INTRODUCTION

Rigorous formulations for the electromagnetic wave scattering by canonical structures (such as wedges, cylinders ..) are the basic knowledge to develop high-frequency techniques that may improve upon electromagnetic modeling tools for estimating the scattering by complex structures. In this framework, a very important role is played by the exact solutions for the scattering at edges in impedance surfaces.

The solution for the electromagnetic scattering at skew incidence by a half plane with surfaces impedance boundary conditions on its faces was pioneered in [1]. A systematic, comprehensive review of most of the available solutions for edges in impedance surfaces may be found in [2], where the formulation is cast in a unique form based on the Maliuzhinets method [3]. In our attempt to develop a rigorous formulation of the Incremental Theory of Diffraction (ITD) [4],[5] for local half and full plane edge impedance discontinuities, it has been found that the available exact solutions for the relevant canonical problems do not explicitly satisfy reciprocity. Indeed, it is seen that those parts of the final expressions that are mainly associated to spectral derivatives and vector projections, do not exhibit the desirable symmetry with respect to incidence and observation aspects. Achieving this property, which is a basic requirement for a rigorous development of an ITD formulation, has motivated the analysis which is presented here.

In this paper, revisited exact solutions are presented for the electromagnetic problems of a half-plane with surface impedance faces and a surface impedance edge discontinuity on a plane, when they are illuminated at skew incidence. They are obtained by introducing a suitable new set of combined spectra that exhibits the desired properties. As a consequence, the final expressions of the spectra, besides satisfying all the Maliuzhinets method conditions and the edge conditions, explicitly satisfy reciprocity and nicely reduce to the original Maliuzhinets solution for normal incidence. The revisited exact solutions are cast in a neat matrix form that provides a rigorous 2D-3D transformation machinery. The achieved symmetry with respect to incidence and observation aspects, is further emphasized in extending the solutions to dipole source illuminations.

REVIEW OF THE AVAILABLE SOLUTIONS

The notation of the review presented here is referred to that in [2]. The geometry at the edge of a wedge is depicted in Fig.1.
There, following the traditional notation introduced by Maliuzhinets, the exterior wedge angle is denoted by \(2\Phi\) and the azimuthal angular coordinates have a zero reference at the bisector of the exterior wedge angle. The formulation presented here refers only to the cases of a half plane with two face impedances \((\Phi = \pi)\) and a surface impedance edge discontinuity on a plane \((\Phi = \pi/2)\).

The Maliuzhinets method suggests to seek for a solution in the Sommerfeld spectral integral form:

\[
(E_z, \zeta H_z) = \frac{1}{2\pi j} \int e^{j(k_\alpha \alpha \beta)} S_{(\phi \beta)}(\alpha + \phi) d\alpha
\]

where

\[
r(\alpha, \beta) = \rho \sin \beta \cos \alpha - z \cos \beta
\]

and \(\Gamma\) denotes the Sommerfeld contour of integration. In (1), an arbitrarily polarized, electromagnetic plane wave at skew incidence \((\phi', \beta')\) is assumed, and the total field is observed at \(P \equiv (r, \beta = \beta', \phi)\). The spectra \(S_{\sigma, \phi}\) were found by Maliuzhinets for \(\beta' = \pi / 2\), when they are subject to uncoupled bc. For \(\beta' \neq \pi / 2\), \(S_{\sigma, \phi}\) are subject to coupled bc.

It was found \([1],[2]\) that a combination \(t_{1,2}(\alpha)\) of these spectra satisfies uncoupled bc for both \(\Phi = \pi, \pi / 2\), that are the specific cases treated here. As a consequence, \(t_{1,2}\) are directly obtained from the original Maliuzhinets solution. They of course involve the Maliuzhinets special functions, and two constants \(C_{1,2}\) that are found by enforcing the residue at \(\alpha = \phi'\) to recover the incident field \((E_z', \zeta H_z') = (e_z, h_z)\). Next, once \(t_{1,2}\) are known, the explicit expressions are directly obtained for the spectra \(S_{\sigma, \phi}\) to be introduced in (1). This procedure introduces non-physical poles that need to be eliminated via somewhat cumbersome calculations. Unfortunately, it is found that this final formulation does not explicitly satisfy reciprocity, and does not lead to the desired symmetry to apply the ITD localization process.

