COMPUTATIONAL COMPLEXITY
OF THE MOMENT METHOD
FOR VARIOUS MATRIX CALCULATION SCHEMES

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Abstract: The computational cost of the square matrix element evaluation, pertaining to the Moment Method (MoM), is estimated for various numerical integration schemes. Both surface and volumetric problems are considered, along with the relevant discretization of the respecting geometries. The Green’s function singularity extraction, aimed at increased accuracy, and its effect on the computational complexity, is also taken into account. The original integrals are transformed into the local (area or volume) coordinate systems, and are subsequently evaluated on the basis of standard numerical quadrature schemes. The resulting overall computational complexity of MoM is estimated, combining both matrix fill and matrix inversion, and direct comparisons among the various schemes are presented.

1. INTRODUCTION
The Moment Method (MoM) [1] is widely used in scattering and radiation problems for the conversion of a surface (or volume) integral equation into the approximate form of a matrix equation. Appropriate discretization schemes with suitable basis functions have been developed both for perfectly conducting (PEC) and dielectric scatterers [2,3], where explicit expressions for the square matrix elements evaluation are developed. The computational complexity of MoM depends on the evaluation of these elements (matrix filling) and on the matrix inversion algorithm. In previous works, the well-known centroid approximation has been used to approximate the integrations over elementary surfaces and volumes.

In this paper, the computational complexity of MoM is estimated, with, or without the centroid approximation, and for various matrix calculation schemes. Very few previous studies have been performed in the past [4,5] focused on the comparison of computational complexities of various numerical methods. Therefore this work is useful to researchers concerned with computational restrictions, associated with the application of MoM, or other related methods, especially to medium-size geometries.

2. ANALYTICAL AND NUMERICAL CONSIDERATIONS
2.1. PEC Case-Surface Integral Equation (SIE). In [2], scattering from arbitrary conducting surfaces using the centroid approximation has been investigated. In this paper, the Green’s function singularity is further taken into account and extracted, whereas the calculation is also performed without any approximation.

2.1.1. Centroid Approximation. The evaluation of matrix elements reduces to the calculation of the two potentials (vector and scalar). The Green’s function singularity in the integral yielding vector potential $\mathbf{A}$ is initially extracted, i.e.

$$\int_{S} \rho_n^+(r') \frac{e^{-jk_{m}R}}{R} dS' = \int_{S} \rho_n^+(r') \frac{e^{-jk_{m}R}}{R} dS' + \int_{S} \frac{\mathbf{e} - \rho}{R} dS' + \int_{S} \frac{\mathbf{e} - \rho}{R} dS'$$

The first integrand in (1) is bounded and thus can be numerically integrated. The last two integrals can be evaluated analytically [6].

Similarly, the integral representing the scalar potential $\Phi$ is written as

$$\int_{S} \frac{e^{-jk_{m}R}}{R} dS' = \int_{S} \frac{e^{-jk_{m}R}}{R} dS' + \int_{S} \frac{1}{R} dS'$$

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Using local area coordinates, the original integrals are transformed into the form

\[
\int \int e^{-jkR} - \frac{1}{R^p} d\zeta d\eta, \int \int e^{-jkR} - \frac{1}{R^p} d\zeta d\eta
\]

(3)

Based on (3), the matrix calculation schemes in [7], combined with Gauss elimination [8] for the matrix inversion, can be shown to be prone to the following computational costs:

- Scheme #1 ([7], p. 148, K=3) \#N_1 \geq 1077N^2 + 1/6(2N^3 + 5N^2 + 9N)
- Scheme #2 ([7], p. 149, Radau, Table 8.2, n=3) \#N_2 \geq 785N^2 + 1/6(2N^3 + 5N^2 + 9N)
- Scheme #3 ([7], p. 151, Hammer et al, Table 8.3, n=3) \#N_3 \geq 300N^2 + 1/6(2N^3 + 5N^2 + 9N)

2.1.2. Exact expression. Without the use of the centroid approximation, the expression of matrix elements for all pq edge-pairs is [9]

\[
Z_{pq}^{pm} = \frac{j\eta_0}{8A_p A_q} \left[ \int e^{-jkR} \left| \begin{array}{cc} \rho_m^+(r) & \rho_m^+(r') \\ e^{-jkR} & R \end{array} \right| dS'dS - \frac{1}{\pi^2} \int e^{-jkR} R dS' dS \right]
\]

(5)

The first double surface integral is essentially the vector potential \(A\), whereas the second is the scalar potential \(\Phi\). Again for the three aforementioned calculation schemes, and on the basis of Gauss elimination for the matrix inversion, we obtain the following results:

- Scheme #1 \#N_4 \geq 8997N^2 + 1/6(2N^3 + 5N^2 + 9N)
- Scheme #2 \#N_5 \geq 6540N^2 + 1/6(2N^3 + 5N^2 + 9N)
- Scheme #3 \#N_6 \geq 520N^2 + 1/6(2N^3 + 5N^2 + 9N)

In the figure below, all computational complexities above (#N_n, i=1,6) are plotted, versus the number of MoM unknowns.

![Fig. 1. Computational costs for various matrix evaluation schemes (SIE).](image)

2.2. Dielectric Case-Volume Integral Equation (VIE). A detailed analysis is carried out by Schaubert et al in [3].

2.2.1. Centroid Approximation. The procedure for tetrahedral elements and dielectric bodies resembles the SIE. Using the centroid approximation, the expressions for the potentials are given in [3], where the integrals are formally analogous to (1) and (2), but covering volume, instead of surface elements.

The three following expressions originate from the invocation of local volume coordinates in the calculation of four independent scalar integrals over each tetrahedron, for the aforementioned matrix calculation schemes:

- Scheme #1 \#M_1 \geq 4157N^2 + 1/6(2N^3 + 5N^2 + 9N)
- Scheme #2 \#M_2 \geq 2220N^2 + 1/6(2N^3 + 5N^2 + 9N)
- Scheme #3 \#M_3 \geq 540N^2 + 1/6(2N^3 + 5N^2 + 9N)

(7)
2.2.2. Exact expression. Without the use of any approximation, the numbers of multiplications for the three calculation schemes become

<table>
<thead>
<tr>
<th>Scheme</th>
<th>#M_i</th>
</tr>
</thead>
<tbody>
<tr>
<td>#1</td>
<td>31150N^2 + 1/6(2N^3 + 5N^2 + 9N)</td>
</tr>
<tr>
<td>#2</td>
<td>17225N^2 + 1/6(2N^3 + 5N^2 + 9N)</td>
</tr>
<tr>
<td>#3</td>
<td>1040N^2 + 1/6(2N^3 + 5N^2 + 9N)</td>
</tr>
</tbody>
</table>

(8)

In the figure below, all computational complexities above (#M_i, i=1,6) are plotted, versus the number of MoM unknowns.

Fig. 2. Computational costs for various matrix evaluation schemes (VIE).

3. CONCLUSIONS

An estimation of the MoM computational complexity for various matrix calculation schemes was performed, with or without the centroid approximation, both for SIE and VIE. Comparative plots were depicted in two figures, for the SIE and VIE respectively. Although for very large numbers of unknowns all schemes behave almost identically (O(N^3)), for moderately large geometries there is a considerable fluctuation of the computational cost among the schemes, which should be taken into account whenever CPU time restrictions are important.

4. REFERENCES