PMU-based Online Change-Point Detection of Imbalance in Three-Phase Power Systems

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Abstract—Voltage imbalances are a costly and potentially damaging phenomenon. In this paper, the problem of online change-point detection of voltage imbalance in a three-phase power system using phasor measurement unit (PMU) data is considered within a sequential hypothesis-testing framework. A general model for the positive-sequence time-domain data from a PMU measurement at off-nominal frequencies is presented. The new formulation enables fast online detection of imbalance. Closed-form expressions of the cumulative sum (CUSUM) and GLR tests are developed for detection of imbalances. The performance of the change-point detection procedures is evaluated using the average-run-length and the expected detection delay. Numerical simulations show that the proposed method can be used for enhanced situational awareness in future grid management systems and demonstrate the ability to inform strategies for advancing grid capabilities by using change-point detection methods.

Index Terms—Phasor measurement unit (PMU), power system monitoring, unbalanced power system, state estimation, online change-point detection

I. INTRODUCTION

The three-phase power system is designed to operate in balanced scenarios [1]. However, in practice, imbalances happen frequently [2]. Imbalances may be a precursor to more serious contingencies leading to possible blackouts [3, 4]. In addition, substantial power imbalance causes excessive losses, overheating, insulation degradation, a reduced lifespan of motors and transformers, and interruptions in production processes [5-8]. Thus, the ability to detect quickly potentially harmful levels of imbalance in various power systems is highly desirable for the benefit of both the utility and customer [4, 6]. To this end, effective algorithms and sophisticated methods are crucial for detecting an abnormal level of imbalance in real time and evaluating the associated effects. It is in this context that modern sensing devices, such as phasor measurement units (PMUs), have the potential to provide rapid detection of contingencies and situational awareness [1].

A. Summary of results

This paper focuses on online detecting a change in the imbalance condition of a three-phase power system. The main contributions of this paper are threefold. First, we express the power system imbalance change-detection as an online hypothesis testing problem based on time-domain samples. The advantage of using time-domain signals is that the PMU constructs the output frequency-domain signals by using a discrete Fourier transform (DFT) operator, which may cause time delay. Second, we apply a cumulative sum (CUSUM) test to solve our problem and a GLR procedure that can be used when the system parameters are unknown. The proposed CUSUM and GLR procedures are well-known in statistics and are specifically tailored for the smart grid structure. Finally, using simulation results for a variety of imbalance conditions, we examine the detection performance of the proposed approaches. In particular, we demonstrate that the performance of the proposed change-point detection procedures are evaluated using the average-run-length (ARL) and the expected detection delay (EDD).

B. Related works

When system imbalance occurs, the PMU’s output exhibits nonstationary frequency deviations [4, 9] and the positive-sequence measurements become non-circular [10, 11]. The main performance measure of imbalances in power systems is the voltage unbalance factor (VUF) [4, 12, 13]. Recently, new parametric imbalance detection methods that outperform the classical parametric methods in terms of probability of error have been proposed. The new methods are based on the generalized likelihood ratio (GLR) test [14, 15] and generalized locally most powerful test (GLMP) [16, 17] approaches. All these methods are performed offline and are based on two- or three-phase data that are not available at the control center. In contrast, the proposed parametric change-detection method is an online method, which employs single-phase data.

In the last decade, modern optimization and statistical methodologies have been shown to be powerful tools in power system problems (see, e.g., [10, 11, 14, 18-21]). In this context, change-point detection, which deals with the detection of an emerging abrupt change point in a time series, seems a promising tool. It is well known that, under some conditions, the optimal change-point detection procedure is the CUSUM procedure (e.g., [22]). However, in the case where there are unknown system parameters, the CUSUM method should be replaced by methods that are also based on a state estimation stage. In the context of power systems, a non-parametric online identification method of the level, location, and effects of voltage imbalance in a distribution network is derived in [6]. Derivation of parametric change-detection methods is expected to improve the detection performance.
II. MEASUREMENT TIME-DOMAIN MODEL

