TRANSIENT RADIATION FROM APERTURE ANTENNAS: EFFICIENT CALCULATION OF TIME-DOMAIN EFFECTIVE HEIGHT

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Abstract - This contribution proposes a new method for the efficient calculation of time-domain aperture effective height. By a suitable use of data fitting and signal deconvolution, the time-consuming calculation of time-dependent radiation integral is avoided. The method is demonstrated by some examples.

INTRODUCTION

Ultra-wide band (UWB) technology is recently originating lots of new applications [1] in communications, measurements and radar identification and imaging. Design of UWB antennas requires to exploit true time domain (TD) electromagnetic models, not only to quickly achieve a broadband antenna response but also to investigate on wavefront propagation and distortion directly in the time domain. To this purpose, time-dependent effective height is commonly used [2] for the complete characterization of the antenna properties. Recently, the authors have proposed a combined application of the Finite-Difference Time-Domain (FDTD) method, data-fitting models and TD Radiation Integral [3] for the transient analysis of aperture antennas such as slots, open-ended waveguides and ridged horns. Following a similar approach, this contribution presents a further development of the method aimed to the efficient calculation of TD effective height avoiding time consuming processing.

DESCRIPTION OF THE METHOD

An aperture $S_a$ on an infinite ground plane in $z = 0$ ($\hat{z}$ is the normal unitary vector on $S_a$) is driven, through a cavity or a waveguide (horn) section by a general-shape probe connected, by means of a transmission line, to a real voltage generator $v_g(t)$ of internal resistance $R_g$. $E_a(\rho, t)$, $\rho \in S_a$, denotes the tangential electric field on the aperture and the vector $r = (r, \theta, \phi)$ ($r = 0$ being the aperture centre) the coordinate of observation points in the half–space $z > 0$. Following the formulation in [2], the antenna far field is expressed in time domain, within the assumption that the voltage generator is matched to the transmission line, as:

$$E(r, t) = -\frac{1}{8\pi c \eta_0 R_g} v_g(\cdot) * h(r, \cdot)(t - t_g)$$

where $\eta_0$ is the free-space impedance, "*" denotes convolution , $t_g$ is the time delay along the line and $h(r, t)$ is the time-domain antenna effective height which depends on time and on the observation point. To simplify the notation, without loss of generality, we hereafter suppose $t_g = 0$, e.g. the voltage source is directly connected to the antenna’s terminals. The time-domain effective height can be formally defined by considering an impulsive voltage generator $v_g(t) = K_0 \delta(t)$ with $K_0 = 1$ V/s for simplicity. In this case, denoting with $E_a^\delta(r, t)$ the corresponding aperture field, it is possible to show that

$$h(r, t) = \frac{4R_g}{\eta_0} \hat{r} \times \int_{S_a} \frac{\partial}{\partial t} E_a^\delta(\rho, t - \frac{r - \hat{r} \cdot \rho}{c}) d\rho \times \hat{z}$$

According to a conventional approach, the above formula is calculated numerically provided that the impulsive aperture field is known in the time domain. Additionally, because of the coupling between space and time parameters, the surface integral needs to be recalculated at any required time $t$ and observation point $(r, \theta, \phi)$. Nevertheless a great simplification in the computational effort may be achieved, for the case of canonical apertures such as rectangular and circular shapes, by using the proposed computation strategy. Starting from the formulation in [3], the FDTD method

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models the antenna together with a small external region close to the aperture. The aperture field is computed when a broadband test signal \(v_0(t)\), for instance a gaussian pulse, excites the antenna. During the FDTD run, aperture field is expanded onto time-independent aperture basis functions \(\{e_p\}\), which are the transverse eigenvectors (modes) of the waveguide having \(S_a\) cross-section:

\[
E_a(\rho, t) \approx \sum_{p=1}^{N} v_p(t)e_p(\rho)
\]  \(\text{(3)}\)

