ELECTROMAGNETIC SCATTERING FROM NATURAL SURFACES: VALIDATION OF A FRACTAL APPROACH

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Abstract: In this paper we present a framework to validate recently developed electromagnetic scattering methods, which employ the fractal geometry to describe the surfaces. In particular, a fractal surface has been designed and built. Its characteristics have been tailored for X-band laboratory experiments and its building was performed by using cheap and light materials (cardboard and aluminum). It has been verified that the surface parameters were compliant with the required characteristics and that their values are in the range of validity of the Kirchhoff approximation theory for the evaluation of the electromagnetic scattered field. Then, the scattered field measurements have been performed in an anechoic chamber using a HP8510 network analyzer. The obtained data are currently being calibrated, in order to make them ready to be compared with the forecasted field.

I. INTRODUCTION
The introduction of the fractal geometry provided researchers involved in the remote sensing community a powerful tool for a reliable natural surfaces description (Mandelbrot [1]). In particular, the fractional Brownian motion (fBm) and the Weierstrass-Mandelbrot function (WM) are the most suitable functions to this aim, and their use in the evaluation of the scattering from natural surfaces allowed to obtain encouraging results (Franceschetti et al. [2]). Their use in conjunction with the Kirchhoff approximation (KA) and the small perturbation method (SPM) (Franceschetti et al.) or the extended boundary condition method (EBCM) (Franceschetti et al. [3] et al.) led to fit measured data better than the same scattering methods did with the classical Gaussian surface description. The main limits to a full comprehension of the obtained results are related to the lack of measured data. In fact, previous experiments did not relate the field scattered by natural surfaces to their fractal parameters, and the comparison between electromagnetic scattering data was inferred by means of best fit procedures.

In this paper we present a measurement campaign planned to give an answer to this lack of data. In particular, in Section II, we define the fBm and the WM processes and we introduce the techniques to numerically design and put up the surface. In addition, the main steps of the surface realization as the superposition of a wrinkled aluminium foil on a cardboard long scale (from 0.5 cm up) topography are presented. Some comments on the analysis performed to verify that the surface characteristics are compliant with the given requirements are also provided. In Section III the expression of the backscattering coefficient in the frame of the KA is provided as a function of the fBm parameters. Then, it is verified that surface parameters are within the range of validity of the KA, and the corresponding electromagnetic scattered field is evaluated and plotted, in order to be compared with measured data. A brief description of the electromagnetic scattering measurements performed in anechoic chamber is also provided. A final discussion and the conclusions are reported in Section IV.

II. SURFACE DESCRIPTION AND BUILDING
It is widely known that the fBm is the most suitable model to account for the natural surfaces properties. It is a continuous everywhere but differentiable nowhere process defined in terms of the pdf of its increments. In particular, a surface \( z(x,y) \) is an fBm if for every \( x, y, x', y' \):

\[
\Pr\{z(x,y) - z(x',y') < \xi\} = \frac{1}{\sqrt{2\pi s\tau^H}} \int_{-\infty}^{\xi} \exp\left(-\frac{\xi^2}{2s^2\tau^{2H}}\right) d\xi
\]

(2.1)

where \( \tau \) is the distance between the points \( (x,y) \) and \( (x',y') \), \( H \) is the Hurst coefficient \((0<H<1)\), related with the fractal dimension \( D \) by means of the relation \( D=3-H \) and \( s \) is a real parameter measured in \([m(1-H)]\),
related to a characteristic length of the fBm surface, called *topothesy* $T$ by the relation $s = T^{(1-H)}$. As most of the natural surfaces do, the fBm has the characteristic power-law spectrum $S(k) = S_0 k^{-\alpha}$, with the $\alpha$ coefficient related with the Hurst coefficient as $\alpha = 2 + 2H$. The synthesis of an fBm process is a controversial issue in literature, and it is strongly related with the application for which the surface is designed. In this paper the surface has been designed as a realization of a band-limited Weierstrass-Mandelbrot (WM) function $z(x,y)$, which can be seen as a frequency sampling of an fBm process (Franceschetti [2]) and can be expressed as:

$$z(x,y) = B \sum_{p=0}^{P-1} C_p \nu^{-H_p} \sin \left[ \kappa_0 \nu^p \left( x \cos \Psi_p + y \sin \Psi_p \right) + \Phi_p \right],$$