The voltages in a three-phase power system are assumed to be pure sinusoidal signals of frequency $\omega_0 + \Delta$, where $\omega_0$ is the known nominal frequency (100π or 120π) and $\Delta$ is the frequency deviation from this nominal value. The magnitudes and phases of the three voltages are denoted by $V_a$, $V_b$, $V_c \geq 0$ and $\varphi_a, \varphi_b, \varphi_c \in [0, 2\pi]$, respectively. The three-phase power system is balanced or symmetrical if $V_a = V_b = V_c$ and $\varphi_a = \varphi_b + 2\pi/3 = \varphi_c - 2\pi/3$. The PMU samples these real signals $N$ times per cycle of the nominal frequency, $\omega_0$, to produce the following discrete-time, noisy measurement model (e.g., [1], pp. 51-52):

$$x[n] = \frac{1}{2}e^{j\omega_0 n}v + \frac{1}{2}e^{-j\omega_0 n}v^* + w[n],$$

for all $n \in \mathbb{Z}$, where $\gamma \triangleq \frac{2\pi}{\omega_0}$ and $v \triangleq [V_a e^{j\varphi_a}, V_b e^{j\varphi_b}, V_c e^{j\varphi_c}]^T$. The noise sequence, $\{w[n]\}_{n \in \mathbb{R}}$, is assumed to be a real white Gaussian noise sequence with a known covariance matrix $\sigma^2 I_3$. We assume that the sequence is given by:

$$x[0], x[1], \ldots, x[\tau - 1], x[\tau], \ldots, x[t],$$

where $1 \leq \tau \leq t$ denotes the change-point location. That is, we assume that the vectors $x[0], \ldots, x[\tau - 1]$ are obtained from a balanced system, while $x[\tau], \ldots, x[t]$ are unbalanced vectors. That is, in the model from (1) the voltage vector $v$ is given by $v_0$ and $v_{\Delta 0}$ before and after the change, respectively.

The positive voltage sequence, i.e., the “space vector” [23], is calculated from three-phase voltages by using the symmetrical component transformation (see, e.g., [1] pp. 63-67):

$$v_+[n] = g^T x[n], \quad n = 0, \ldots, t,$$

where $g \triangleq \frac{1}{3} [1, \alpha, \alpha^2]^T$ and $\alpha = e^{j2\pi/3}$. For the special case of a perfectly balanced system, the three-phase voltages satisfy $V_a = V_b = V_c$ and $\varphi_a = \varphi_b + 2\pi/3 = \varphi_c - 2\pi/3$. Therefore, it can be verified that for this case, i.e., before the change, $g^T v = 0$. By using this result and substituting the model from (1) in (3), one obtains that before the change:

$$v_+[n] = \frac{1}{2} e^{j\omega_0 n} C_+^v + \mu_+[n],$$

where $\forall n = 0, \ldots, \tau - 1$, and after the change

$$v_+[n] = \frac{1}{2} e^{j\omega_0 n} C_+^v + \frac{1}{2} e^{-j\omega_0 n} (C_0^v)^* + \mu_+[n],$$

where $\forall n = \tau, \ldots, t$, where $C_+^v \triangleq g^T v$, $C_0^v \triangleq g^T v^*$, and $C_0^v \triangleq g^T v$. In addition, the noise sequence, $\mu_+[n] \triangleq g^T w[n]$, $n = 0, 1, \ldots, t$, is time-independent sequence and satisfies

$$\mu_+[n] \sim NC\left(0, \frac{\sigma^2}{3}\right).$$

III. ONLINE CHANGE-POINT DETECTION FORMULATION

A. The hypothesis-testing problem

The objective of this study is to develop a method for change-point detection based on the PMU output of the positive sequence components. Our goal is to detect the change point as soon as possible after it occurs. For instance, for the positive sequence, which is shown in Fig. 1, single-phase imbalance occurs at time $n = 50$, and with the parameters: $C_+^v = 0.3709 + 0.3353j$, $C_0^v = 0.8654 + 0.7825j$, and $C_0^v = 0.1400 - 0.6518j$, $N = 48$, $\omega_0 = 60$ Hz, and $\Delta = 1$ Hz. The goal is to use the PMU voltage positive-sequence measurements to locate this change point. It can be seen that this task becomes easier as the signal-to-noise ratio (SNR) increases.

