COUPLED WAVES IN NONLINEAR GUIDED-WAVE OPTICS:
SPATIAL-TEMPORAL DAVEY-STEWARTSON SOLITONS IN
SECOND-ORDER CASCADE

J. M. Arnold

Department of Electronics and Electrical Engineering
University of Glasgow, Glasgow G12 8LT, UK.

Abstract

The spatial-temporal second-order cascade nonlinearity in 2-dimensional nonlinear optics is approximated by an exactly integrable system in 2+1 dimensions, the Davey-Stewartson equations. An essential component of this integrable system is the explicit appearance of the optical rectification term which naturally accompanies the second-order nonlinearity but has been universally neglected in theoretical treatments of the cascade effect. Conditions for this reduction are identified, and are shown to require a sequence of matching relations between the fundamental-second-harmonic phase mismatch, the RF-fundamental velocity mismatch, and the linear dispersion parameter of the fundamental. This result ensures that spatial-temporal solitons with the full properties of integrability can be generated in the second-order cascade interaction under appropriate conditions. These appear to be realisable with realistic values of material parameters.

INTRODUCTION

Considerable interest surrounds the phenomenon of the second-order cascade effect, a nonlinear optical interaction in which the generated second harmonic wave mixes with the original fundamental pump wave to produce a further component at the fundamental frequency whose phase differs from that of the original wave. Here it is shown that the second-order cascade effect can support true spatial-temporal solitons that satisfy the Davey-Stewartson equations, which are well known in hydrodynamics as a completely integrable system in 2+1 dimensions. An essential role is played in this wave type by the optical rectification which necessarily accompanies any second-order interaction, but which is always neglected in analytical treatments of the phenomena since it is believed to play no part in the generation of second-harmonic radiation.

The Davey-Stewartson equations derived here are essentially a coupled system of interacting waves at three frequencies, having proper soliton solutions which decay exponentially both spatially and temporally. They occur in a planar medium which supports electromagnetic waves polarised parallel to the plane. Such a planar medium can be realised to a good approximation by a planar waveguide propagating TE-polarised modes at all three frequencies, with an appropriate tensor to couple the waves together. Assuming practical values for the optical parameters of such a planar waveguide, it appears that this class of spatial-temporal solitons can be achieved in practical devices with spatial beamwidths of a few tens of microns, and temporal pulsewidths of a few hundred femtoseconds.

FORMULATION

We consider a 2-dimensional medium, approximated for instance by a planar waveguide. This medium supports electromagnetic waves polarised in the plane of the medium. We select one coordinate, $z$, as the propagation direction, and one coordinate, $x$, as the transverse direction. Time-dependent electromagnetic waves in this medium are described by the electrodynamical Maxwell wave equation

$$\nabla^2 \mathbf{E} - \frac{1}{c^2} \partial_t^2 \mathbf{E} = \frac{1}{\varepsilon_0 c^2} \left( \partial_t^2 \mathbf{P} - c^2 \nabla \nabla \cdot \mathbf{P} \right). \quad (1)$$

The polarisation $\mathbf{P}$ is defined in terms of the electric fields using the standard frequency-domain representation.
in terms of nonlinear susceptibilities

\[ \tilde{P}_j(\omega) = \epsilon_0 \sum_{n=1}^{\infty} (2\pi)^{n+1} \int_{-\infty}^{\infty} \cdots \int_{-\infty}^{\infty} \chi^{(n)}_{j_1 \ldots j_n}(\omega; \omega_1, \ldots, \omega_n) \]

\[ \times \delta(\omega_1 + \cdots + \omega_n - \omega) \tilde{E}_{k_1}(\omega_1) \cdots \tilde{E}_{k_n}(\omega_n) d\omega_1 \cdots d\omega_n \]  

(2)

where \( \tilde{E} = \tilde{E}_x u_x + \tilde{E}_y u_y + \tilde{E}_z u_z \) is the macroscopic electric field in the dielectric, \( \tilde{P} = \tilde{P}_x u_x + \tilde{P}_y u_y + \tilde{P}_z u_z \) is the polarisation density induced in the dielectric, \( \chi^{(n)}_{j_1 \ldots j_n} \) is the order-\( n \) susceptibility of the medium, and the delta function in the integrations picks out the combination of frequencies \( \omega = \sum_{r=1}^{n} \omega_r \). The tensor indices range over \( j, k_1, \ldots, j \) and \( x, y \).

