Abstract: A new higher-order method of moment (MoM) technique is presented for volume-surface integral equations (VSIE) for electromagnetic modeling of composite metallic and dielectric objects. The higher-order MoM scheme comprises higher-order hierarchical Legendre basis functions and an accurate representation of the object by higher-order curvilinear elements. Due to the orthogonal nature of the basis functions the continuity condition at the interface between metal and dielectric is satisfied explicitly. By utilizing the continuity condition the number of unknowns can be significantly reduced, especially for metal objects with thin dielectric coating.

INTRODUCTION
The coupled volume-surface integral equation (VSIE) is often employed for radiation and scattering analysis of composite metal and dielectric objects. In the VSIE formulation, microstrip antennas can be simulated taking into account the finite-size effects of the dielectric substrate. The VSIE is also suitable for modeling of scattering by metal objects with dielectric coating. Due to its volume integral equation part the VSIE treats inhomogeneous dielectric materials more accurately than the piece-wise constant approximation required by the standalone surface integral equation.

Straightforward method of moment (MoM) solutions of integral equations become ineffective as the size and complexity of the object under investigation increases. Consequently, accelerated integral equation solvers, such as the fast multipole method (FMM/MLFMM), have been developed. These solvers, which are usually based on low-order basis functions (RWG, rooftop, or pulse), can operate on a huge number of unknowns demanding a moderate amount of computer memory.

In this paper we suggest to reduce the number of unknowns by utilizing higher-order basis functions. Defined on rather large geometry elements these functions generally require much less unknowns per wavelength than low order basis functions. Here we use higher-order hierarchical Legendre basis functions [1] to solve the volume-surface integral equation. Previously, these functions have been applied to the analysis of metallic objects in free space [1] and in layered media [2] by the surface integral equations, and to the analysis of dielectric objects by the volume integral equations [3]. Being near-orthogonal the higher-order hierarchical Legendre basis functions allow a low condition number of the MoM matrix to be achieved. Moreover, since the electric charge is expanded in orthogonal functions, the continuity condition at the boundary between metal and dielectric can be enforced explicitly, which further reduces the number of unknowns. The reduction can be significant especially for metal objects with thin dielectric coating.

FORMULATION
The volume-surface integral equation comprises two coupled equations for a dielectric volume $V$ and a perfectly electrically conducting (PEC) surface $S$

$$
E'(\vec{r}) = \overline{E}(\vec{r}) - E'_i(\vec{r}) - E'_s(\vec{r}), \quad \vec{r} \in V;
$$

$$
E'(\vec{r}) = -E'_i(\vec{r}) - E'_s(\vec{r}), \quad \vec{r} \in S
$$

where $\overline{E}(\vec{r})$ is the total electric field, $E'_i(\vec{r})$ is the incident electric field, $E'_s(\vec{r})$ and $E'_s(\vec{r})$ denote the electric field scattered by the unknown induced electric volume current density $J'_v(\vec{r})$ and the electric surface current density $J'_s(\vec{r})$, respectively. It is more convenient to use the electric flux density $D(\vec{r})$ rather than $J'_v(\vec{r})$ as the unknown because the normal component of $\overline{D}(\vec{r})$ is continuous across the boundary between two different dielectric materials.

Higher-Order MoM: Following the higher-order discretization procedure, the geometry of the object is represented by higher-order curvilinear elements; hexahedra for dielectric volumes, and quadrilaterals for...
PEC surfaces. A unique mapping between the curvilinear coordinate system \((u, v, w)\) \((u, v)\) for quadrilaterals) and the physical space \((x, y, z)\) is defined using Lagrange interpolation. The components of the unknown vector functions \(\mathbf{D}(\mathbf{r})\) and \(\mathbf{J}_S(\mathbf{r})\) at each element are expanded in terms of the higher-order hierarchical Legendre basis functions as \([1, 3]\)

\[
\mathbf{D}^\xi = \frac{1}{A} \sum_{m=0}^{M} \sum_{n=0}^{N} \sum_{q=0}^{Q} a_{mnq}^\xi \tilde{P}_m(\xi)P_n(\eta)P_q(\zeta), \\
\mathbf{J}_S^\xi = \frac{1}{A} \sum_{m=0}^{M} \sum_{n=0}^{N} \sum_{q=0}^{Q} b_{mnq}^\xi \tilde{P}_m(\xi)P_n(\eta),
\]

\((\xi, \eta, \zeta)\) is \((u, v, w)\), \((v, w, u)\), or \((w, u, v)\), \(-1 \leq u, v, w \leq 1\).

\[
\tilde{P}_m(\xi) = \begin{cases} 1 - \xi, & m = 0 \\ 1 + \xi, & m = 1 \\ P_m(\xi) - P_{m-2}(\xi), & m \geq 2. \end{cases}
\]

Herein, \(A\) is the Jacobian of the parametric transformation, \(P_m(\xi)\) are Legendre polynomials, \(a_{mnq}^\xi\) and \(b_{mnq}^\xi\) are unknown coefficients for the electric flux density and surface current density, respectively. \(M^\xi, N^\eta,\) and \(Q^\zeta\) denote the expansion orders along the parametric directions.