The change-point detection problem can be mathematically formulated as the following composite sequential hypothesis testing problem:

$$\begin{cases} H_0: \text{The system is balanced till (at least) } n = t \\ H_1: \text{There is a change point at time } \tau \end{cases}, \quad (7)$$

where the measurement model is according to (4) and (5). Hypothesis $H_0$ represents the scenario in which there is no change, i.e., $\tau \to \infty$, and the system is balanced at any time. The change-point detection problem comprises estimation of the change time $\tau$ from the measured sequences, $v_+[0], \ldots, v_+[t]$. This is a well-studied problem in statistical signal processing and the CUSUM procedure is a popular algorithm in the literature that has some optimality properties for known system parameters [22].

B. CUSUM approach

The desired stopping rules are determined by likelihood ratios, as described in the following. By using (4) and (5), as well as the noise statistics from (6) and the hypothesis testing from (7), the log-likelihood ratio of hypothesis $H_1$ versus $H_0$ for a given change-point time, $1 \leq \tau \leq t$, is given by:

$$L_\tau(\tau, C_+^v, C_0^v, C_0^v) \triangleq \log f(\nu_+; H_1) - \log f(\nu_+; H_0)
\quad = Q_0(C_+^v) - Q_1(\tau, C_+^v, C_0^v, C_0^v), \quad (8)$$

Fig. 1. Positive-sequence measurement of balanced system and unbalanced system before and after the change, respectively, where the change point is at $n = 50$. 


where \( f(\nu_+; \mathcal{H}_i) \) is the pdf of \( \nu_+ \) under hypothesis \( \mathcal{H}_i \),
\[
Q_0 \left( C_+^v \right) \triangleq \frac{3}{2\sigma^2} \sum_{n=0}^{t} \left| v_+[n] - \frac{1}{2} e^{j\gamma_{\nu_+} n} C_+^v \right|^2,
\]
and
\[
Q_1 \left( \tau, C_+^v, C_+^e, C_-^e \right) \triangleq \frac{3}{2\sigma^2} \times \left( \sum_{n=0}^{\tau-1} \left| v_+[n] - \frac{1}{2} e^{j\gamma_{\nu_+} n} C_+^v \right|^2 + \sum_{n=\tau}^{t} \left| v_+[n] - \frac{1}{2} e^{j\gamma_{\nu_+} n} C_+^v - \frac{1}{2} e^{-j\gamma_{\nu_+} n} C_-^e \right|^2 \right). \tag{10}
\]

By substituting (9) and (10) in (8), one obtains the decision function:
\[
\mathcal{L}_t(\tau, C_+^v, C_+^e, C_-^e) = S[t] - S[\tau - 1], \tag{11}
\]
where the cumulative sum from 0 to \( k \) is defined as:
\[
S[k] \triangleq \sum_{n=0}^{k} l[n], \quad k \geq 0 \tag{12}
\]
and
\[
l[n] \triangleq \frac{1}{4} \left( |C_+^v|^2 - |C_+^e|^2 - |C_-^e|^2 \right) - \text{Real} \left\{ (C_+^v - C_+^e) e^{j\gamma_{\nu_+} \omega_0} v_+^*[n] \right\} + \text{Real} \left\{ C_+^e e^{j\gamma_{\nu_+} \omega_0} v_+^*[n] \right\} - \frac{1}{2} \text{Real} \left\{ C_+^e C_-^e e^{2j\gamma_{\nu_+} \omega_0} \right\}, \forall n \geq 0. \tag{13}
\]

