SCATTERING OF ELECTROMAGNETIC WAVES BY INHOMOGENEOUS DIELECTRIC GRATINGS WITH PERFECTLY CONDUCTING STRIP

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Abstract: In this paper, we have proposed a new method for the scattering of electromagnetic waves by inhomogeneous dielectric gratings with perfectly conducting strip for TM waves using the combination of improved Fourier series expansion method and point matching method. This method also can be applied to the dielectric gratings having an arbitrarily periodic structures combination of dielectric and metallic materials.

INTRODUCTION
Scattering and guiding problems of the inhomogeneous dielectric gratings such as optoelectronic devices, photonic bandgap crystals and frequency selective devices have been considerable interest, and many analytical and numerical methods which are applicable to the gratings having an arbitrarily structures combination of dielectric and metallic materials[1] by the development of manufacturing technology of optical devices.

In this paper, we proposed a new method for the scattering of electromagnetic waves by inhomogeneous dielectric gratings with perfectly conducting strip using the combination of improved Fourier series expansion method[2] and point matching method[3]. This method also can be applied to the dielectric gratings having an arbitrarily periodic structures combination of dielectric and metallic materials.

METHOD OF ANALYSIS
We consider inhomogeneous dielectric gratings with perfectly conducting strip as shown in Fig.1(a). The grating is uniform in the y-direction and the permittivity \( \varepsilon(x,z) \) is an arbitrary periodic function of \( z \) with period \( p \) with respect to the position \( (x,w) \). The permeability is assumed to be \( \mu_0 \). The time dependence is \( \exp(-i\omega t) \) and suppressed throughout. In the formulation, the TM wave is discussed. When the TM wave (the magnetic field has only the \( y \)-component) is assumed to be incident from \( x > 0 \) at the angle \( \theta_0 \), the magnetic fields in the regions \( S_1(x \geq 0) \) and \( S_2(x \leq -d) \) are expressed[2] as

\[
S_1(x \geq 0) : \quad H_y^{(1)} = e^{ik_y(x \sin \theta_0 - z \cos \theta_0)} + e^{i\theta_0 x} \sum_{n=-N}^{N} r_n^{(1)} e^{i(k_1^{(1)}x + 2\pi nz/p)} ; \quad k_1 \triangleq \omega \sqrt{\varepsilon_0 \mu_0} \tag{1}
\]

\[
S_2(x \leq -d) : \quad H_y^{(3)} = e^{i\theta_0 x} \sum_{n=-N}^{N} r_n^{(3)} e^{ik_3^{(3)}(x+d)-2\pi nz/p} \tag{2}
\]

\[
k_n^{(j)} \triangleq \sqrt{k_0^2 \varepsilon_j / \varepsilon_0 - \gamma_n^2} ; \quad \gamma_n \triangleq k_0 \sin \theta_0 + 2\pi n/p , \quad k_0 \triangleq 2\pi / \lambda , \quad j = 1, 3.
\]

where \( \lambda \) is the wavelength in free space, \( r_n^{(1)} \) and \( r_n^{(3)} \) are unknown coefficients to be determined by boundary conditions.

Fig.1 Structure of inhomogeneous dielectric grating with perfectly conducting strip.
(a) Coordinate system, (b) Approximated inhomogeneous layers.
Main process of our method to treat these problems is as follows (see Fig.1(b)):

1. First, the grating layer \(-d < x < 0\) is approximated by an assembly of \(M\) stratified layers of modulated index profile with step size \(d_s (\equiv d/M)\) approximated to step index profile \(\varepsilon'^{(l)}(z) (\equiv \varepsilon(l+0.5)d_s, z)\), \(l = 1 \sim M\), and the magnetic fields are expanded appropriately by a finite Fourier series.

\[
S_z(-d < x < 0): H_y^{(l)(z)} = \sum_{p=1}^{2N_z} \left[ A_{\nu} e^{i\varepsilon^{(l)}(z)(x-l)\nu} + B_{\nu} e^{i\varepsilon^{(l)}(z)(x+l)\nu} \right] e^{i\nu_{\rho}\sin\theta_{\rho}} \sum_{n=-N}^{N} u_{n}^{(l)} e^{2\pi n\rho/p}\]

(3)

where \(h_n^{(l)}\) is the propagation constant. \(h_n^{(l)}\) is the propagation constant in the \(x\)-direction.

