Two-dimensional relativistic longitudinal Green’s function in the presence of a moving planar dielectric–magnetic discontinuity

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The current contribution is concerned with obtaining the relativistic two-dimensional (three-dimensional in relativity jargon) Green’s function of a time-harmonic line current that is embedded in a moving dielectric–magnetic medium with a planar discontinuity. By applying a plane-wave (PW) spectral representation for the relativistic electromagnetic Green’s function of a dielectric–magnetic medium that is moving in a uniform velocity, the exact reflected and transmitted (refracted) fields are obtained in the form of a spectral integral over PWs in the so-called laboratory and comoving frames. We investigate these spectral representations, as well as their asymptotic evaluations, and discuss the associated relativistic wave phenomena of direct reflected/transmitted rays and relativistic head waves (lateral waves).

1. INTRODUCTION AND STATEMENT OF THE PROBLEM

Green’s functions of electromagnetic (EM) wave propagation in a moving medium have been a subject of a continuing research [1–4]. The dyadic Green’s functions of an homogeneous isotropic dielectric–magnetic medium have been derived in closed form in [5] and an alternative simple derivation was presented in [6].

Plane-wave (PW) spectral representations have been an important tool for solving relativistic scattering and diffraction problems. Such spectral representations have been utilized for scattering of a PW by a uniformly moving perfectly conducting half-plane [7], for the two-dimensional problem of the uniformly moving perfectly conducting cylinder [8], for solving the EM field radiated by an infinitely long thin wire antenna that uniformly translates in a direction parallel to a plane interface [9], for the EM radiation from fractal antennas [10], and for EM wave scattering from moving rough surfaces [11].

EM scattering from moving objects is of fundamental importance in antenna and scattering theory, cellular and satellite communication, radar applications, inverse scattering, remote sensing, and more. Although impressive advances have been made in this field, little effort has been made to adjust basic stationary wave theories [such as the geometrical theory of diffraction (GTD), the uniform theory of diffraction, and direct time-domain methods] to the scatterer dynamics. Basic wave phenomena of relativistic scattering, such as high-frequency/short-pulsed canonical forms and diffraction coefficients, are not yet fully understood. Furthermore, little attention has been given to problems concerning scattering in a moving dielectric medium where the electrodynamics introduces wave phenomena that are unique to special relativity and that are not found in classical (stationary) GTD, such as relativistic lateral waves and their wave phenomena, Čerenkov-type scattering and its high-frequency forms, and more.

Recently, a PW spectral representation of the relativistic electric and magnetic dyadic Green’s functions of a uniformly moving dielectric–magnetic medium was obtained in [12]. By applying a simple coordinate transformation, scalarization of the EM vectorial problem was obtained in which the EM dyads are evaluated from Helmholtz’s free-space (isotropic) scalar Green’s function. The spectral PW representations of the dyadic Green’s functions were obtained by applying the spatial two-dimensional (2D) Fourier transform to the scalar Green’s function in both the under and over phase-speed medium velocity regimes.

In the current work, we apply the spectral representations in [12] in order to obtain the EM Green’s function of a time-harmonic line current that is located at \((y, z) = (0, 0)\) and is embedded in a uniformly moving dielectric–magnetic medium with a planar discontinuity. In \(K\)-frame, the medium is moving in a constant translation velocity \(v = v\hat{z}\), where we assume that the velocity is in the direction of the \(z\) axis. Unit vectors in the conventional Cartesian coordinate system, \((x, y, z)\), are denoted by a hat over bold fonts. Under the framework of special relativity, a three-dimensional (space–time) event \((y, z, t)\) in the so-called laboratory frame (\(K\)-frame) is mapped to the event \((y', z', t')\) in the comoving frame (\(K'\)-frame) by the Lorentz transformation (LT) and the inverse LT (ILT), which are defined for the \(K'\)-frame velocity of \(v = v\hat{z}\) by

\[
y' = y, \quad z' = y(z - \beta ct), \quad t' = y(t - \beta z/c) \quad (1)
\]

\[
y = y', \quad z = y(z' + \beta ct'), \quad t = y(t' + \beta z'/c) \quad (2)
\]

where \(c = 1/\sqrt{\epsilon_0\mu_0}\) is the speed of light in vacuum and
Here and henceforth, all physical quantities in the $K'$-frame are denoted by a prime. The EM field transformation that is corresponding to $v = i\hat{z}$ is given by [13]

$$E'(r', t') = \mathbf{V} \cdot [E + v \times B],$$

$$B'(r', t') = \mathbf{V} \cdot [B - v \times E/c^2],$$

$$D'(r', t') = \mathbf{V} \cdot [D + v \times H/c^2],$$

$$H'(r', t') = \mathbf{V} \cdot [H - v \times D],$$

where $r' = (y', z')$, the $r = (y, z)$ and $t$ coordinates are transformed into $(r', t')$ via ILT in Eq. (2), and the (diagonal) dyadic $\mathbf{V}$ is given by

$$\mathbf{V} = \text{diag}\{\gamma, \gamma, 1\}.$$  

Here and henceforth, underlined boldfaced letters denote matrices/dyads. The inverse of Eq. (4) is form invariant and thus can be obtained by interchanging all primed and unprimed quantities where $\mathbf{V} = \mathbf{V}$ and $v = -\mathbf{v}$.

We are aiming at obtaining the relativistic 2D (3D in relativity jargon) Green’s function of the time-harmonic line current,

$$J(r, t) = I_0 \delta(z) \delta(y) \exp(i\omega t) \hat{x},$$

that is located in the so-called laboratory $K$-frame at $r = (y, z) = (0, 0)$ (see Fig. 1). The source is embedded in a uniformly moving dielectric–magnetic medium with a planar discontinuity. The medium’s discontinuity is located at $z = z_0 > 0$ at time $t = 0$ in $K$-frame. The medium is assumed to be a linear isotropic dielectric–magnetic in the frame at rest (the “comoving” $K'$-frame) with $\varepsilon'_{12}$ and $\mu'_{12}$ denoting its permittivity and permeability, where indices 1 and 2 refer to either side of the discontinuity. Thus, the constitutive relations in $K'$-frame are

$$D' = \varepsilon'_1 E', \quad B' = \mu'_1 H', \quad z' < z_0,$$

$$D' = \varepsilon'_2 E', \quad B' = \mu'_2 H', \quad z' > z_0,$$

where $z'_0 = \gamma z_0$ denotes the interface location in $K'$-frame, which is obtained from LT in Eq. (1). In the current contribution, we assume that the medium is lossless and dispersion free and that its velocity does not exceed the $K'$-frame phase velocities of $c/\sqrt{\varepsilon'_1 \mu'_{12}}$.

The outline of the paper is as follows: in Section 2 we briefly describe the spectral representation of the incident field that was derived in [12] and, in Section 3, the spectral representation of the Green’s function in the presence of a moving discontinuity is obtained by applying Maxwell’s boundary conditions in $K'$-frame. The spectral representation is obtained in both $K'$- and $K$-frames of reference in Subsections 3.A and 3.B, respectively. The asymptotic evaluations of the incident, reflected, and transmitted (refracted) waves in the high-frequency regime are obtained in Sections 4 and 5 for fields in $K'$- and $K$-frames, respectively.