Denote by \( \hat{\tau}_C \), the time at which the CUSUM algorithm declares an imbalance condition. Then, under the assumption that the parameters \( C_+^v, C_+^e \) and \( C_-^e \) are known, \( \hat{\tau}_C \) is given by
\[
\hat{\tau}_C = \arg \max_{1 \leq \tau \leq t} \mathcal{L}_t(\tau, C_+^v, C_+^e, C_-^e), \tag{14}
\]
such that \( \mathcal{L}_t(\tau, C_+^v, C_+^e, C_-^e) > \eta \), where \( \eta > 0 \) is a threshold set by the user. By substituting (11) in (14), we obtain that the change-time estimator from (14) can be rewritten as
\[
\hat{\tau}_C = \arg \min_{1 \leq \tau \leq t} S[\tau - 1], \tag{15}
\]
such that \( G[t] > \eta \), where
\[
G[t] \triangleq S[t] - \min_{1 \leq \tau \leq t} S[\tau - 1], \quad \forall t > 0. \tag{16}
\]
It is shown in [24] that (16) can be rewritten as
\[
G[t] = \{ G[t-1] + l[t] \}^+, \tag{17}
\]
where \( \{ z \}^+ = \max\{z, 0\} \) and \( l[t] \) is defined in (13).

The idea is that before any imbalance scenario occurs, the mean of the log-likelihood ratio from (8) is negative. As a result, \( G[t] \) would remain close to or at 0 prior to the outage. On the other hand, when the system becomes unbalanced, the mean of the log-likelihood ratio from (8) is positive. As a result, \( G[t] \) increases this change. Hence, the CUSUM algorithm declares the occurrence of an imbalance situation at the first time that \( G[t] \) reaches the pre-determined threshold, \( \eta \). The algorithm is summarized in Algorithm 1, below.

**Algorithm 1: CUSUM for imbalance change detection**

**Input:** Positive sequence observations \( v_+^*[n], n = 0, \ldots, t \), phasors \( C_+^v, C_+^e, C_-^e \), and the nominal frequency \( \omega_0 \).

**Output:** Change-point decision

**Initialization:**
- Set the detection threshold \( \eta > 0 \)
- \( S[-1] = G[-1] = 0 \)
- Fix \( n = 0 \)

**for** \( n = 0, \ldots, t \) **do**

**while** the algorithm is not stopped **do**

1. Set \( l[n] \) according to (13) and by using \( v_+^*[n] \)
2. Set \( S[n] = S[n-1] + l[n] \)
3. Evaluate the function \( G[n] = \{ G[n-1] + l[n] \}^+ \)
4. **if** \( G[n] > \eta \) **then**
   - The change time estimator from (15) is given by
   \[
   \hat{\tau}_C = \arg \min_{1 \leq \tau \leq n} S[\tau - 1]
   \]
   - Stop or reset the algorithm

**end if**

**end**

**end**

**end**

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**IV. GLR TEST**

In practice, the phasors \( C_+^v, C_+^e \) and \( C_-^e \) are unknown. Therefore, the CUSUM algorithm cannot be implemented since the likelihood function depends on the unknown parameters. In this context, we can apply the GLR approach. In this approach, we compute the LR statistics combined with maximization w.r.t. the unknown parameters. In order to implement the sequential GLR test, first we derive the ML state estimators under each hypothesis in Section IV-A and then we develop the corresponding GLR procedure based on these estimators in Section IV-B.