Although the expansions of \(\mathbf{D}(\mathbf{r})\) and \(\mathbf{J}_S(\mathbf{r})\) in (2) are not perfectly orthogonal, they yield orthogonal expansions of the corresponding charge densities. For instance, the surface charge density associated with the current component \(\xi_{SJ}\) is expressed as

\[
\rho_s^\xi = \frac{1}{\omega} \frac{d}{d\xi} \left( AJ_S^\xi \right) = \frac{1}{\omega} A \sum_{n=0}^{N^\eta} \left\{ b_{0n}^\xi P_n(\eta) \right\} + \frac{1}{\omega} A \sum_{m=2}^{M^\xi} \sum_{n=0}^{N^\eta} \left\{ b_{mn}^\xi (2m-1)P_{m-1}(\xi)P_n(\eta) \right\}.
\]

Continuity condition. If the object contains interfaces between dielectric and PEC surfaces, the continuity condition

\[
\hat{n} \cdot \mathbf{D} = \rho_s,
\]

at these boundaries must be satisfied, where \(\hat{n}\) is a unit vector normal to the dielectric-PEC interface. Due to the orthogonality of the expansion (3), the continuity condition (4) can be satisfied explicitly. Thus, if one of the faces of a dielectric hexahedral element is shared with a PEC quadrilateral, the coefficients \(a_{mnq}^\xi\) of the electric flux density over this face can be expressed in terms of coefficients \(b_{mnq}^\xi\) associated with the electric currents on the PEC quadrilateral. For instance, unknowns associated with the cell face \(w=1\) can be removed simply by using

\[
a_{mnq}^w = \frac{1}{2\omega} \begin{cases} b_{00}^w - b_{00}^q + b_{01}^q - b_{00}^w, & n = q = 0 \\ (2n+1)b_{w+1}^w + b_{w1}^q - b_{00}^w, & n > 0, q = 0 \\ b_{w1}^q - b_{00}^q + (2q+1)b_{w(q+1)}^w, & n = 0, q > 0 \\ (2n+1)b_{w+1}^w (2q+1)b_{w(q+1)}^w, & n > 0, q > 0. \end{cases}
\]

Consequently, the number of unknowns in the MoM system reduces. It is worth noting that for other types of higher-order hierarchical basis functions, which do not possess the favorable feature of orthogonality as in (3) (for instance power basis functions), it is much more difficult to satisfy the continuity condition explicitly. The continuity condition can be enforced explicitly also with the RWG basis functions [4]. However, in accelerated solvers it is usually satisfied only numerically for the sake of implementation simplicity [5].

**NUMERICAL RESULTS**

The first example is a coated PEC sphere of radius 1.5\(\lambda_0\) illuminated by a plane wave. The dielectric coating has the permittivity \(\varepsilon_r = 1.5 - j0.5\) and thickness 0.1\(\lambda_0\). The surface of the PEC sphere and the dielectric coating are approximated by 54 second-order quadrilateral and 54 second-order hexahedral elements, respectively. The surface current \(\mathbf{J}_S(\mathbf{r})\) is expanded with the maximum order \(M^\xi = 4\). For the electric flux
density $D_\ell (\mathbf{r})$ expansion orders $M^z = 4$ and $M^z = 1$ are used for tangential and radial directions, respectively. The bistatic radar cross section (RCS) computed with 4,236 unknowns is presented in Fig. 1 along with the exact Mie series reference result. Due to (5), 828 unknowns have been eliminated. Note, that this problem is solved by MLFMM [6] using 19,992 unknowns.

In the second example a coated PEC tangent ogive is analyzed. The length of the ogive is $L = 3.5m$, and base diameter is $d = 0.9m$. The dielectric coating is 0.075$m$ thick, and made of dielectric with $\varepsilon_r = 3.0 - j0.09$. The monostatic RCS computed in the meridional plane of the ogive at 0.25GHz is plotted in Fig. 2. Corresponding parameters used in the simulation are summarized in Table 1. The result by MLFMM [6] is also presented for comparison.

### CONCLUSIONS

The higher-order method of moment is applied to solve the volume-surface integral equation for scattering by composite metal and dielectric objects. Higher-order curved quadrilateral and hexahedral elements are used for accurate modeling the object geometry. The unknown electric surface currents on the PEC surfaces and electric flux density in the dielectric are expanded in terms of higher-order hierarchical Legendre basis functions. Orthogonal representation of the charge by these functions allows the continuity condition at the metal-dielectric interfaces to be forced explicitly, and, consequently, reduce the number of unknowns. The presented approach is validated by the exact results for scattering by the layered sphere. A significant reduction in the number of unknowns is achieved in comparison with the known MLFMM implementations.

### REFERENCES