\section{SPECTRAL REPRESENTATION OF THE INCIDENT FIELD}

The incident EM field in $K$-frame is the field that is radiated by the current line source in Eq. (6). The line current is embedded in a uniformly moving dielectric–magnetic medium of $(K')$ permittivity and permeability of $\varepsilon'_i$ and $\mu'_i$, respectively (for all $r'$). In $K$-frame, the corresponding constitutive relations can be stated as [6]

$$D = \varepsilon_i \mathbf{a} \cdot \mathbf{E} + c^{-1} m \hat{z} \times \mathbf{E},$$

$$B = \mu_i \mathbf{a} \cdot \mathbf{H} - c^{-1} \hat{m} \hat{z} \times \mathbf{H},$$

where

$$m = \beta \sqrt{n_i^2 - 1}, \quad n_i = c \sqrt{\varepsilon_i \mu'_i},$$

and $\mathbf{a}$ is the diagonal matrix

$$\mathbf{a} = \text{diag}\{\alpha, \alpha, 1\}, \quad \alpha = \frac{1 - \beta^2}{1 - n_i^2}.$$  

The spectral representation of the incident EM field that is denoted by $\mathbf{E}(r, t)$ can be obtained by formulating the current source Green’s function in $K'$-frame where the constitutive relations have simple expressions, while the source density function involves distributions that are moving along the $z$ axis with velocity $v$ (see, for example, [14]). In the current contribution we use the results that were obtained in [12] in which the incident EM field was derived directly in $K$-frame by using the normalized longitudinal coordinate and wavenumber

$$\bar{z} = \sqrt{\alpha} z, \quad \bar{k}_1 = \omega / \alpha \sqrt{\varepsilon'_i \mu'_i}.$$  

Note that $m$ in Eq. (9) and $\alpha$ in Eq. (10) are positive (assuming that the medium velocity does not exceed the medium’s phase velocity of $c/n_i$) so that $\sqrt{\alpha} > 0$ is real.

By using these definitions, the incident electric field is decomposed into PWs in the form

$$\mathbf{E}(r, t) = \frac{1}{2\pi} \int dk_y \mathbf{E}(r, t; k_y),$$

where the incident electric field spectral PWs that are denoted by $\mathbf{E}'$ are given by

$$\mathbf{E}'(r, t; k_y) = -\frac{i \omega \mu'_i \sqrt{\alpha}}{2k_z} \exp(\beta \Psi') - \frac{\omega}{2k_z} - k_y \Phi + kmz,$$

\section{CHAP. 1 - PHYSICAL CONFIGURATION OF THE RELATIVISTIC GREEN'S FUNCTION OF THE TIME-HARMONIC LINE CURRENT}

\begin{figure}[h]
\centering
\includegraphics[width=0.5\textwidth]{Fig1.png}
\caption{Physical configuration of the relativistic Green's function of a line current that is embedded in a dielectric–magnetic medium with a planar discontinuity. The medium is uniformly moving in the $z$ direction with a velocity of $v = rz$ in the laboratory frame and the discontinuity is located in $z = z_0$ at $t = 0$. The medium is an isotropic dielectric–magnetic in the comoving frame with $\varepsilon'_i$ and $\mu'_i$ denoting its permittivity and permeability on either side of the discontinuity.}
\end{figure}
where \( k = \omega/c \) is the wavenumber in vacuum, and the longitudinal wavenumber that is denoted by \( k_z1 \) is given by

\[
\frac{\omega}{c} = \sqrt{k_z1^2 - k_y^2}, \quad \text{Re} k_z1 \geq 0, \quad \text{Im} k_z1 \leq 0. \tag{14}
\]

Here and henceforth, over tilde (\( \sim \)) denotes PW constituents. The integration contour is described in Fig. 2 where the upper Riemann sheet is defined by \( \text{Re} k_z1 \geq 0 \).

The integral in Eq. (12) represents the incident electric field as a superposition of the spectral PWs in \( z > 0 \) half-space. Propagating PWs occur for spectral wavenumbers \( |k_y| \leq k_1 \). For all other \( k_y \) values, \( k_{z1} \) is imaginary and the PWs are evanescent and decay exponentially away from the source location. The incident magnetic field that is denoted by \( \mathbf{H}(r, t) \) is given by the PW spectral integral

\[
\mathbf{H}(r, t) = \frac{1}{2\pi} \int dk_y \tilde{\mathbf{H}}(r, t; k_y). \tag{15}
\]

over the magnetic spectral PWs

\[
\tilde{\mathbf{H}}(r, t; k_y) = \frac{I_0}{2k_z1} (\tilde{k}_{z1} \hat{y} + \sqrt{ak_y} \hat{z}) \exp(i\Psi). \tag{16}
\]

where \( \Psi \) is given in (13).

The scattering wave is obtained by transforming the incident EM field spectral PWs in Eqs. (13) and (16) to the \( K' \)-frame and applying Maxwell’s boundary conditions at the interface. By inserting Eqs. (12) and (15) into Eq. (8), we obtain the electric displacement and magnetic flux density spectral PW representations, which are required for the EM field transformation in (4). Thus

\[
\mathbf{D}(r, t) = \frac{1}{2\pi} \int dk_y \tilde{\mathbf{D}}(r, t; k_y),
\]

\[
\mathbf{B}(r, t) = \frac{1}{2\pi} \int dk_y \tilde{\mathbf{B}}(r, t; k_y), \tag{17}
\]

where the corresponding spectral PW fields are

\[
\tilde{\mathbf{D}}(r, t; k_y) = -\frac{I_0}{2k_z1} \left( \omega \mu_0^\prime \epsilon_0^\prime c^{3/2} - \frac{mk_z1}{c} \right) \hat{x} \exp(i\Psi),
\]

\[
\tilde{\mathbf{B}}(r, t; k_y) = -\frac{\mu_0^\prime \sqrt{\alpha_0}}{2k_z1} \left[ (\sqrt{ak_z1} - \frac{m\alpha^\prime}{c}) \hat{y} - k_y \hat{z} \right] \exp(i\Psi). \tag{18}
\]

3. SPECTRAL REPRESENTATION OF GREEN’S FUNCTION

In this section we derive the spectral representations of the reflected and transmitted waves on either side of the interface, \( z \leq z_0 \).