**A. State estimation**

Let us define
\[
z_{t_1, t_2} \triangleq \sum_{n=t_1}^{t_2} e^{-j\gamma_{\nu_+} \omega_0} v_+^*[n], \tag{18}
\]
\[
y_{t_1, t_2} \triangleq 2 \sum_{n=t_1}^{t_2} e^{-j\gamma_{\nu_+} \omega_0} v_+^*[n], \tag{19}
\]
\[
\Psi_{\tau} \triangleq \sum_{n=\tau}^{t_2} e^{2j\gamma_{\nu_+} \omega_0}, \quad \forall 1 \leq \tau \leq t, \tag{20}
\]
for any \( t_1, t_2 \in [0, t] \). According to the hypothesis problem in (7), the ML estimator of \( C_+^v \) maximizes the likelihood function.
under hypothesis $\mathcal{H}_0$, i.e., minimizes $Q_0(\hat{C}_+^n)$ from (9). By
equating the derivative of (9) w.r.t. $\hat{C}_+^n$ to zero, one obtains:

$$\hat{C}_+^{\tau_0} = \frac{z_0}{t + 1}$$ \tag{21}

The notation $\hat{C}_+^{\tau_0}$, $i = 0, 1$ denotes the fact that this is
the estimator of $C_+^\tau$ under hypothesis $\mathcal{H}_i$. Similarly, the
ML estimators of $C_{+,i}^0$, $C_{+,i}^{\tau}$, and $C_{+,i}^\tau$ maximize the likelihood
function under hypothesis $\mathcal{H}_i$, and are obtained by equating the
derivative of (10) w.r.t. $C_{+,i}^\tau$ to zero:

$$\hat{C}_{+,i}^{\tau_0} = \frac{z_{\tau_0}}{t + 1 - \tau}$$ \tag{22}

$$\hat{C}_{+,i}^{\tau_1} = \frac{z_{\tau_1}}{t + 1 - \tau} - \frac{(\hat{C}_{+,i}^{\tau})^* \Psi_{\tau}^{*}}{t + 1 - \tau}$$ \tag{23}

and

$$\hat{C}_{-,i}^{\tau_0} = \frac{y_{\tau_0}}{t + 1 - \tau} - \frac{(\hat{C}_{-,i}^{\tau})^* \Psi_{\tau}^{*}}{t + 1 - \tau}$$ \tag{24}

for any $1 \leq \tau \leq t$. Equations (23) and (24) imply that

$$\hat{C}_{+,i}^{\tau_1} = \frac{(t + 1 - \tau)z_{\tau_1} - y_{\tau_1}^* \Psi_{\tau}^{*}}{(t + 1 - \tau)^2 - |\Psi_{\tau}^{*}|^2}$$ \tag{25}

and

$$\hat{C}_{-,i}^{\tau_1} = \frac{(t + 1 - \tau)y_{\tau_1} - z_{\tau_1}^* \Psi_{\tau}^{*}}{(t + 1 - \tau)^2 - |\Psi_{\tau}^{*}|^2},$$ \tag{26}

for any $1 \leq \tau \leq t$. It can be seen that, as expected, for
$
\tau = t + 1$, i.e., there is no change, $\hat{C}_{+,i}^{\tau_1} = \hat{C}_{-,i}^{\tau_1} = 0$ and
$\hat{C}_{+,i}^{\tau_0} = \hat{C}_{+,i}^{\tau_0}$.

**B. GLR method**

In this section, the CUSUM method from Algorithm 1 is replaced
with the GLR method, in which the unknown parameters in the CUSUM
terms are replaced by their ML estimators. For this case, the relevant statistic
from (8) is replaced by the following GLR formulation:

$$\hat{\tau}_{GLR} = \arg \max_{1 \leq \tau \leq t} \hat{E}_t(\tau, \hat{C}_{+,i}^{0,\tau}, \hat{C}_{+,i}^{1,\tau}, \hat{C}_{-,i}^{0,\tau}, \hat{C}_{-,i}^{1,\tau}),$$ \tag{30}

where the r.h.s. should be higher than the threshold $\eta$. Similar
to the recursive CUSUM formulation in (15), the recursive
version of the GLR procedure can be written as

$$\hat{\tau}_{GLR} = \min \{ t : \max_{1 \leq \tau < t} \hat{E}_t(\tau, \hat{C}_{+,i}^{0,\tau}, \hat{C}_{+,i}^{1,\tau}, \hat{C}_{-,i}^{0,\tau}, \hat{C}_{-,i}^{1,\tau}) > \eta \}.$$ \tag{31}