We get the following eigenvalue equation\(^2\) in regard to \(h_n^{(l)}\)

\[
\Phi_{\nu}^{(l)} \equiv \begin{bmatrix} h_n^{(l)} \end{bmatrix}^2 \Lambda_{\nu} \Phi_{\nu}^{(l)}; \quad \Lambda_{\nu} \equiv \begin{bmatrix} \eta_{n,\nu}^{(l)} \end{bmatrix}, \quad \Lambda_{\nu} \equiv \begin{bmatrix} \varepsilon_{n,\nu}^{(l)} \end{bmatrix}, \quad l = 1 \sim M,
\]

where \(\Phi_{\nu}^{(l)} \equiv \left[ u_{n}^{(l)}, \ldots, u_{N}^{(l)}, \ldots, u_{-N}^{(l)} \right]^T\), \(T\) : transpose,

\[
\varepsilon^{(l)}(z) \equiv k_0^2 \varepsilon(z) - \gamma_n \left( \gamma_n^{(l)} + 2\pi(n - m)\nu_{\rho}/p \right), \quad \gamma_n^{(l)} \equiv (k, \sin\theta_{\rho} + 2\pi n/p),
\]

(5)

By using matrix algebra in Eq.(9), we get following matrix form.

\[
\text{where} \quad \Lambda_{\nu} \Phi_{\nu}^{(l)} = \begin{bmatrix} h_n^{(l)} \end{bmatrix}^2 \Lambda_{\nu} \Phi_{\nu}^{(l)}
\]

(4)

For the TM case, the permittivity profile approximated by a Fourier series of index profile with step size \(d_s (\equiv d/M)\) approximated to step index profile \(\varepsilon'^{(l)}(z) (\equiv \varepsilon(l+0.5)d_s, z)\), we get following equation.

\[
\text{In the Eq.(8), the boundary condition at } E_z^{(i,j)} = E_z^{(i,j+1)} \text{, it is satisfied in all matching points. Therefore, rearranging after multiplying both sides } \varepsilon_j(z) \cdot \varepsilon_{j+1}(z) \text{ in Eq.(8) by using the orthogonality properties of } \{e^{2\pi n\rho/p}\}, \text{we get following equation.}
\]

\[
\sum_{p=1}^{2N_z} \left[ A_{\nu} e^{i\varepsilon^{(l)}(z)(x+l)\nu} - B_{\nu} \right] \psi_{n,\nu}^{(l)} = \sum_{p=1}^{2N_z} \left[ A_{\nu} e^{i\varepsilon^{(l)}(z)(x-l)\nu} - B_{\nu} \right] \psi_{n,\nu}^{(l)}
\]

(9)

By using matrix algebra in Eq.(9), we get following matrix form.

\[
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\]

\[
\Phi_{\nu}^{(l)}[D^{(l)}A^{(l)} - B^{(l)}] = \Psi_{\nu}^{(l)}[C^{(l)}][A^{(l)} - D^{(l)}B^{(l)}]
\]

(10)

where \(\Phi_{\nu}^{(l)} \equiv \left[ \eta_{n,\nu}^{(l)} \right], \Psi_{\nu}^{(l)} \equiv \left[ \psi_{n,\nu}^{(l)} \right], \quad C^{(l)} \equiv \left[ h_n^{(l)} \cdot \delta_{n+N(i+1),\mu} \right], \quad D^{(l)} \equiv \left[ e^{i\varepsilon^{(l)}(z)} \cdot \delta_{n+N(i+1),\mu} \right], \quad \Theta \equiv \left[ \delta_{n+N(i+1),\mu} \right], \text{Kronecker's delta.}
\]

We get following matrix form combined with Eq.(6) and Eq.(7).

\[
H^{(l)}[D^{(l)}A^{(l)} + D^{(l)}B^{(l)}] = H^{(l)}[A^{(l)} + D^{(l)}B^{(l)}]
\]

(11)
\[ V^{(j)}_1 \mathbf{A}^{(j)} + V^{(j)}_2 \mathbf{B}^{(j)} = V^{(j+1)}_1 \mathbf{A}^{(j+1)} + V^{(j+1)}_2 \mathbf{B}^{(j+1)} \]

where

\[
H^{(j)} = \begin{bmatrix}
  e^{-\text{ijk}} & \cdots & e^{-\text{ijk}} \\
  \vdots & \ddots & \vdots \\
  e^{-\text{ijk}} & \cdots & e^{-\text{ijk}}
\end{bmatrix} \mathbf{z}_j \in \mathbb{C}, \quad \mathbf{U}^{(j)} = \begin{bmatrix}
  \cdots & \cdots & \cdots \\
  \vdots & \ddots & \vdots \\
  \cdots & \cdots & \cdots
\end{bmatrix} \mathbf{z}_j \in \mathbb{C},
\]

\[
H^{(j+1)} = \begin{bmatrix}
  0 & \cdots & 0 \\
  \vdots & \ddots & \vdots \\
  0 & \cdots & 0
\end{bmatrix} \mathbf{z}_j \in \mathbb{C}, \quad \mathbf{U}^{(j+1)} = \begin{bmatrix}
  \cdots & \cdots & \cdots \\
  \vdots & \ddots & \vdots \\
  \cdots & \cdots & \cdots
\end{bmatrix} \mathbf{z}_j \in \mathbb{C}
\]