A. Fields in \( K' \)-Frame

By inserting the spectral representations in Eqs. (12), (15), and (17) into the field transformation in Eq. (4) and using the LT in Eq. (1), we obtain the spectral integral of the incident EM field in \( K' \)-frame:

\[
\mathbf{E}^\prime(r', t') = \frac{1}{2\pi} \int dk_y \tilde{\mathbf{E}}^\prime(r', t'; k_y),
\]

\[
\mathbf{H}^\prime(r', t') = \frac{1}{2\pi} \int dk_y \tilde{\mathbf{H}}^\prime(r', t'; k_y). \tag{19}
\]

where the spectral PWs are given by

\[
\tilde{\mathbf{E}}^\prime(r', t'; k_y) = -\mathbf{E}_0 \hat{x} \exp(i\Psi^\prime),
\]

\[
\tilde{\mathbf{H}}^\prime(r', t'; k_y) = -\frac{I_0 \omega \mu_0^\prime \epsilon_0^\prime c^{3/2}}{2k_z1} \left[ \gamma \alpha \left( \frac{k_z1}{k_1} - n_1^\prime \beta \right) \hat{y} - \frac{k_y \sqrt{\alpha}}{k_1} \hat{z} \right] \exp(i\Psi^\prime). \tag{20}
\]

Here \( \Psi^\prime \) is given in Eq. (9), \( k_{z1} \) is given by Eq. (14), the amplitude \( \mathbf{E}_0 \) is given by

\[
\mathbf{E}_0 = \frac{I_0 \omega \mu_0^\prime \epsilon_0^\prime c^{3/2}}{2k_z1} \gamma (1 - n_1^\prime \beta k_z1/k_1), \tag{21}
\]

and the phase \( \Psi^\prime \) is given by

\[
\Psi^\prime(r', t'; k_y) = \omega' r' - k_y y' - k_{z1}' z', \tag{22}
\]

where

\[
\omega' = \omega \alpha (1 - n_1^\prime \beta k_z1/k_1), \quad k_{z1}' = \sqrt{\alpha \gamma (k_z1 - n_1^\prime \beta k_1)} \tag{23}
\]

are identified as the temporal frequency (Doppler shift) and the longitudinal wavenumber of the spectral PW in \( K' \)-frame.

Next we define normalized \( z' \) and \( t' \) coordinates:

\[
z' = \sqrt{\alpha} z', \quad t' = \sqrt{\alpha} t', \tag{24}
\]

and recast Eq. (22) in the form

\[
\Psi^\prime(r', t'; k_y) = \omega' t' - k_y y' - k_{z1}' z', \tag{25}
\]

where we define the normalized frequency and longitudinal wavenumber:

\[
\omega' \equiv \omega' / \sqrt{\alpha}, \quad k_{z1}' \equiv k_{z1}' / \sqrt{\alpha} = \left( k_z1 - n_1^\prime \beta k_1 \right). \tag{26}
\]

The reflected and transmitted fields on either side of the (stationary) interface at \( z' = z_0' \) can be obtained from the well-known results of PW scattering from stationary dielectric–magnetic planar discontinuity (i.e., the Fresnel coefficients) since, for a given \( k_y \), the incident spectral PW is a time-harmonic field that carries the temporal frequency of \( \omega' \) in Eq. (23). Thus, the reflected EM field, which is denoted by \( \mathbf{E}^\prime \) and \( \mathbf{H}^\prime \), is given by a spectral representation similar to Eqs. (12) and (15):
\[
\mathbf{E}''(r', t') = \frac{1}{2\pi} \int dk_y \mathbf{E}''(r', t'; k_y),
\]
\[
\mathbf{H}''(r', t') = \frac{1}{2\pi} \int dk_y \mathbf{H}''(r', t'; k_y),
\]  
(27)

over their spectral PWs

\[
\mathbf{E}''(r', t'; k_y) = -\mathbf{E}_0 (k_y) \mathbf{x} \exp(i \Psi''),
\]
\[
\mathbf{H}''(r', t'; k_y) = \frac{1}{\eta_1} \Gamma(k_y) \mathbf{H}_0 y' \exp(i \Psi''),
\]  
(28)

Here \(\omega'\) is given in Eq. (23) and the phase \(\Psi''\) is given by

\[
\Psi''(r', t'; k_y) = \omega' t - k_y y' - k_z' (2z' - z),
\]

with \(z'\) in Eq. (24) and \(z'_0 = \sqrt{\omega' z_0}\). In Eq. (28), \(\Gamma(k_y)\) denotes the Fresnel reflection coefficient [13]:

\[
\Gamma(k_y) = \frac{\mu_2 k'_2 - \mu_1 k'_{z}}{\mu_2 k'_2 + \mu_1 k'_{z}},
\]  
(30)

where

\[
k'_2 = \sqrt{\omega^2 \varepsilon_2 \mu'_2 - k_y^2 / \alpha r'^2},
\]

(31)

and \(\omega'\) and \(k'_2\) are given in Eq. (26).

Similarly, the transmitted field that is propagating in \(z' > z'_0\) is given by the spectral integrations

\[
\mathbf{E}''(r', t') = \frac{1}{2\pi} \int dk_y \mathbf{E}''(r', t'; k_y),
\]
\[
\mathbf{H}''(r', t') = \frac{1}{2\pi} \int dk_y \mathbf{H}''(r', t'; k_y),
\]  
(32)

over the transmitted spectral PWs

\[
\mathbf{E}''(r', t'; k_y) = -\mathbf{E}_0 T(k_y) \mathbf{x} \exp(i \Psi''),
\]
\[
\mathbf{H}''(r', t'; k_y) = -\frac{1}{\omega' \mu'_2} \mathbf{E}_0 T(k_y) (k'_2 y - k_y z) \exp(i \Psi''),
\]  
(33)

where the phase \(\Psi''\) is given by

\[
\Psi''(r', t'; k_y) = \omega' t - k_y y' - k'_2 (z' - z'_0) - k_z' z_0,
\]

(34)

and the transmission coefficient \(T(k_y)\) of the interface at \(z' = z'_0\) is given by

\[
T(k_y) = 1 + \Gamma(k_y).
\]  
(35)

The spectral PWs in Eq. (33) are propagating in \((y', z' > z'_0)\) half-space in the direction of the unit vector \(\hat{i}'' = \hat{y} \sin \phi'' + \hat{z} \cos \phi''\) where, by using the phase \(\Psi''\) in Eqs. (34) and (31), we obtain

\[
\sin \phi'' = \frac{k_y}{\omega' \sqrt{\varepsilon_2 \mu'_2}}.
\]  
(36)

B. Fields in \(K\)-Frame

The spectral integral of the reflected EM fields in \(K\)-frame are obtained by inserting Eqs. (27) and (28) with Eq. (7) into Eq. (4) and using the ILT in Eq. (2). The resulting spectral integrals of the EM fields are

\[
\mathbf{E}'(r, t) = \frac{1}{2\pi} \int dk_y \mathbf{E}'(r, t; k_y),
\]
\[
\mathbf{H}'(r, t) = \frac{1}{2\pi} \int dk_y \mathbf{H}'(r, t; k_y),
\]  
(37)

over the spectral PWs

\[
\mathbf{E}'(r, t; k_y) = -\mathbf{E}_0 \Gamma(k_y) \left(1 - n_1' \sqrt{\alpha \omega' / \omega_1} k_y \right) \mathbf{x} \exp(i \Psi'),
\]
\[
\mathbf{H}'(r, t; k_y) = \frac{\mathbf{H}_0}{\eta_1} \left(\frac{\omega \sqrt{\alpha k_z' - \omega'} n_1' k_y}{\alpha k_1 - \alpha n_1' \beta \sqrt{\alpha k_z'}} y + \omega k_y t' k_0 \right) \times \exp(i \Psi').
\]  
(38)

Here \(\omega'\) is given in Eq. (26) and the phase \(\Psi'\) is given by

\[
\Psi'(r, t; k_y) = \sqrt{\alpha \varepsilon_2 (\omega' - \beta k_z') t - k_y y} - \sqrt{\alpha \varepsilon_2 (\omega' - \beta k_z') z + k'_2 (z_0 - z)}.
\]  
(39)

The spectral integrals in Eq. (37) are evaluated asymptotically in Section 5.