Second-order nonlinear interactions, in which three waves at frequencies 0, \( \Omega \) and 2\( \Omega \) given by

\[ E(z, t) = E_0(z, t) + \text{Re}\{E_1(z, t)e^{i(kz-\Omega t)} + E_2(z, t)e^{i(2kz-2\Omega t)}\} \]  

(3)

undergo parametric interaction, satisfy the wave equations

\[ (\partial_x^2 + \partial_y^2 + \frac{1}{c_0^2} \partial_z^2)E_0 = \frac{1}{c^2} \chi_0(\partial_x^2 - c_0^2 \partial_z^2)(E_1 E_1^*) \]  

(4)

\[ 2i\kappa_1(\partial_x + v_{g1}^{-1} \partial_t)E_1 = -\partial_x^2 E_1 + \kappa_1 D_1 \partial_z^2 E_1 - \frac{\Omega^2}{c_1^2} (\chi_1 E_2 E_1^* + \chi_0 E_0 E_1) \]  

(5)

\[ 4i\kappa_1(\partial_x + v_{g2}^{-1} \partial_t)E_2 = 4\kappa_\Delta \kappa E_2 - \partial_x^2 E_2 + 2\kappa_2 D_2 \partial_z^2 E_2 - 2\frac{\Omega^2}{c_2^2} \chi_2 E_1^2 \]  

(6)

where \( \kappa_j = n(\Omega_j)/c, \kappa = \kappa_1, \Omega_j = j\omega, \Delta \kappa = 2\kappa_1 - \kappa_2 \) is the phase-mismatch parameter, \( \chi_0 = \chi^{(2)}(0; -\Omega, \Omega), \chi_1 = \chi^{(2)}(\Omega; 2\Omega, -\Omega), \chi_2 = \chi^{(2)}(2\Omega, \Omega, \Omega) \). Here equations (5) and (6) are the usual paraxially approximated expressions for the fundamental and second harmonic waves, whereas (4) describes optical rectification of the fundamental. In the guided-wave case, the complex fields \( E_0, E_1, E_2 \) represent the amplitudes of suitable modes of a planar guiding structure with appropriate tensor coupling between the modes. For example, the second-order nonlinear susceptibility in a cubic symmetry medium such as gallium arsenide couples TE fundamental modes to TM second-harmonic and optically rectified modes; the optical modes can be supported in a planar dielectric waveguide and the RF modes can be supported in a metallised coplanar structure deposited on the top surface of the dielectric. The various \( \chi^{(2)} \) parameters in the wave equation then represent averaged couplings of the relevant tensor coefficients.

We consider first equation (6) in the limit of large phase mismatch, \( \Delta \kappa \to \pm \infty \). Then the induced second harmonic wave has small amplitude, and can be approximated by

\[ E_2 = \frac{\Omega^2 \chi_2}{\Delta \kappa c_2^2} E_1^2. \]  

(7)

Substituting for \( E_2 \) in (5) gives

\[ 2i\kappa_1(\partial_x + v_{g1}^{-1} \partial_t)E_1 = -\partial_x^2 E_1 + \kappa_1 D_1 \partial_z^2 E_1 - \frac{\Omega^2}{c_1^2} (\frac{\Omega^2 \chi_1 \chi_2}{\Delta \kappa c_2^2} E_1^2 E_1^* + \chi_0 E_0 E_1). \]  

(8)

Next we introduce scalings for the lengths, times and field strengths involved in these interactions. Introduce the retarded scaled time by \( \tau = T^{-1}(t - v_{g1}^{-1} z) \). Let \( z/L \to z \) and \( \sqrt{\kappa L} x \to x \). Here \( T \) and \( L \) are arbitrary time and length parameters which will be fixed later in the analysis. Introduce scaled fields by

\[ \psi_0 = \sigma a \chi_0 (E_0 + \chi_0 |E_1|^2), \quad \psi_1 = a \chi_0 \sqrt{\frac{1 - b}{|b|}} E_1, \]  