By using matrix algebra in Eq.(11) and (12), we get the following matrix relationship between \( \mathbf{A}^{(j)}, \mathbf{B}^{(j)} \) and \( \mathbf{A}^{(M)}, \mathbf{B}^{(M)} \).

\[
\begin{bmatrix}
  \mathbf{A}^{(j)} \\
  \mathbf{B}^{(j)}
\end{bmatrix} = \begin{bmatrix}
  \mathbf{S}_{j}^{(1)} \\
  \mathbf{S}_{j}^{(2)}
\end{bmatrix} \begin{bmatrix}
  \mathbf{A}^{(j+1)} \\
  \mathbf{B}^{(j+1)}
\end{bmatrix},
\]

where

\[
\begin{bmatrix}
  \mathbf{S}_{j}^{(1)} \\
  \mathbf{S}_{j}^{(2)}
\end{bmatrix} = \begin{bmatrix}
  \cdots & \cdots & \cdots \\
  \vdots & \ddots & \vdots \\
  \cdots & \cdots & \cdots
\end{bmatrix} \mathbf{S}_{j}^{(1)} = \begin{bmatrix}
  \cdots & \cdots & \cdots \\
  \vdots & \ddots & \vdots \\
  \cdots & \cdots & \cdots
\end{bmatrix} \mathbf{S}_{j}^{(2)} = \begin{bmatrix}
  \cdots & \cdots & \cdots \\
  \vdots & \ddots & \vdots \\
  \cdots & \cdots & \cdots
\end{bmatrix}.
\]

Finally, we obtain the relationship between \( \mathbf{A}^{(1)}, \mathbf{B}^{(1)} \) and \( \mathbf{A}^{(M)}, \mathbf{B}^{(M)} \) using boundary condition at \( x = -l \cdot d_a (l = 1 \sim M - 1) \).

\[
\begin{bmatrix}
  \mathbf{A}^{(1)} \\
  \mathbf{B}^{(1)}
\end{bmatrix} = \begin{bmatrix}
  \mathbf{S}_{1}^{(1)} \\
  \mathbf{S}_{1}^{(2)}
\end{bmatrix} \begin{bmatrix}
  \mathbf{A}^{(M)} \\
  \mathbf{B}^{(M)}
\end{bmatrix} = \begin{bmatrix}
  \mathbf{S}_{1}^{(1)} \mathbf{S}_{2}^{(1)} \cdots \mathbf{S}_{M}^{(1)} \mathbf{S}_{1}^{(2)} \cdots \mathbf{S}_{M}^{(2)}
\end{bmatrix} = \begin{bmatrix}
  \mathbf{S}_{1}^{(1)} \mathbf{S}_{2}^{(1)} \cdots \mathbf{S}_{M}^{(1)} \mathbf{S}_{1}^{(2)} \cdots \mathbf{S}_{M}^{(2)}
\end{bmatrix} = \begin{bmatrix}
  \mathbf{S}_{1}^{(1)} \mathbf{S}_{2}^{(1)} \cdots \mathbf{S}_{M}^{(1)} \mathbf{S}_{1}^{(2)} \cdots \mathbf{S}_{M}^{(2)}
\end{bmatrix}.
\]

Using Eq.(14), we get the following homogeneous matrix equation in regard to \( \mathbf{A}^{(M)} \).

\[ W \cdot \mathbf{A}^{(M)} = \mathbf{F}, \quad W \mathbf{S} = \{ Q S_1 + Q S_2 - (Q S_1 + Q S_2) Q S_1 Q S_2 \} \]

The mode power transmission coefficients \( |T_{n}^{(M)}|^2 \) is given by

\[ |T_{n}^{(M)}|^2 \approx \varepsilon_1 \text{Re} \left\{ k_{n}^{(1)} / k_{n}^{(2)} \right\}^2 \left\{ \varepsilon_1 k_{n}^{(1)} \right\}.
\]

CONCLUSIONS

In this paper, we have proposed a new method for the scattering of electromagnetic waves by inhomogeneous dielectric gratings with perfectly conducting strip using the combination of improved Fourier series expansion method and point matching method. This method also can be applied to the dielectric gratings having an arbitrarily periodic structures combination of dielectric and metallic materials.