The spectral integrals of the transmitted EM field in \(K\)-frame are obtained by inserting Eqs. (32) and (33) with Eq. (7) into Eq. (4) and using ILT in Eq. (2). This procedure yields

\[
\mathbf{E}'(r, t) = \frac{1}{2\pi} \int dk_y \mathbf{E}'(r, t; k_y),
\]
\[
\mathbf{H}'(r, t) = \frac{1}{2\pi} \int dk_y \mathbf{H}'(r, t; k_y),
\]  
(40)

where the spectral PWs are given by

\[
\mathbf{E}'(r, t; k_y) = -\mathbf{E}_0 T(k_y) \left(1 + c \mu'_2 \frac{k_z'}{\alpha' \omega'} \right) \mathbf{x} \exp(i \Psi'),
\]
\[
\mathbf{H}'(r, t; k_y) = \frac{\mathbf{H}_0}{\eta_2} \left(\gamma n_2' + \frac{c k_z'}{n_2' \alpha'} \right) y - \frac{c k_y}{n_2' \alpha'} z \times \exp(i \Psi').
\]  
(41)

Here \(k'_z\) is given in Eq. (31) and the phase \(\Psi'\) is given by

\[
\Psi'(r, t; k_y) = \sqrt{\alpha \varepsilon_2 (\omega' + c \beta k_z') t - k_y y} - \sqrt{\alpha \varepsilon_2 (\omega' + c \beta k_z') z + (k'_1 - k'_z) z_0}.
\]  
(42)

Note that, by evaluating \(\tilde{\mathbf{E}}'\) and \(\tilde{\mathbf{H}}'\) in the same manner, the resulting spectral PWs and the ones in Eq. (41) satisfy the constitutive relations in Eq. (8) by replacing \(\varepsilon'_1\) and \(\mu'_1\) with \(\varepsilon'_2\) and \(\mu'_2\), respectively.

4. ASYMPTOTIC EVALUATION IN \(K\)-FRAME

In the high-frequency regime, the dominant contributions to the scattered field spectral integrals in Eqs. (19), (27), and (32) arise from the vicinity of spectral saddle points. In order to evaluate the spectral integrals asymptotically, it is
convenient to apply here the spectral angle \( \tilde{\phi} \), which is defined for a given spectral wavenumber \( k_y \) by the transformation

\[
k_y = k_1 \sin \tilde{\phi}.
\]

The spectral angle eliminates the branch points \( k_y = \pm k_1 \) of the spectral \( k_y \) integrals since \( k_{z1} = k_1 \cos \tilde{\phi} \). Next we evaluate the incident and scattered fields in \( K \)-frame asymptotically and identify the associated relativistic wave phenomena.

### A. Incident Wave

To simplify the form of spectral integrals in Eq. (45), we introduce the incident wave coordinates, \((R', \phi')\), that are defined for a given event \((y', z', t')\) by the transformation [see Fig. 4(a)]

\[
R' \cos \phi' = z' + v't', \quad R' \sin \phi' = y'.
\]

By using these coordinates and \( \tilde{\phi} \) in Eq. (43), we recast Eq. (19) in the Fourier integral form:

\[
E'(r', t') = -\hat{\mathbf{\Phi}}(r', t') \exp(i\Psi_0) \cos(\tilde{\phi} - \tilde{\phi}'), \quad \Psi_0 = \gamma \omega t' + \beta n'_1 k_1 z'.
\]

In Eq. (45), \( I'(r', t') \) denotes the spectral integral

\[
I'(r', t') = \frac{1}{2\pi} \int_C d\tilde{\phi} f'(\tilde{\phi}) \exp[i\Psi'(r', t'; \tilde{\phi})]
\]

over the integration contour in Fig. 3, where the phase \( \Psi'(\tilde{\phi}) \) and the amplitude \( f'(\tilde{\phi}) \) are given by

\[
\Psi'(r', t'; \tilde{\phi}) = -k_1 R' \cos(\tilde{\phi} - \phi'), \quad f'(\tilde{\phi}) = I_0 \mu' / (\sqrt{\alpha} / 2).
\]

Here \( \omega' \) that is defined in Eq. (23) is a function of \( \tilde{\phi} \):

\[
\omega'(\tilde{\phi}) = \omega \alpha t' (1 - n'_1 \beta \cos \tilde{\phi}).
\]

Upon setting \( \partial_\tilde{\phi} \Psi' = k_1 R' \sin(\tilde{\phi} - \phi') = 0 \), we find that the phase in Eq. (47) has a saddle point \( \tilde{\phi}'_S = \phi' \). The amplitude \( f'(\tilde{\phi}) \) has no singularities near the saddle point and the integral in Eq. (46) can be evaluated asymptotically along the steepest descent path (SDP) using the isolated saddle point contribution [15]:

\[
I'(r', t') \sim I'_{SDP} = \frac{1}{2\pi \sqrt{\alpha}} \left| \frac{1}{\partial_\tilde{\phi} \Psi'(\tilde{\phi}')} \right| f'(\tilde{\phi}') \exp[i\Psi'(\tilde{\phi}')] \pm \frac{j \pi}{4},
\]

where the \( \pm \) sign corresponds to \( \tilde{\phi}^\ast \Psi'(\tilde{\phi}') \leq 0 \).

By inserting Eqs. (47) into Eq. (49), we obtain

\[
I'_{SDP} = \sqrt{\alpha I_0} \omega'(\phi') \exp(-j k_1 R' - j \pi / 4),
\]

where \( \omega'(\phi') \) is obtained from Eq. (48).

In a similar manner, using \( \mathbf{H}' \) in Eq. (20) as well as \( \tilde{\phi} \) in Eq. (43) and the coordinates in Eq. (44), we obtain

\[
\mathbf{H}'(r', t') \sim \frac{1}{\eta'_1} I'_{SDP} \exp(j \Psi'_0) \mathbf{h}'(\phi'),
\]

\[
\mathbf{h}'(\phi') = \frac{-\mathbf{j} (\cos \phi' - n'_1 \beta) + z \sin \phi' \sqrt{\alpha}}{(1 - n'_1 \beta \cos \phi')}.
\]

The physical interpretation of the asymptotic field in Eq. (50) is plotted in Fig. 4(a). At time \( t' = \sqrt{\alpha t} \), the source is located in \( z' = -vt \) over the \( z' \) axis. The source is moving at constant speed away from the stationary plane \( z' = \tilde{z}' \). Here \( R' \) in Eq. (44) is identified as the distance from the moving source to the observation space–time event \((y', z', t')\). The asymptotic scalar field \( I'_{SDP} \) resembles the 2D asymptotic scalar Green’s function in free space in which the phase is modulated along \( R' \). Thus the main contribution to the integral in Eq. (19) arises from the ray that emanates from the moving source and moves along the straight line to the observation point. Apart from \( R' \) dependence on \( t' \), the electric field in Eq. (45) exhibits harmonic oscillations of \( \exp(\gamma \alpha t') \). Note that the field accumulates an additional \( \phi' \)-dependent phase term in Eq. (45) due to the \( z' \)-directed medium velocity.