The time-dependent coefficients \(\{v_p(t)\}\) are obtained by discretization of internal products \(v_p(t) = \iint_{S_a} E_a(\rho, t) \cdot e_p(\rho) d\rho\). A similar representation is also considered for the impulsive aperture field \(E_a^p(\rho, t)\) according to \(\{g_p(t)\}\) coefficients which have the meaning of impulse responses between the antenna voltage test source \(v_0(t)\) and the amplitude \(\{v_p(t)\}\) of the aperture patterns and they are retrieved by means of the Moment Expansion deconvolution method \(\text{[4]}\). We further introduce the following data-fitting model of the impulse response \(g_p(t)\):

\[
g_p(t) \approx \sum_{k=-K_p}^{K_p} g_{pk}e^{s_{pk}t}U(t - t_p)
\]  \(\text{(4)}\)

where the complex pole \(s_{pk}\) and the corresponding residue \(g_{pk}\) are estimated numerically \(\text{[5]}\). Above expansion is extremely fast convergent in the late transient, when the complex \(\{g_{pk}\}\) take into account the oscillating contributes due to the complex natural resonances \(\{s_{pk}\}\) of the source and of the multiple diffraction from the guiding section (or cavity) and the aperture wedges. It is expected that a larger number of exponentials are required to fit the early transient.

By combining (2) with (3) and (4), an approximate expression of the effective height is:

\[
h(r, t) \approx -\frac{4R_d}{\eta_0} \left[ \sum_{p,k} g_{p,k}I_{pk}(r, t) \right]
\]  \(\text{(5)}\)

which is computationally advantageous over the direct calculation of (2) provided that the following integral may be computed analytically:

\[
I_{pk}(r, t) = \hat{r} \times \frac{\partial}{\partial t} \iint_{S_a} e^{s_{pk}(t - \frac{\rho}{c} - t_0)} U(t - \frac{r - \hat{r} \cdot \rho}{c} - t_0)e_p(\rho)d\rho \times \hat{z}
\]

Denoting with \(F_{t,p}(\theta, \phi, \omega) = \iint_{S_a} e_p(\rho)e^{j\frac{\omega}{c}\rho}d\rho\) the transverse component of the \(p\)-th modal space factor \(\text{[3]}\), e.g. the spectral Fourier transform of the \(p\)-th aperture field pattern, it is possible to recognize that for times \(t > t_r = \frac{\pi + \theta_0}{c} + t_0\), \(\rho_{\text{max}}\) being the maximum distance of source points from the aperture centre, the integral \(I_{pk}\) is directly obtained from the modal space factor when \(\omega\) is replaced by \(-js_{pk}\). By recalling that \(F_p = F_{t,p} - \tan(\theta)(F_{x,p} \cos \phi + F_{y,p} \sin \phi)\hat{z}\), after simple vectorial manipulations the following expression is obtained:

\[
I_{pk}(r, t) = s_{pk} \frac{e^{s_{pk}(t - \frac{\rho}{c} - t_0)}}{2\pi c} F_{pk}(\theta, \phi) \cos \theta
\]  \(\text{(6)}\)

Closed form expressions of \(F_{pk}(\theta, \phi)\) have been found in \(\text{[3]}\) for both rectangular and circular apertures. As it will be shown later on, the transient far field phenomenology is therefore described by only a small number of \(\{s_{pk}, g_{pk}\}\) couples.

**NUMERICAL EXAMPLE**

To characterize the effective height of the pyramidal ridged horn in Fig.1a, the antenna is preliminary sourced by a test voltage gaussian pulse \(v_0(t)\) with -3dB bandwidth [0,6.5 GHz]. Effective
heights, obtained from FDTD-computed aperture field, deconvolution and equation (6), and corresponding to some aperture basis functions are shown in Fig.1b. As expected, it is possible to note that the oscillation frequency increases with the aperture basis functions’ order. To demonstrate the accuracy of the method, the transient radiated field, corresponding to a more complex excitation signal \( v_{in}(t) \), has been computed by the proposed method involving the convolution in (1) and the approximate expression in (5). A reference solution has been produced by means of an FDTD model which is directly sourced by \( v_{in}(t) \) and extends within a larger region including the far field test points. Comparison in Fig.1d shows a good agreement between reference solution and data computed by the new method, with a correlation of 0.99 when 30 pole-residue couplets are used.

**CONCLUSIONS**

The presented method permits an efficient characterization of aperture radiation for real transmitting signals through a simplified representation of TD effective height which requires the storage of a small amount of data. The allowed bandwidth for input signals is determined by the accuracy of the FDTD model and by the bandwidth of the test signal used to calculate the aperture impulsive field. Finally, thanks to the exponential form of the effective height, the convolution in (1) can be performed in a very efficient way.

**REFERENCES**


