SCATTERERS LOCALIZATION IN THE CIRCULAR SCANNING GEOMETRY

O. M. Bucci, A. Capozzoli, G. D’Elia, M. Santojanni

Dipartimento di Ingegneria Elettronica e delle Telecomunicazioni
Università di Napoli Federico II – Via Claudio, 21 80125 Napoli Italia
E-mail: g.delia@unina.it, Phone: +39-081-7683115, Fax: +39-081-5934448

Abstract: This paper describes the extension to the circular scanning case of a recently proposed approach to source/scatterers localization problem from field data. The method is based on the concepts of Point Source Spectral Content and Local Spectral Content that allows the determination of the number and the location of a set of sources/scatterers without requiring the solution of full inverse problem, i.e., the inversion of the radiation or scatterer operator. The numerical analysis shows that the achievable performance are often better than those obtained with a rectilinear scanning.

1. Introduction

The estimation of the number and the location of electromagnetic radiators (sources or scatterers) from field data is a problem of relevant theoretical and practical interest. To collect concise information about the geometry of the observed system has two main applications. It can represent the real goal of the inversion procedure. It can represent the result of a pre-processing step that provides key a priori information on the observed system, useful to improve the performances of full inversion algorithms.

However, to attain beneficial effects on the ill-position and the ill-conditioning, the geometrical information about the radiators should be extracted from the field data using a strategy that does not require the solution of the full inverse problem.

Recently, an effective localisation technique, following the above philosophy, has been presented in [1-2]. The method exploits two properties of the electromagnetic field that depend only on the geometrical characteristics of the radiators: the Point Source Spectral Content (PSSC) and the Local Spectral Content (LSC) [3] of the electromagnetic field. The PSSC has been successfully exploited also in convex hull estimation [4] and shape reconstruction [5] of radiating or scattering systems.

Since now, the localization technique has been applied only to a 2D geometry and rectilinear observation curves [1-2]. However, since the rectilinear scanning line does not allow to collect the scattered field from the full scatterers surface, shadowed scatterers cannot be well localized.

The aim of this communication is to investigate, by means of an extensive numerical analysis, the effectiveness of the approach achievable when the method is applied to a 2D geometry and circular observation curves.

The method and the numerical tests, here presented, refer only to the case of radiating sources. Anyway, the approach can applied also to the case of metallic and dielectric scatterers, as long as the induced or polarisation currents are considered.

The results show that, according to the forecast, the method is able of good performances, often better than those achieved with the rectilinear scanning [1-2].

2. Statement of the problem and solution strategy

In the following we consider a set of radiating sources and a scalar 2D geometry. The exp(jωt) time dependence is assumed and dropped out.

Let us consider a point wise source located in \( \mathbf{r}'=(x',y') \) and a circular observation line \( \Gamma \) described by the polar eq. \( \mathbf{r} = \mathbf{r}(\theta) \), enclosing the source (see Fig.1). The field on \( \Gamma \), due to the source at \( \mathbf{r}' \), is given by:

\[
E(\theta, \mathbf{r}') = E_0(\mathbf{r}(\theta), \mathbf{r}') \exp(-j\beta|\mathbf{r}(\theta) - \mathbf{r}'|)
\]

where \( E_0 \) is a slow varying term related to the free space Green function.

We can now consider the sliding window Fourier transform \( \tilde{E}(\mathbf{r}', \mathbf{r}') \) of \( E(\theta, \mathbf{r}') \):
\( \tilde{E}(\kappa, \vartheta, \mathbf{r}') = \int g(\vartheta' - \vartheta) \cdot E(\vartheta', \mathbf{r}') \exp(-j\kappa \vartheta') d\vartheta' = \int g(\vartheta' - \vartheta) E_0(\vartheta', \mathbf{r}') \exp\left(-j\beta |\mathbf{r}' - \mathbf{r}| - j\kappa \vartheta'\right) d\vartheta' \) \hspace{1cm} (2)