### B. Reflected Wave

The asymptotic evaluation of the reflected wave is obtained by introducing the reflected wave polar coordinates, \((R'', \phi'')\), which are defined for a given event \((y', z', t')\) by the transformation [see Fig. 4(b)]

\[
R'' \cos \phi'' = 2z_0 - z' + v't', \quad R'' \sin \phi'' = y'.
\]

By using Eq. (52), we recast the spectral integrals in Eq. (27) in the form
\[
\mathbf{E}''(r', t') = -i\tilde{\mathbf{I}}'(r', t') \exp(i\Psi''_0), \\
\Psi''_0 = \gamma a t' - \beta n'_i k_1(z' - 2z'_0).
\] (53)

In Eq. (53), \(I'(r', t')\) denotes the spectral integral

\[
I'(r', t') = \frac{1}{2\pi} \int_C d\tilde{\varphi} \tilde{f}'(\tilde{\varphi}) \exp[i\Psi''(r', t'; \tilde{\varphi})].
\] (54)

over the integration contour in Fig. 3 with

\[
\Psi''(r', t'; \tilde{\varphi}) = -k_1 R'' \cos(\tilde{\varphi} - \varphi''), \quad f''(\tilde{\varphi}) = \Gamma(\tilde{\varphi}) f'(\tilde{\varphi}),
\] (55)

with \(f'(\tilde{\varphi})\) in Eq. (47). \(\Gamma(\tilde{\varphi})\) denotes the reflection coefficient

\[
\Gamma(\tilde{\varphi}) = \frac{\mu'_2 (\cos \tilde{\varphi} - n'_i \beta) - \mu'_1 \sqrt{n_{21}^2 (1 - n'_i \beta \cos \tilde{\varphi})^2 - \sin^2 \tilde{\varphi}/(\alpha^2)}}{\mu'_2 (\cos \tilde{\varphi} - n'_i \beta) + \mu'_1 \sqrt{n_{21}^2 (1 - n'_i \beta \cos \tilde{\varphi})^2 - \sin^2 \tilde{\varphi}/(\alpha^2)}}.
\] (56)

with \(n'_{21} \equiv n'_2/n'_1\). The reflection coefficient has two branch points that are denoted by \(\varphi_{b1,2}\). By setting the square root in Eq. (56) to zero, we obtain in range \(|\varphi_{b1,2}| \leq \pi/2\)

\[
\cos \varphi_{b1,2} = \frac{n'_{21} n'_2 \alpha \gamma^2 \pm \sqrt{(n'_{21} n'_2 \alpha \gamma^2)^2 - (n'_{21} n'_2 \alpha^2) + 1(n'_{21} n'_2 \alpha^2 - 1)}}{n'_{21} n'_2 \alpha^2 + 1}.
\] (57)

The branch points define two relativistic critical angles for which the spectral transferred PWs propagate parallel to the interface [see Eq. (36)].

The integral in Eq. (54) has the stationary point \(\tilde{\varphi}' = \varphi''\). We distinguish two scattering regimes: under critical angle incidence in which \(\varphi_{b1} < \tilde{\varphi}' < \varphi_{b2}\) and over critical angle incidence in which either \(\tilde{\varphi}' < \varphi_{b1}\) or \(\tilde{\varphi}' > \varphi_{b2}\).

1. Under Critical Angle Incidence

The integration contour in this regime can be deformed into integral along the SDP as plotted in Fig. 5(a). The reflected wave is evaluated asymptotically by using the isolated saddle point contribution in (49). This procedure yields

\[
I' \sim I'_{\text{SDP}} = \Gamma(\varphi'') \sqrt{2\mu'_1} a'(\varphi'') \frac{I_0 \exp(-j k_1 R'' - j\pi/4)}{\sqrt{8\pi k_1 R''}},
\] (58)

where, from Eq. (48), \(a'(\varphi'') = a(1 - \beta n'_i \cos \varphi'')\). In a similar manner, by using \(H''\) in Eq. (28) we obtain

\[
H''(r', t') \sim \frac{1}{n'_{21}} I'_{\text{SDP}} \exp(i\Psi''_0) \tilde{h}''(\varphi'').
\] (59)

The physical interpretation of the asymptotic EM field in Eq. (58) is plotted in Fig. 4(b). The source, which is located in \(z' = -n' t\) over the \(z'\) axis, is moving at speed \(v\) away from the stationary interface. Here \(R''\) in Eq. (52) is identified as the length of the reflected ray from the interface to the observa-

![Fig. 5. Integration contour in \(\tilde{\varphi}\) plane for the reflected field (a) under and (b) over critical angle incidence.](image)

The reflected field asymptotic evaluation for over critical angle incidence is performed by deforming the integration path of \(I'\) in Eq. (54) to the integration contour in Fig. 5(b). The electric field is consist of two contributions

\[
\mathbf{E}''(r', t') = \mathbf{E}_{\text{SDP}}''(r', t') + \mathbf{E}_b''(r', t').
\] (60)

where \(\mathbf{E}_{\text{SDP}}''\) denotes the contribution of the integration along the SDP and \(\mathbf{E}_b''\) denotes the contribution of the integration around the branch cut of \(\varphi_{b1}\) (or \(\varphi_{b2}\) in Eq. (57)). The asymptotic SDP contribution that arises from the vicinity of the stationary point in the upper Riemann sheet is given in Eq. (53) with Eq. (58).

