APPLICATION OF MOMENT METHOD WITH MULTIGRID TO 
SCATTERING OF A GAUSSIAN BEAM BY A DIELECTRIC CYLINDER

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Abstract: Scattering of a Gaussian beam by a dielectric cylinder is analyzed by using the multilevel moment method. The shape of the cylinder is arbitrary. The scattered fields can be obtained by the volume equivalent theorem. The integral equation is converted to the matrix form by the moment method. As numerical examples, the scattering of a Gaussian beam by a circular dielectric cylinder is considered. The computation time and residual norm are examined for V-cycle, W-cycle and saw-cycle of the multigrid. The scattered near fields are also calculated and are compared with the exact solution.

INTRODUCTION
Scattering problem is one of the important issues in electromagnetic theory. So far, many books [1]–[3] and papers have been published. Recently, the problems for the scattering of the large size compared to the wavelength and of the random medium are interested from the practical point of view. In this case, the large size matrix equation has to be solved as fast as possible. FMM (Fast Multipole Method) is one candidate to solve these problems [4]. Multigrid method [5]–[7] is another fast solver. This method is applied to various problems such as the analysis of corner reflector antenna [8], the analysis of the eigenvalue problem in anisotropic microstrip line [9], [10], and the scattering from the rough surface [11] and so on.

In this article, the moment method with multigrid is applied to the scattering of a Gaussian beam by a dielectric cylinder. The scattered field is obtained by the volume equivalent theorem. The integral form is converted into the matrix equation by the moment method. The multigrid method is applied to the matrix equation to solve as fast as possible. The effect of the number of the cycle indices and cycles on the residuals are examined numerically. Also, the scattered near fields are calculated for various parameters of the multigrid method.

SCATTERING BY A DIELECTRIC CYLINDER
Consider the scattering of a Gaussian beam by a dielectric cylinder as shown in Fig. 1. The relative permittivity \( \varepsilon_r \) is assumed to be inhomogeneous. The incident electric field is polarized in the \( z \) axis. In this case, the scattered field is expressed in the integral form as follows [3]:

\[
E_s^z(r) = -j\omega\mu_0 \int_{S'} G(r, r') J_{eq}(r') dS'
\]  

(1)

where the equivalent current \( J_{eq}(r) \) is given by

\[
J_{eq}(r) = j\omega\varepsilon_0 (\varepsilon_r - 1) E_z^n(r)
\]

(2)

and Green’s function \( G(r, r') \) is given by the Hankel function of the second kind of order 0. After adding the incident field to the both side in Eq. (1) and applying the moment method, the following equation is obtained.

\[
\sum_{n=1}^{N} C_{mn} E_n = E_m^i, \quad m = 1, \ldots, N
\]

(3)

where

\[
C_{mn} = \delta_{mn} + \frac{jk^2}{4} \left( \varepsilon_r(n) - 1 \right) \int_{cell \ n} H_0^{(2)}(k\rho) dx'dy'
\]

(4)

and \( E_n \) and \( \varepsilon_r(n) \) are the total electric field and the relative permittivity at \( n \)th cell. \( N \) is the total number of unknowns, \( \rho = \sqrt{(x_m - x')^2 + (y_m - y')^2} \), and \( E_m^i \) is the incident field at \( (x_m, y_m) \), which is the center of the cell \( m \).

MULTIGRID METHOD
The matrix equation (3) is solved by W-cycle algorithm of the multigrid method with the Kaczmarz iteration method. Since the matrix equation obtained here is not the diagonal dominance, the Gauss-Seidel method is
not suitable. Kaczmarz method has been proved to converge for any system of linear equations [12].

The outline of the multigrid method is as follows [5]–[7]. At first, the matrix equation is solved on the finest grid. The residual is restricted to the coarser grid and the corrected matrix equation is solved by using the restricted residual. The correction is interpolated to the finer grid and the guess is modified. Then, the matrix equation is solved by Kaczmarz method with the modified guess and the next approximation is obtained.

**NUMERICAL RESULTS**

As numerical examples, the scattered near field of a Gaussian beam by a dielectric circular cylinder is calculated and compared with the previous results in order to examine the validity. The smallest spot size of the incident beam is \(w_0 = \lambda\) and its location is \(x_0 = -2\lambda, y_0 = 0\). The wavelength \(\lambda = 1.55\mu m\). The incident Gaussian beam is expressed by the complex-source-point method. The radius of the circular cylinder is \(a = \lambda\) and the relative permittivity is \(\varepsilon_r = 4.0\). The total number of unknowns \(N\) is \(48 \times 48\). Three-level scheme of the multigrid method is applied.

In order to check the accuracy of the scheme, the residual norm is defined as follows

\[
\text{residual norm} = \sqrt{\sum_{s=1}^{N} |E_{is} - \sum_{m=1}^{N} C_{sm} E_{im}|^2}
\]  

\(\text{(5)}\)

Figure 2 shows the residual norm and computation time by using Kaczmarz method, which corresponds to one-level scheme. As is shown, when the number of the iteration increases, the residual norm decreases and it takes the computation time.

The computation time for various number of the cycles \(m\) as a function of the cycle index \(\gamma\) in W-cycle is shown in Fig. 3. The case of \(\gamma = 1\) corresponds to V-cycle. Figure 4 indicates the residual norm. For these figures, the number of iterations \(l = 5\) is assumed. The residual norm is improved when \(m\) changes to 2 from 1, but the improvement is not found for \(m > 3\). The scattered near field at \(x = 2\lambda\) is shown in Fig. 5 for various number of the cycles. The number of iterations and the cycle index are set as \(l = 5\) and \(\gamma = 40\). It is confirmed that the scattered near field for \(m = 2\) becomes almost the same as the exact one. The exact solution is obtained by the eigenfunction expansion method. The computation time for \(l = 5, m = 2\) and \(\gamma = 40\) is 47 sec, which is about 1/4 of LU decomposition.

Finally, the residual norm and the computation time for saw cycle are indicated in Fig. 6. The solid lines and the solid circles indicate the residual norm and the computation time, respectively. The solid circles with the same colors as the residual correspond to the same \(l\). It is seen that when the number of the cycles \(m\) increases, the residual norm decreases and it takes much computation time. It is also found that saw cycle requires much computation time than W-cycle in the present model.

**CONCLUSIONS**

The scattering of a Gaussian beam by a dielectric cylinder has been examined by the moment method with multigrid. It is shown that W-cycle is more desirable in this model. We will investigate the scattering by the nonlinear cylinder and a chiral cylinder.

**REFERENCES**