(2.2)

where $H$ is the Hurst coefficient ($0 < H < 1$), related with the fractal dimension $D$ by means of the relation $D = 3 - H$; $B$ is a vertical height scaling factor; $\nu$ is an irrational parameter ($\nu > 1$) accounting for the frequency spacing between tones; $\kappa_0$ is the fundamental wavenumber; $C_p$, $\Phi_p$, $\Psi_p$ are random amplitude, phase and direction coefficients. The tones that are included in the sum depend on the scales involved in the interaction with the incident field. In particular, the parameter values have been chosen in order to optimise an X band measurement campaign (hence the electromagnetic wavelength $\lambda$ is about 3 cm). This set the range of scales to be considered (from about 50 $\lambda$ =1.5m to about $\lambda/10$=3mm) and, once the $\nu$ value has been set (choice of 0.5e guarantees a reliable frequency sampling of an fBm process), the number of tones. The chosen values are presented in Table 1. The number of tones $P$ turns out to be $P=20$.

<table>
<thead>
<tr>
<th>$\kappa_0$ [m$^{-1}$]</th>
<th>$B$ [m]</th>
<th>$H$</th>
<th>$\nu$</th>
<th>$T$ [m]</th>
</tr>
</thead>
<tbody>
<tr>
<td>5.71</td>
<td>0.01</td>
<td>0.7</td>
<td>0.5e</td>
<td>$7.3 \times 10^{-3}$</td>
</tr>
</tbody>
</table>

Table 1: Surface parameters.

Once the surface has been numerically synthesized, an innovative technique has been performed to set up the surface. A crucial point for a cheap and controlled surface was the choice of the materials. Hence a realization of cardboard slices shaped according with the contour curves of the surface, with width of 0.5 cm allowed to realize the long scale component of the surface. On this topography, a layer of wrinkled aluminum foils has been added, giving the object the small scale roughness and a fully reflecting property. The chosen materials allowed us to build a cheap and light solution. A photograph with the top view of the realized circular (with 1.5m diameter) surface is shown in Figure 1.

Figure 1: Top view of the built surface.
An optical measurement of the surface height with an accuracy of 80 micrometers was carried out in order to perform a deep surface analysis confirming that the surface properties are those imposed by our simulation. The variogram and the spectral analysis methods allowed to extract the realized surface characteristics and to verify that they are compliant with the required characteristics (i.e., $H$ and $B$ theoretical values reported in Table I).

III. THE ELECTROMAGNETIC METHOD

In this Section we present the electromagnetic scattering method we employ in conjunction with the fBm model to be tested. A comparison between the surface parameters and the validity limits defined in Franceschetti [2] leads to theoretically predict the field scattered from our surface in the frame of the Kirchhoff approximation. Accordingly, the backscattering coefficient can be expressed as:

$$
\sigma_{pp}^0 = \frac{R_p(\theta)}{2} k^2 T^2 \cos^2 \theta \sum_{n=0}^{\infty} \frac{(-1)^n \Gamma \left( \frac{n+1}{H} \right)}{(n)!^2} \left( \frac{H}{\sin \theta} \right)^{2n+2} \left( \frac{2\sqrt{kT \cos \theta}}{H} \right)^{2n},
$$

where $k$ is the electromagnetic wavelength, $R_p$ are the Fresnel coefficients that depend on polarisation, incidence and scattering angles, and complex dielectric constant, $\theta$ is the look angle, $\Gamma$ is the Euler function. A plot of the forecasted scattered field as a function of the incidence angle is presented in Figure 2.

![Figure 2: Theoretical backscattering coefficient in the frame of the fBm method for HH polarization.](image)

Measurements of the polarimetric radar backscattering of the synthesized fractal surface took place in a 10 m x 7.5 m x 7.5 m (l x w x h) anechoic chamber at X band using a HP8510 network analyzer. The set of measurements were carried out using a frequency range of 7 to 12 GHz, a range of incidence angles from 0 to 70º and a complete 360º rotation of the disc with angular steps of 2º. Data calibration is currently in progress. Preliminary results of comparison between measured and predicted data are promising. A complete analysis will be provided in the final presentation.

IV. DISCUSSION AND CONCLUSIONS

In this paper a procedure for the validation of theoretical methods for the evaluation of the field scattered by fractal surfaces has been presented. A surface representing a single realisation of a fractal process has been built according with strict requirements in terms of weight, cost and dimension. Its characteristics have been optimised for X band measurements. The field scattered by this surface has been predicted in the frame of the KA theory, according with the surface properties. The above described steps define a complete measurement procedure. Full analysis and comparison of scattering measured data and theoretical predictions are currently in progress, and will be presented at the conference.

REFERENCES


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