In the high-frequency regime, the main contribution to \(\mathbf{E}_b''\) arises from points in the vicinity of the corresponding branch point in the upper Riemann sheet [16]. Therefore, in order to evaluate \(\mathbf{E}_b''\) asymptotically, we apply a first-order Taylor approximation to \(\Gamma(\tilde{\varphi})\) in Eq. (56) about the branch point. Thus, for \(\tilde{\varphi} = \tilde{\varphi}_{b1,2}\),

\[
\Gamma(\tilde{\varphi}) \simeq 1 + j2\mu'_1 \frac{\sqrt{2 \sin \varphi_{b1,2} \varphi - \varphi_{b1,2}}}{\gamma \sqrt{\alpha}(\cos \varphi_{b1,2} - n'_i \beta)(1 - n'_i \beta \cos \varphi_{b1,2})}.
\] (61)

Equation (54) with Eq. (61) has the generic form of the branch-cut integral [16]:

\[
I_b = \int_{P_b} d\tilde{\varphi} \left( a + b \sqrt{\tilde{\varphi}_0 - \varphi} \right) \exp[i\Omega q(\tilde{\varphi})] \sim \frac{b \sqrt{\pi}}{|-4q(\tilde{\varphi}_0)|^{1/2}} \exp[i\Omega q(\tilde{\varphi}_0)],
\] (62)

where \(P_b\) is a contour encircling the branch cut in the positive sense [see Fig. 5(b)], \(\Omega\) is a large parameter, \(a\) and \(b\) are
constants, \( q(\bar{\phi}) \) is a regular in the vicinity of \( \bar{\phi}_0 \), and \( \exp[0q(\bar{\phi})] \) decays along the contour. Thus, by applying Eq. (62) to Eq. (54) with Eqs. (57) and (61) for large real \( k_1 \) values we obtain:

\[
E_n''(\mathbf{r}', t') \sim -\hat{x} E_{b1}' D'E''_{b2} E''_{b3} \exp(j\gamma \phi') \tag{63}
\]

where

\[
E_{b1}' = \frac{I_0 \sqrt{\alpha r_n} e^{\alpha (q')^2}}{\sqrt{8\pi k_1 L_1'}} \exp(j\Psi_{b1}). \tag{64}
\]

with

\[
\Psi_{b1} = -k_1 L_1 - (\pi/4) + \beta n'_1 k_1 z_0', \quad L_1 = (z_0' - vt') / \cos \theta_c. \tag{65}
\]

By comparing Eq. (64) to Eqs. (45) with (54), \( E_{b1}' \) is identified as the incident ray that propagates from the source and impinges on the interface at the critical angle (angle of total reflection) \( \theta_c \) that is defined by

\[
\sin \theta_c = n_2 \tag{66}
\]

(see Fig. 6). The \( D' \) term in Eq. (63), which is given by

\[
D' = \frac{2j \sin^2 \theta_c \sqrt{k_1 L_1'}}{\sqrt{\alpha r_n} \cos \theta_c \sqrt{1 - n_1'/\cos \theta_c}}, \tag{67}
\]

is identified as the relativistic diffraction coefficient. The \( E_{b2}' \) and \( E_{b3}' \) terms in Eq. (63) are given by

\[
E_{b2}' = \frac{\exp(-jk_2 L_2')}{(k_2 L_2')^{1/2}}, \quad L_2' = y - L_1' \sin \theta_c + (z' - z_0') \tan \theta_c, \tag{68}
\]

where \( k_2 = a_v \sqrt{\alpha c^2 \mu_2} \), and

\[
E_{b3}' = \exp(-jk_2 L_2') \exp[j\beta n'_1 k_1(z_0' - z')],
\]

\[
L_3' = (z_0' - z') / \cos \theta_c. \tag{69}
\]

The \( E_{b2}' \) field term in Eq. (68) describes the relativistic lateral ray that is propagating along \( L_2' \) parallel to the interface in the \( z' = z_0' \) medium. The lateral ray \( (k_2 L_2')^{-1/2} \) decay along its trajectory is due to the \( E_{b2}' \) field radiation back to \( n_1' \) medium. By comparing Eqs. (55) with (58) to Eq. (68), the \( E_{b3}' \) field term is identified as the ray that is propagating from the interface at angle \( \theta_c \) to the observation point \( (y', z') \) along \( L_3' \) (see Fig. 6).

Finally, by sampling the amplitude of \( H''_b \) in Eq. (28) at \( \bar{\phi} = \bar{\phi}_{b2} \) and applying the same analytic procedure, we obtain the asymptotic head-wave magnetic field:

\[
H''_b(\mathbf{r}', t') \sim \frac{1}{\eta_1} E_{b1}' D'E''_{b2} E''_{b3} \exp(j\gamma \phi') H''(\bar{\phi}_{b2}), \tag{70}
\]

\[
\mathbf{h}''(\bar{\phi}_{b2}) = \frac{\mathbf{y}(\cos \phi_{b2} - \beta n_1' \sin \phi_{b2}) + \hat{\mathbf{z}} \sin \phi_{b2}/(\gamma \sqrt{\mu})}{(1 - \beta n_1' \cos \phi_{b2})}. \tag{71}
\]

C. Transmitted Field

The exact transmitted EM field in \( K' \)-frame is given by a spectral representation in Eq. (32). To simplify the form of the spectral integrals, we introduce the transmitted wave polar coordinates, \( (R'_1, R'_2, \phi_1', \phi_2') \), that are defined for a given space–time event \( (y', z', t') \) by the transformation (see Fig. 7)

\[
R'_1 \cos \phi_1' = z'_1 + vt', \quad R'_2 \cos \phi_2' = z' - z_0', \tag{71}
\]

where angles \( \phi_1' \) and \( \phi_2' \) satisfy

\[
\tan \phi_2' = \frac{(n_2 \alpha r_0'^2)^{-1} \sin \phi_1' - n_1'(1 - n_1' \cos \phi_2') \tan \phi_1'}{(1 - n_1' \cos \phi_1')^2 - \sin^2 \phi_1' / n_2 \alpha r_0'^2}. \tag{72}
\]

Equation (72) is identified later as the relativistic Snell’s law [see discussion following Eq. (83)]. Note that, by inverting the transformation in Eq. (71), we obtain

\[
y' = R'_1 \sin \phi_1' + R'_2 \sin \phi_2'. \tag{73}
\]

By using Eq. (71) and the complex angle \( \bar{\phi} \) in Eq. (45), we recast the spectral integrals in Eq. (32) in the form

\[
E''(\mathbf{r}', t') = -\hat{x} \exp[j\Psi''_0(\mathbf{r}', t')] F'(\mathbf{r}', t'), \tag{74}
\]

\[
F'(\mathbf{r}', t') = \frac{1}{2\pi} \int d\bar{\phi} f'(\bar{\phi}) \exp[j\Psi''_0(\mathbf{r}', t'; \bar{\phi})], \tag{74}
\]

where \( \Psi''_0 \) is given in Eq. (45), the amplitude

\[
f' (\bar{\phi}) = T' (\bar{\phi}) f' (\bar{\phi}), \tag{75}
\]

with \( f' \) in Eq. (47), and the spectral phase \( \Psi''(\bar{\phi}) \) is given by

Fig. 6. Geometrical optics interpretation of the lateral (head) wave in Eq. (63).

