right)$. ∎

Corollary 2: Any sequential update dynamics of the hierarchical multi-channel ALOHA (HMCA) game $\Gamma_{HMCA}(K, P_{max,1}, P_{max,2})$ converges to a NEP in finite time, starting from any point. Specifically, the proposed sequential best response algorithm converges to a NEP in finite time.

A. Determining $P_{max,1}$ and $P_{max,2}$

Next, we discuss the choice of $P_{max,1}$ and $P_{max,2}$. Assume again that $P_{max,1} = \alpha_1/N$ and $P_{max,2} = \alpha_2/N$. As explained in the beginning of this section, typically, $P_{max,2}$ has to be sufficiently small to limit interference to the primary users caused by secondary users, such that QoS requirements are satisfied. A natural criterion is to maximize the secondary user expected rates subject to an expected target rate of a primary user. To simplify the analysis, we focus on the case where all users select their best collision-free channels, which is the optimal solution when the number of users N approaches infinity (by a straightforward extension of Theorem 4) and is the initial point of the proposed best-response algorithm. This simplification provides an approximate solution for large N. However, the network controller can dynamically update $P_{max,1}$, $P_{max,2}$ (and inform the users) according to the channels load to improve performance, while satisfying the QoS requirements (using a Stackelberg setup, for instance). We assume again that matrix U, defined in (2), satisfies the general conditions specified in Theorem 4. Let $N_p = |\mathcal{N}_p|$ and

$N_s = |\mathcal{N}_s|$ be the number of primary and secondary users in the network, respectively. The achievable rates of primary and secondary users that select their best collision-free channel k^* are given by:

$$R_n = u_n(k^*)\frac{\alpha_1}{N}\left(1-\frac{\alpha_1}{N}\right)^{\tilde{N}_p(k^*)}\left(1-\frac{\alpha_2}{N}\right)^{N_s(k^*)}, \quad \forall n \in \mathcal{N}_p ,$$

$$R_n = u_n(k^*)\frac{\alpha_2}{N}\left(1-\frac{\alpha_1}{N}\right)^{N_p(k^*)}\left(1-\frac{\alpha_2}{N}\right)^{\tilde{N}_s(k^*)}, \quad \forall n \in \mathcal{N}_s ,$$

where $N_p(k^*) \sim B(N_p, 1/K)$, $\tilde{N}_p(k^*) \sim B(N_p - 1, 1/K)$, $N_s(k^*) \sim B(N_s, 1/K)$, $\tilde{N}_s(k^*) \sim B(N_s - 1, 1/K)$ and $\mathbf{E}\left\{\left(1-\frac{\alpha}{N}\right)^{N(k^*)}\right\} = \mathbf{E}\left\{e^{N(k^*)\log\left(1-\frac{\alpha}{N}\right)}\right\} = M_{N(k^*)}\left(\log\left(1-\frac{\alpha}{N}\right)\right)$, where $M_x(t) = (1-p+pe^t)^N$ is the moment generating function of a Binomial r.v $x \sim B(N,p)$ at t [41].

Therefore, the expected rates are given by:

$$\mathbf{E}\{R_n\} = \mathbf{E}\{u_n(k^*)\}\frac{\alpha_1}{N}\left(1-\frac{\alpha_1}{NK}\right)^{N_p-1}\left(1-\frac{\alpha_2}{NK}\right)^{N_s}, \quad \forall n \in \mathcal{N}_p ,$$

$$\mathbf{E}\{R_n\} = \mathbf{E}\{u_n(k^*)\}\frac{\alpha_2}{N}\left(1-\frac{\alpha_1}{NK}\right)^{N_p}\left(1-\frac{\alpha_2}{NK}\right)^{N_s-1}, \quad \forall n \in \mathcal{N}_s . \qquad (31)$$

Let

$$R_n^* = \max_{\alpha_1}\left\{\mathbf{E}\{u_n(k^*)\}\frac{\alpha_1}{N}\left(1-\frac{\alpha_1}{NK}\right)^{N_p-1}\right\}$$

be the maximal achievable rate of a primary user $n \in \mathcal{N}_p$ when $\alpha_2 = 0$, i.e., when the secondary users are not allowed to transmit. Differentiating with respect to α_1 and equating to zero yields $\alpha_1 = KN/N_p$ and

$$R_n^* = \mathbf{E}\{u_n(k^*)\}\tfrac{K}{N_p}\left(1-\tfrac{1}{N_p}\right)^{N_p-1}.$$