It can be seen that the change-time estimator, $\hat{\tau}_{GLR}$, is a
function of the amplitude of the estimator of the negative
phasor, $\hat{C}_{-,i}^{1,\tau}$, and of the difference between the positive
sequence before and after the change, $\hat{C}_{+,i}^{0,\tau} - \hat{C}_{-,i}^{1,\tau}$. That is,
the GLR detector in (30) can be interpreted as a detector of the
presence of a negative-sequence phasor and the presence of
a change in the positive-sequence phasor, which fits a typical
situation in which the system becomes unbalanced. However,
recursive implementation of a GLR test in a CUSUM way is
intractable in the presence of unknown parameters. Therefore,
an altered window-limited version of the GLR test from (30)
has to be used in practical real-time settings.

**V. SIMULATIONS**

In this section, the performance of the proposed real-time
unbalanced system detection using the CUSUM algorithm in
Algorithm 1 and the GLR rule from (30) is evaluated via
10,000 Monte-Carlo simulations. We consider a single PMU
and a sampling rate of $N = 48$ time-domain samples per
hour, with a nominal frequency, $\omega_0 = 60$ Hz. We assume
a frequency deviation of $\Delta = 1$ Hz, and a variety of values
of noise variance $\sigma^2$. For each scenario, the authorized level
of imbalances $\delta$ is set equal to 0.04. The SNR is defined as

$$\text{SNR} = \frac{3 \hat{C}_{+,i}^2}{2 \sigma^2}.$$

To analyze the detection performance of the proposed
methods, we consider a single-phase amplitude and/or phase
imbalance. That is, the three voltage phases, named $a$, $b$, and
c satisfy $\phi_a = 0.234\pi$, $\phi_b = \phi_a - \frac{2\pi}{3}$ and $\phi_c = \phi_a + \frac{2\pi}{3}$, with
voltage amplitudes fixed at 1 p.u. To introduce imbalance, the $c$
phasor is multiplied by the complex number $\beta e^{j\phi}$, where $\beta$
and $\epsilon$ are real numbers. Values of $\beta$ that are different from 1
cause magnitude imbalance while non-zero values of $\epsilon$
cause phase imbalance. A single-phase voltage magnitude and
angle imbalance is implemented by setting $\beta > 1$ and $|\epsilon| > 0$.

Two standard performance metrics used to characterize
the performance of a change detection algorithms are: the
expected duration in between two false alarms, called the
ARL, and the EDD, which is the expected time to stop in
the extreme case when the change occurs immediately at
$\tau = 0$. It is desired that the ARL quantity will be as small as
possible to minimize the reaction time of the change-detection
method. In general, a tradeoff occurs between high accuracy
and minimum delay.

In Figs. 2 and 3 the ARL ($k = 25$) and the EDD are
presented, respectively, versus SNR, where $0 < \text{SNR} \leq \frac{3 \hat{C}_{+,i}^2}{2 \sigma^2}$. Fig.
2 shows that, in terms of ARL for a change that happens at
for the LR, GLR, LMP, and GLMP tests.

$k = 25$, the GLR achieves the CUSUM performance for high SNR, while for low SNR it has a higher ARL. Fig. 3 shows that the EDD decreases as the SNR increases. These figures demonstrate that the performance of the CUSUM procedure, which assumes perfect knowledge of the unknown parameters, can be used as a lower bound to the ARL of the GLR decision rules that are based on the estimation of the unknown parameters.

ACKNOWLEDGMENT

The work of T. Routtenberg was partially supported by the ISRAEL SCIENCE FOUNDATION (ISF), grant No. 1173/16. The work of Y. Xie is partially supported by NSF CCF-1442635 and CMMI-1538746.

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