**THE REVISITED EXACT SOLUTION**

It is found that the definition of combined spectra such as \(t_{1,2}\), has a degree of freedom, so that any combined spectra

\[
\tau_{(1,2)}(\alpha) = \lambda(\alpha) t_{(1,2)}(\alpha)
\]

in which \(\lambda(\alpha \pm \Phi) = \lambda(-\alpha \pm \Phi)\) with a \(2\Phi\) periodicity, still satisfy uncoupled boundary conditions. Therefore, also \(\tau_{1,2}\) are obtained from the original Maliuzhinets solution.

In order to achieve the desired form of the solution, the choice of \(\lambda(\alpha)\) is subject to the conditions

i) \(\lambda(\alpha) = 1\) for \(\beta' = \pi / 2\);

ii) the residue contribution at \(\alpha = \phi'\) recovers the incident field;

iii) the final form explicitly satisfies reciprocity.

Next, it is found that the definition

\[
\lambda(\alpha) = \lambda^+(\alpha) = \frac{\cos(\alpha - \Phi)}{\sin \beta \cos(\alpha - \Phi) \pm j \cos \beta}
\]

exhibits just the desired properties. The relevant final forms for the spectra \(S_{\sigma, \phi}\) satisfy all the Maliuzhinets method conditions. In order to satisfy the extinction principle, certain poles occurring in \(S_{\sigma, \phi}\), that are introduced by \(\lambda(\alpha)\), are eliminated; however, this is done in closed form. Also, these expressions nicely reduce to the original Maliuzhinets solution for \(\beta' = \pi / 2\) and explicitly satisfies reciprocity.

The achieved symmetry with respect to the aspects of incidence and observation is further emphasized by extending the solutions obtained so far to the case of dipole source illumination of the edge. It is a rather simple matter to obtain their spectral integral representations for a \(z\)-oriented electric \(p^z\) and
magnetic \( p^n \) dipole illuminations. Indeed, by analytic continuation, the solutions obtained for plane wave incidence can directly be used to weight the plane wave spectra of the \( E_z \), \( H_z \) components of the incident fields from the \( p^r \hat{z} \) and \( p^n \hat{z} \) dipoles. Thus, the relevant expressions in the spectral spherical domain are obtained as

\[
(E_z, H_z) = \frac{1}{2\pi j} \int \int \int \Sigma_{(e,h)}(\alpha + \varphi, \alpha' + \varphi'; \theta) e^{jkr(\alpha', \theta) + r(\alpha, \theta)} \sin \theta d\alpha' d\alpha d\theta \tag{5}
\]

where \( r^{(\alpha; \theta)} \) are defined as in (2), the contours of integration in \( \alpha, \alpha', \theta \) are defined along \( C \equiv (-j\infty, \pi + j\infty) \). The spectra \( \Sigma_{(e,h)} \) can be cast in the neat matrix form

\[
\Sigma(\alpha + \varphi, \alpha' + \varphi'; \theta) = \Lambda(\alpha + \varphi) \Lambda(\alpha' + \varphi') \overline{R(\alpha + \varphi)} \overline{G(\alpha + \varphi, \alpha' + \varphi')} \overline{R^*}(\alpha' + \varphi') \overline{f}_z \tag{6}
\]

which provides a rigorous 2D-3D transformation machinery. In (6)

\[
\overline{f}_z = \begin{pmatrix} e_z \\ h_z \end{pmatrix} \quad \overline{G(y', y)} = \begin{pmatrix} G_{\alpha}(y', y) & 0 \\ 0 & G_{\alpha}(y, y') \end{pmatrix} \tag{7}
\]

in which \( G_{\alpha(h)} \) are obtained directly from the original Maliuzhinets solution, and \( (e_z, h_z) \) denote the \( z \)-components of the incident fields from the \( p^r \hat{z} \) and \( p^n \hat{z} \) dipoles. Furthermore,

\[
\Delta(\xi^{(\gamma)}) = \cos^2 \xi^{(\gamma)} + \cos^2 \theta \sin^2 \xi^{(\gamma)} \quad \Delta(\xi^{(\gamma)}) = \cos^2 \xi^{(\gamma)} + \cos^2 \theta \sin^2 \xi^{(\gamma)} \tag{8}
\]

Finally, it is noted that the above expressions clearly show that each spectral component nicely satisfies the edge condition [3],[6] even in the tricky \( \sin \theta = 0 \) case.

REFERENCES