(9)

where

\[ a = \frac{\Omega^2 L}{2\kappa c^2}, \quad b = \frac{\chi_0^2}{\chi_1 \chi_2}, \quad \sigma = \text{sign}(b). \]  

(10)
After some manipulation, the scaled equations (8) can be brought to the form

$$i \partial_t \psi_1 = -\frac{1}{2}(a_1 \partial_x^2 + \partial_z^2)\psi_1 - \sigma(|\psi_1|^2 + \psi_0)\psi_1$$

\[ (a_0 \partial_x^2 - \partial_z^2)\psi_0 = -2a_2 \partial_x^2|\psi_1|^2. \tag{12} \]

where

$$a_0 = \frac{\delta L}{\kappa c_0 T}, \quad a_1 = \frac{D_1 L'}{T^2}, \quad a_2 = \frac{1}{2n_0^2} \frac{b(1-\delta)}{\delta(1-b)}, \quad \delta = 1 - \frac{\kappa_0^2}{n_0^2}. \tag{13}$$

If $\kappa L \gg 1$ and $\Omega T \gg 1$, the last two terms on the right-hand side of (12) are both negligible. When these terms are neglected the equations become

$$i \partial_t \psi_1 = -\frac{1}{2}(a_1 \partial_x^2 + \partial_z^2)\psi_1 - \sigma(|\psi_1|^2 + \psi_0)\psi_1$$

\[ (a_0 \partial_x^2 - \partial_z^2)\psi_0 = -2a_2 \partial_x^2|\psi_1|^2. \tag{14} \]

With $a_0 = a_1 = a_2 = \sigma = \pm 1$ the equations (14)-(15) are the Davey-Stewartson equations; the upper sign (+1) gives Type 1 and the lower sign (-1) Type 2. The Davey-Stewartson equations arrived at in this way can be simplified by dividing out the parameter $\sigma$ to obtain

$$ \pm i \partial_x \psi_1 = -\frac{1}{2}(\partial_x^2 \pm \partial_z^2)\psi_1 - (|\psi_1|^2 + \psi_0)\psi_1 $$

\[ (\partial_x^2 \pm \partial_z^2)\psi_0 = -2\partial_x^2|\psi_1|^2, \tag{17} \]

with the upper or lower signs for Type 1 or Type 2 respectively.

The Davey-Stewartson equations are integrable by the Inverse Scattering Transform (IST) \[1\], which means that they may exhibit soliton solutions. The solitons of Davey-Stewartson Type 1 are exponentially decaying at infinity in both the $\tau$- and $x$-variables. The Type 2 solitons are algebraically decaying at both limits. The Davey-Stewartson equations obey a simple scale invariance, given by $\tau \to \eta \tau, x \to \eta x, z \to \eta^2 z, \psi_0 \to \eta^2 \psi_0$ and $\psi_1 \to \eta \psi_1$ for $\eta$ an arbitrary real positive constant.

The conditions $a_0 = a_1 = a_2 = \pm 1$ imply determination of some of the free parameters in the scalings. The coordinate scales $L$ and $T$ are completely free, and by waveguide design we can also regard the two mismatch parameters $\delta$ and $\Delta$ as free parameters. The three conditions required to fix the Davey-Stewartson equations therefore determine three of these four parameters, the fourth remaining free. In this way we deduce the following conditions for the Davey-Stewartson form:

$$ \delta = -\kappa c_0^2 D_1, \quad T = \sqrt{|D_1|L}, \quad \frac{\Delta \kappa}{\kappa} = \frac{\delta}{\eta_0^2(1+\delta)} = \frac{\chi_1 \chi_2}{\chi_0^2}, $$

\[ \tag{18} \]

with either $T$ or $L$ chosen as the single remaining free parameter.

**CONCLUSIONS**

Optical rectification supported by a suitable guiding structure enhances spatial-temporal soliton formation in a planar dielectric waveguide under conditions of second-order cascade nonlinear optics. A coupled-wave analysis exhibits appropriate parametric scalings to isolate this effect.

**REFERENCES**