Fig. 7. Geometrical optics interpretation of asymptotic transferred field in Eq. (80).
\[ \Psi^\prime(r', t'; \bar{\varphi}) = -k_1 \left[ R'_1 \cos(\bar{\varphi} - \varphi'_1) + R'_2 \left( \sin \varphi'_2 \sin \bar{\varphi} + n'_1 \cos \varphi'_2 \sqrt{(1 - n'_1 \beta \cos \bar{\varphi})^2 - \sin^2 \bar{\varphi}/n'^2_{21} \alpha^2} \right) \right]. \] (76)

The Fresnel transmission coefficient in the amplitude in Eq. (75) in terms of the spectral angle \( \bar{\varphi} \) is given by

\[ T(\bar{\varphi}) = \frac{2 \mu'_2 (\cos \bar{\varphi} - n'_1 \beta)}{\mu'_2 (\cos \bar{\varphi} - n'_1 \beta) + \mu'_1 n'_21 \sqrt{(1 - n'_1 \beta \cos \bar{\varphi})^2 - \sin^2 \bar{\varphi}/n'^2_{21} \alpha^2}}. \] (77)

The stationary point that is denoted by \( \bar{\varphi}_{s1} \) is obtained by setting \( \partial_{\bar{\varphi}} \Psi^\prime(\bar{\varphi}) \vert_{\bar{\varphi}_{s1}} = 0 \). By using Eqs. (71), (75), and (76), we find that \( \bar{\varphi}_{s1} \) satisfies

\[ y' - \tan \bar{\varphi}_{s1}(z_0 + v t) - \tan \bar{\varphi}_{s2}(z' - z_0) = 0, \] (78)

where \( \bar{\varphi}_{s2} \) is related to \( \bar{\varphi}_{s1} \) by

\[ \tan \bar{\varphi}_{s2} = \frac{(n'_2 \alpha^2)^{-1} \sin \bar{\varphi}_{s1} - n'_2(1 - n'_1 \beta \cos \bar{\varphi}_{s1}) \beta \tan \bar{\varphi}_{s1}}{\sqrt{(1 - n'_1 \beta \cos \bar{\varphi}_{s1})^2 - \sin^2 \bar{\varphi}_{s1}/n'^2_{21} \alpha^2}}. \] (79)

By comparing Eqs. (78) and (79) with Eqs. (72) and (73), we deduce that \( \bar{\varphi}_{s1} = \varphi'_1 \) and, therefore, \( \bar{\varphi}_{s2} = \varphi'_2 \).

By inserting Eqs. (78), (79) with (77) and (76), (75) into (49), we obtain the asymptotic expression of the transferred field:

\[ E^\prime(r', t') \sim -\hat{x} E'_1 E'_2 \exp(j \omega \gamma t'), \] (80)

where

\[ E'_1 = \sqrt{\alpha} \mu'_1 \omega' (\varphi'_1) \frac{I_0 \exp(-jk_1 R'_1) + j \beta n'_1 k_1 z_0 - j x/4}{\sqrt{8 \pi k_1 R'_1}} \] (81)

and

\[ E'_2 = \frac{T(\varphi'_1) \exp(-jk_2 R'_2)}{\sqrt{1 + E'^2_2/\rho'^2}}. \] (82)

where \( T(\varphi'_1) \) is given in Eq. (77), and

\[ \rho' = \frac{R'_1}{\sin \varphi'_2 \cos \varphi'_1} \times \left[ \frac{n'_{12}[1 - \beta (n'_2 - n'_2)] \sin \varphi'_1 - n'_1 \beta \tan \varphi'_1}{n'_{12}[1 - \beta (n'_1 - n'_2)] - n'_2 \beta / \cos \varphi'_1 + n'_{12} \tan^2 \varphi'_2} \right]. \] (83)

Equation (78) sets the ray path that is shown in Fig. 7, where observation event \((y', z', t')\) is represented by \( \varphi'_2, \varphi'_2, R'_1, R'_2 \) via Eq. (71). Following this definition, we identify Eq. (78) as the path from the source along a straight line with angles \( \varphi'_1, \varphi'_2 \) and lengths \( R'_1, R'_2 \) in \( z' < z'_0 \) or \( z' > z'_0 \), respectively. Thus Eq. (79) for Eq. (72) describes the relativistic Snell’s law for this specific scattering scenario. This relation adjusts the angles to the source velocity. The spectral Doppler shift in Eq. (23) changes the conventional (stationary) Snell’s law, which is obtained for \((K\text{-frame})\) time-harmonic excitation. Note that, by setting \( \beta = 0 \) in Eq. (79), we obtained the well-known (stationary) Snell’s law.

The transferred field in Eq. (80) consists of \( E'_1 \) in Eq. (81) and \( E'_2 \) in Eq. (82). By using Eq. (80) in Eq. (45), \( E'_1 \) is identified as the incident electric field at point \( P \) over the interface. This point is the intersection of the incident ray, which is emanating from the source with departure angle \( \varphi'_0 \), and the interface at \( z = z'_0 \). Thus, \( E'_1 \) is identified as the principal radius of curvature of the transferred wavefront at point \( P \).

The corresponding magnetic transferred field is obtained from Eq. (35), giving

\[ \mathbf{H}^\prime(r', t') = \frac{1}{\mu'_2} E'_1 E'_2 \exp(j \omega \gamma t') \mathbf{h}'(\varphi'_1), \]

\[ \mathbf{h}'(\varphi'_1) = -\hat{y} \cos \varphi'_1 + \hat{z} \sin \varphi'_1 / (n'_{12} \sqrt{\alpha}) \] (84).

5. ASYMPTOTIC FIELDS IN K-FRAME

A. Incident Field

The asymptotic fields in \( K\text{-frame} \) are transformed to \( K\text{-frame} \) by applying the inverse field transformation of Eq. (4) and ILT Eq. (2) to Eqs. (45), (50), and (51) with \( K\text{-frame} \) constitutive relations in Eq. (7). This results in

\[ E^\prime(r, t) \sim -\hat{x} E^\prime_r(r) \exp(j \omega \gamma t), \]

\[ E^\prime_r(r) = I_g u'_{12} a'_{12} \exp(-jk_1 R^i - j \pi/4) \exp(j k m z), \]

\[ H^\prime(r, t) \sim \frac{1}{\mu_1} E^\prime_r(r) \exp(j \omega \gamma t) -\hat{y} \cos \varphi' + \hat{z} \sqrt{\alpha} \sin \varphi' \] (85)

where \( R^i = \sqrt{y^2 + z^2} \) and \( \cos \varphi' = z/R^i \). These expressions are used later for identifying the incident ray field contribution to the reflected and transmitted fields.

B. Reflected Field

In Subsection 4.B we distinguished two reflection regimes in which the \( K\text{-frame} \) incident ray is impinging on the interface
with an angle that is larger or smaller than the critical angle. In this subsection, we identify the $K$-frame relativistic wave phenomena associated with these two scattering regimes.