The normalized expected target rate is set to $R_t = \omega R_n^*/\mathbf{E}\{u_n(k^*)\}$, where $0 \le \omega \le 1$. Thus, the primary users tolerate a decline in performance to allow secondary users to use the channels. We need to find α_1, α_2 that maximize the secondary user expected rate such that the primary user achieves at least the expected target rate. Thus, we solve the following optimization problem:

$$\begin{aligned} \max_{\alpha_1,\alpha_2} \quad & \frac{\alpha_2}{N}\left(1-\frac{\alpha_1}{NK}\right)^{N_p}\left(1-\frac{\alpha_2}{NK}\right)^{N_s-1} \\ \text{s.t.} \quad & \frac{\alpha_1}{N}\left(1-\frac{\alpha_1}{NK}\right)^{N_p-1}\left(1-\frac{\alpha_2}{NK}\right)^{N_s} \ge R_t . \end{aligned} \qquad (32)$$

The optimization problem in (32) is log-concave and can be simply solved by the network controller over only two variables α_1, α_2. Complexity does not depend on the number of channels K or the number of users N. Note that for the optimal solution, the rate is achieved on the boundary of the rate constraint, since reducing α_1 increases the objective function. In addition, the primary users achieve higher rates by the proposed best-response algorithm. Therefore, the network controller can further reduce ω dynamically according to the channel load to improve performance while satisfying the QoS requirements.

Remark 4: In this section we have discussed the case where secondary users are allowed to interfere with primary users, as long as QoS requirements are satisfied. Another possible scenario is when secondary users are not allowed to interfere with primary users. In this scenario, an extension of our model can be made for networks, where the primary users implement the multi-channel ALOHA protocol while the secondary users use CSMA to avoid interference to primary users. Such a model for a single-channel scenario was introduced in [23], and can be extended to multi-channel systems. Analysis of this extension is however beyond the scope of this paper.

VI. Simulation Results

In this section we provide numerical examples to illustrate the algorithm's performance. Here, we focus on the constrained rate maximization. We simulated a network with the following parameters unless otherwise specified: the number of channels was set to $K = 10$. The channels were distributed according to a Rayleigh fading distribution, i.i.d across users and channels. The bandwidth W of each channel was set to 10MHz. The entries of the collision-free rate matrix U were $u_n(k) = W\log(1 + \text{SNR})$Mbps. We set $P_{max} = K/N$. We compared three algorithms: a random access algorithm, where each user selects a channel randomly, a totally greedy algorithm, in the sense that each user transmits over the channel that maximizes its collision-free rate $u_n(k)$ without considering the channel utilization, and the proposed best response algorithm given in Table I. We initialized the proposed algorithm by the totally greedy algorithm solution, as described in Table I.

In Fig. 1 we present the average rates achieved by the proposed best response algorithm as a function of P_{max} for $N = 30$ and SNR= 20dB. It can be seen that setting $P_{max} = K/N = 1/3$ maximizes the rate for small N, as well.

In the following simulation results we present the ratio to the rate of a random access scheme. In Fig. 2 we present the rate gains of the proposed best response and the totally greedy algorithms over the random access scheme as a function of the number of users for SNR= 0dB and SNR= 20dB. We point out that the rate variance of the proposed algorithm is much lower than the rate variance of the totally greedy algorithm. It can be seen that the proposed best response algorithm significantly outperforms the totally greedy algorithm for small N. However, it approaches the totally greedy algorithm as N increases, as discussed in Section IV-A. Furthermore, the gain over the random access algorithm decreases with SNR. However, the performance gain of the best response algorithm over the totally greedy algorithm increases with SNR. In Fig. 3 we present the convergence of the proposed best response algorithm to the average gain over random access for $N = 30$ and SNR= 20dB. It can be seen that convergence is achieved in less than 12 iterations for almost all realizations.

In Table II we compare the algorithms' performance with the optimal centralized performance for $N = 10$, $K = 3$ and

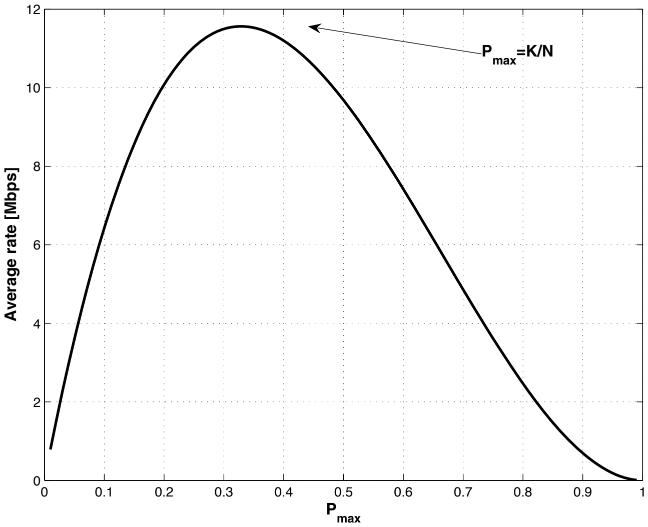

Fig. 1. Average rate achieved by the proposed best response algorithm as a function of P_{max}. Simulation parameters: $N = 30$, SNR=20dB.

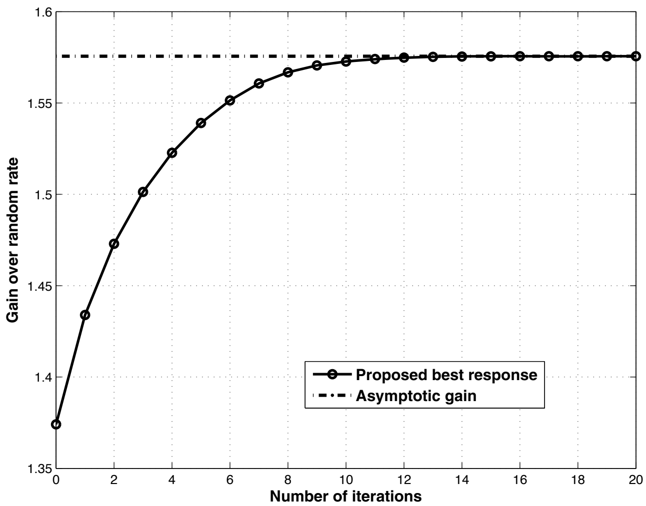

Fig. 3. Convergence of the proposed best response algorithm. Simulation parameters: $N = 30$, SNR=20dB.

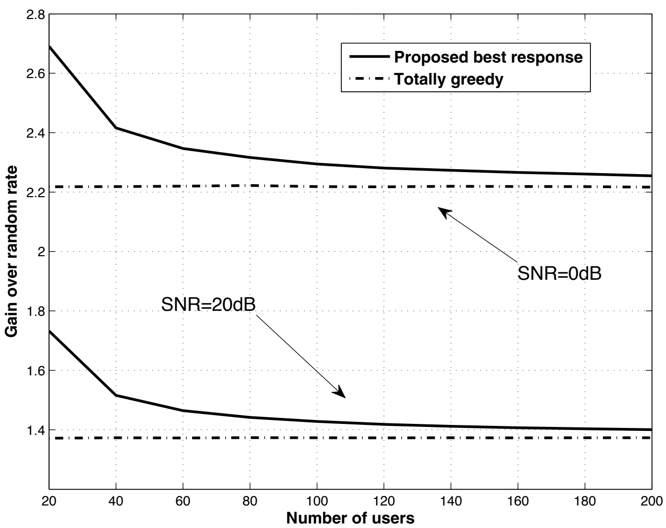

Fig. 2. Rate gain of the proposed best response and the totally greedy algorithms over the random access scheme as a function of the number of users.

TABLE II
PERFORMANCE COMPARISON FOR $N = 10$, $K = 3$ AND SNR= $20dB$

	Rate gain over random access
Centralized solution	1.36
Proposed algorithm	1.33
Totally greedy	1.23

SNR= 20dB. The rate gains of the proposed best response, the totally greedy algorithms and the optimal centralized solution over the random access scheme are presented. It can be seen that the average rate achieved by the proposed best response algorithm outperforms the average rate achieved by the other distributed algorithms and almost achieves the average rate achieved by the optimal centralized solution.

Finally, we consider the case where 15 primary users and 15 secondary users co-exist in the same frequency band. In Table III we compare the algorithms performance for SNR= 20dB. We set the target rate to $0.8R_n^* = 16.4$Mbps; i.e., $\omega = 0.8$, where R_n^* is the maximal achievable rate of primary users that select their best collision-free channel when secondary users are not allowed to transmit. As discussed in Section V-A, the solution to the optimization problem in (32) is accurate for the totally greedy algorithm and is a good approximation for the proposed best response algorithm for sufficiently large N. For the proposed best response algorithm, the primary users achieved 21.7Mbps with $\omega = 0.8$. Thus, we further reduce ω to increase the secondary user rates while still satisfying the primary user rate constraint (practically, this can be done dynamically by the system controller). It can be seen that for expected target rate of 16.4Mbps for the primary users, the proposed best response algorithm achieves roughly a 84% relative performance gain over the totally greedy algorithm in terms of the achievable expected rate of the secondary users.