1. Under Critical Angle Incidence

The under critical angle scattering regime for which in $K$-frame $\vec{\omega}_b < \vec{\psi}_c < \vec{\omega}_b$ was investigated in Subsection 4.B.1. By applying ILT we obtained that, in $K$-frame, the stationary point is given by

$$\vec{\psi}_c = \cos^{-1}\left[\frac{\sqrt{\lambda^2 + \lambda_d^2(2(z_0 + vt) - (1 + \beta^2))}}{\sqrt{\lambda^2 + \lambda_d^2(2(z_0 + vt) - (1 + \beta^2))^2}}\right].$$  \hfill (86)

By inserting Eqs. (53) and (59) with Eq. (58) into Eq. (7) and then into the field transformation in Eq. (4) and using ILT in Eq. (2), we obtain the reflected field (isolated saddle point contribution) in $K$-frame in the form

$$E'(r, t) \sim -\mathbf{x}\Psi'(r, t) \exp(j\Psi'),$$ \hfill (87)

where

$$\Psi' = r^2\alpha(1 + \beta^2n_i^2) \omega t - \beta r^2 \sqrt{\alpha}k_1[z(1 + n_i^2)/n_i - 2n_i^2 z_0],$$

$$E'_y(r, t) = j\omega n_i^2 r^2e^{\gamma/2}G'(\Psi') \frac{I_0}{\sqrt{\tan k_1 R^2}}$$ \hfill (88)

with

$$R = \sqrt{\lambda^2 + \lambda_d^2(2(z_0 + vt) - z(1 + \beta^2))^2},$$

$$\cos\Psi' = r^2\sqrt{\lambda^2(2(z_0 + vt) - z(1 + \beta^2))}\tan k_1 R^2,$$

$$G'(\Psi') = (1 - 2\beta n_i^2 \cos\Psi' + \beta^2n_i^2)^\gamma G(\Psi').$$ \hfill (89)

$\Gamma$ is given in Eq. (56).

In a similar manner,

$$H'(r, t) \sim \frac{1}{\gamma_i} E'_x \exp(j\Psi')H'(\Psi'),$$

$$H'(\Psi') = \frac{\sqrt{\lambda^2 + \lambda_d^2(2(z_0 + vt) - 2n_i^2 z_0)} \tan k_1 R^2}{1 - 2\beta n_i^2 \cos\Psi' + \beta^2 n_i^2} \frac{\sin\Psi' \sqrt{\lambda^2}}{\tan k_1 R^2}.$$ \hfill (90)

2. Over Critical Angle Incidence

The over critical angle scattering regime for which in $K$-frame, $\vec{\omega}_b < \vec{\psi}_c$ or $\vec{\psi}_c > \vec{\omega}_b$ was investigated in Subsection 4.B.2. $\vec{\psi}_c$ in $K$-frame is given in Eq. (86). Following the discussion preceding Eq. (60), the reflected electric field consists of two contributions

$$E'(r, t) = E'_{SDP}(r, t) + E'_b(r, t).$$ \hfill (91)

where $E'_{SDP}$ denotes the contribution of the integration along the SDP, which is given in Eq. (87), and $E_b$ denotes the contribution of the integration around the branch cut of $\vec{\omega}_b$ (or $\vec{\omega}_b$) in Eq. (57). Following the $K$-frame representation in Eq. (62), we define

$$L_1 = \sqrt{\alpha^2(z_0 + \beta^2z - vt)/\cos\theta_c},$$

$$L_2 = \sqrt{\alpha^2(z_0 - z + vt)/\cos\theta_c},$$

$$L_3 = R \sin\phi' - L_1 \sin\theta_c + \sqrt{\alpha^2(z - z_0 - vt)} \tan\theta_c.$$ \hfill (92)

By inserting Eqs. (63) and (70) into Eq. (7) and then into the field transformation in Eq. (4) and using ILT in Eq. (2), we obtain

$$E'_b(r, t) \sim \mathbf{x}\Psi'_b(r, t) \exp[j\omega^2 n(t - \beta z/c)],$$ \hfill (93)

where

$$E'_b = \frac{I_0 r^2 e^{\gamma/2} \mu_0 n t \omega(\Psi')}{\sqrt{8\pi k_1 R^2}}(1 - 2\beta n_i^2 \cos\Psi' + \beta^2 n_i^2)\exp(j\Psi'_b).$$ \hfill (94)

with

$$\Psi'_b = -k_1 L_1 - \pi/4 + \beta n_i^2 k_1 \sqrt{\alpha^2 z_0}.$$ \hfill (95)

By comparing Eq. (64) to Eqs. (45) with (50), $E'_b$ is identified as the incident ray that propagates from the source and impinges on the interface at the critical angle $\theta_c$ in Eq. (66).

The $D$ term in Eq. (93), which is given by

$$D = \frac{2j \sin\theta_c \sqrt{k_1 L_1}}{\sqrt{\alpha^2(\cos\theta_c)^2/2} \cos\theta_c - n_i^2 \beta \sqrt{1 - n_i^2 \beta} \cos\theta_c},$$ \hfill (96)

is identified as the relativistic diffraction coefficient. The $E'_y$ and $E'_b$ terms in Eq. (93) are given by

$$E'_y = \frac{\exp(-j k_2 L_2)}{(k_2 L_2)^{3/2}},$$ \hfill (97)

$$E'_b = \exp(-j k_2 L_2) \exp[j \alpha n_i^2 k_1 \sqrt{\alpha^2 (z_0 - z + vt)}].$$ \hfill (98)

C. Transferred Field

By applying the field transformation to Eq. (30), we obtain the asymptotic refracted fields in $K$-frame:

$$E'_r(r, t) \sim -\mathbf{x}\bar{E}'_1(r, t)E'_y(r, t) \exp[j \omega^2 n(t - \beta z/c)],$$ \hfill (99)

where

$$E'_1 = I_0 r^2 e^{\gamma/2} \mu_0 \omega(\Psi'_1)\exp(-j k_1 R'_1 + j k_2 n_i^2 \beta \sqrt{\alpha^2 z_0 - j z/4})/\sqrt{8\pi k_1 R'_1},$$

$$E'_2 = \frac{T(\Psi'_1) \exp(-j k_2 R'_2)}{\sqrt{1 + R'_2 / \beta^2}}.$$ \hfill (100)

with $T(\Psi'_1) = T(1 - n_i^2 \beta \cos\phi'_1 + n_i^2 \beta \cos\phi'_2)$. $T'$ is given in Eq. (77), and

$$\cos\phi'_1 = \sqrt{\alpha^2(z_0 + vt - \beta^2 z)/R'_1},$$

$$\cos\phi'_2 = \sqrt{\alpha^2(z - z_0 - vt)/R'_2}.$$ \hfill (101)
\[
R_1^t = \sqrt{y_P^2 + \alpha \gamma^4(z_0 + vt - \beta^2 z)^2},
\]
\[
R_2^t = \sqrt{(y - y_P)^2 + \alpha \gamma^4(z_0 - vt - z)^2},
\]

where \(y_P = R_0^t \sin \phi_t\) denotes the \(y\)-axis value of the intersection point \(P\) of the incident ray and the interface (see Fig. 7).

6. CONCLUSIONS

In this paper we have investigated the canonical problem of the 2D longitudinal Green’s function of a uniformly moving (lossless and dispersion-free) dielectric–magnetic medium with a planar discontinuity. The exact solution in the form of PW spectral integrals, as well as asymptotic solutions, were obtained in both the laboratory frame and the comoving frame. Interpretation of the relativistic asymptotic solutions in the form of ray fields was given. New canonical ray forms were derived for the relativistic incident fields, reflected fields, lateral wave, and transmitted (refracted) fields. This research and future investigations can eventually lead to establishing a relativistic geometrical theory of diffraction.

REFERENCES

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