VII. CONCLUSION

In this paper we investigated the problem of distributed rate maximization of networks applying the multi-channel ALOHA random access protocol. We characterized the NEPs of the network when users solve the unconstrained rate maximization. In this case, for any NEP, we obtained that each user tries to access a single channel with probability 1 and does not try to access other channels. Next, we limited each user's total access probability and solved the problem under a total probability constraint, to overcome the problem of collisions when the number of users is much larger than the number of channels. We characterized the NEPs when user rates are subject to a total transmission probability constraint. We proposed a simple best-response algorithm that solves the constrained rate maximization, where each user updates its strategy using its

TABLE III
PERFORMANCE COMPARISON FOR $N = 30$ AND SNR$= 20dB$

	Proposed algorithm	Totally greedy
Expected rate of primary users [Mbps]	16.4	16.4
Expected rate of secondary users [Mbps]	7.15	3.88
Average number of iterations	7.4	1

local CSI and by monitoring channel utilization. We used the theory of potential games to prove convergence of the proposed algorithm. We formulated the problem of choosing the access probability as a leader-followers Stackelberg game, where a single cognitive user is chosen to be the leader to manage the network. A fully distributed setup has been applied to approximately optimize the network throughput for a large number of users. We would like to point out that extending the Stackelberg model to the case of multiple-leaders multiple-followers may be examined in other network models that are not discussed in this paper (for instance, in networks containing selfish groups of users, where each group of users follows its own leader). However, this extension is beyond the scope of this paper.

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Kobi Cohen received the B.Sc.(cum laude) in electrical engineering from Bar-Ilan University, Ramat Gan, Israel, in 2007. He is currently working towards the Ph.D. degree in the School of Engineering, Bar-Ilan University, Ramat Gan, Israel.

His main research interests include resource-constrained signal processing techniques, communication protocols, and dynamic spectrum management for wireless and wireline networks.

Amir Leshem (SM'06) received the B.Sc.(cum laude) in mathematics and physics, the M.Sc. (cum laude) in mathematics, and the Ph.D. in mathematics, all from the Hebrew University, Jerusalem, Israel, in 1986, 1990, and 1998, respectively. From 1998 to 2000, he was with the faculty of Information Technology and Systems, Delft University of Technology, The Netherlands, as a postdoctoral fellow working on algorithms for the reduction of terrestrial electromagnetic interference in radio-astronomical radio-telescope antenna arrays and signal processing for communication. From 2000 to 2003, he was director of advanced technologies with Metalink Broadband, where he was responsible for research and development of new DSL and wireless MIMO modem technologies and served as a member of ITU-T SG15, ETSI TM06, NIPP-NAI, IEEE 802.3, and 802.11. From 2000 to 2002, he was also a visiting researcher at Delft University of Technology. He is one of the founders of the new school of electrical and computer engineering at Bar-Ilan University, where he is currently a Professor and Head of the signal processing track. From 2003 to 2005, he was also the technical manager of the U-BROAD consortium developing technologies to provide 100 Mbps and beyond over DSL lines. He was the leading guest editor for a special issue of the IEEE JOURNAL ON SELECTED TOPICS IN SIGNAL PROCESSING, dedicated to signal processing for space research and for a special issue of *IEEE Signal Processing Magazine*, dedicated to signal processing in astronomy and cosmology. Since 2008, he has been an associate editor for the IEEE TRANSACTIONS ON SIGNAL PROCESSING.

His main research interests include multichannel wireless and wireline communication, applications of game theory to dynamic and adaptive spectrum management of communication and sensor networks, array and statistical signal processing with applications to multiple element sensor arrays and networks in radio-astronomy, brain research, wireless communications and radio-astronomical imaging, set theory, and the logic and foundations of mathematics.

Ephraim Zehavi received his B.Sc. and M.Sc. degrees in electrical engineering from the Technion-Israel Institute of Technology in Haifa, in 1977 and 1981, respectively, and his Ph.D. degree in electrical engineering from the University of Massachusetts, Amherst, in 1986.

In 1985, he joined Qualcomm Inc. in San Diego, California. From 1988 to 1992, he was a faculty member at the Department of Electrical Engineering, Technion-Israel Institute of Technology. In 1992, he rejoined Qualcomm Inc. as a principal engineer. Upon his return to Israel in 1994, he became an assistant general manager of engineering for Qualcomm Israel, Ltd., and later became the general manager. In 1999, he co-founded Mobilian Inc. and served as CTO until 2002. In 2003, he joined the University of Bar Ilan faculty, where he is now Vice Dean of the Faculty of Engineering and head of the communication track.

He is the co-recipient of the 1994 IEEE Stephen O. Rice Award and is named as an IEEE fellow for his contribution to pragmatic coding and bit interleaving; he also holds more than 40 patents in the area of coding and wireless communication. His main research interests include wireless communications, coding technology, and applications of game theory to communication.