where \( g \) is a properly chosen windowing function and \( \kappa \) is the conjugate variable of \( \vartheta \). For\( \beta \to \infty \), \( \tilde{E} \) can be asymptotically evaluated by the stationary phase method [4]. Accordingly, for any given value of \( \vartheta \), the spectrum is significantly different from zero only for \( \kappa \) given by [4]:

\[
\frac{\partial}{\partial \vartheta} (|\mathbf{r}' - \mathbf{r}|) + \dot{\vartheta} = -h(\vartheta, \mathbf{r}') + \kappa = 0 \hspace{1cm} (3)
\]

The function \( h(\vartheta, \mathbf{r}') \) in eq. (3) is the PSSC function and defines the spectral content of the field on the observation circle due to a point wise source located at \( \mathbf{r}' \). In particular, if \( \mathbf{r}(\vartheta) = (dcos(\vartheta), dsin(\vartheta)) \), its expression is given by:

\[
h(\vartheta, \mathbf{r}') = -\beta \frac{-dsin \theta \cdot (dcos \theta - x') + dcos \theta \cdot (dsin \theta - y')}{\sqrt{(dcos \theta - x')^2 + (dsin \theta - y')^2}} \hspace{1cm} (4)
\]

Accordingly, the spectral content of the sliding Fourier transform of the observed field (the Local Spectral Content (LSC) of the field) is centred on the function \( h \) and can be theoretically predicted from eq. (4), once the source location \( \mathbf{r}' \) is given. As a consequence, the source location can be determined by comparing the PSSC estimated from the Windowed Local Fourier Transform of the field data with the PSSC theoretically evaluated from (4).

When a set of point like sources are involved, the LSC of the field incorporates the contributions due to the PSSC of each elementary radiator and, as long as the sources are at least few wavelength far from each other, the sliding-window spectrum consists of distinguishable components. As a consequence, the sources can be found by looking for that set of sources whose PSSCs match at best the LSC obtained from the local Fourier transform of the observed field data. The matching is obtained by exploiting an Evolutionary Algorithm [6].

The main steps of the localisation procedure can be summarised as follows:
1) The local windowed Fourier transform of the simulated field on the observation line is evaluated and the LSC of the field estimated.
2) A trial set of scatterers is considered and their theoretical PSSCs on the observation circle are evaluated.
3) A comparison between the LSC of the field and all the PSSCs of the trial configuration of the radiators is made according to a properly defined quality factor Q.
4) A new trial set of radiators is considered and steps i)-iv) are repeated until a minimum of Q is attained.
5) The number of radiators is increased and steps i)-v) are repeated until no further improvements of Q are obtained.

3. Localization examples.

Let us present now an example of the numerical analysis. Four elementary sources, whose co-ordinates and excitations are reported in Table I, have been considered. The field has been simulated on a circular observation curve of radius \( d=50\hat{a} \) and centre \((0,0)\). The Sliding Window Fourier transform of the field and the theoretical PSSCs of the four sources (eq. (4)) are shown in Fig.2. The observation line, the true and the estimated (Table II) coordinates of the sources are shown in Fig.3 as a dotted line, circles and crosses, respectively. As can be seen, the sources are located within an error of \( \hat{a} \). It is worth noting that, for linear scanning a 1-2 \( \hat{a} \) uncertainty has been observed [1,2].
References


<p>| Table I. True source coordinates and excitations |</p>
<table>
<thead>
<tr>
<th>x/λ</th>
<th>y/λ</th>
<th>excitations</th>
</tr>
</thead>
<tbody>
<tr>
<td>S1</td>
<td>-25</td>
<td>-25</td>
</tr>
<tr>
<td>S2</td>
<td>0</td>
<td>0</td>
</tr>
<tr>
<td>S3</td>
<td>10</td>
<td>-40</td>
</tr>
<tr>
<td>S4</td>
<td>20</td>
<td>-15</td>
</tr>
</tbody>
</table>

<p>| Table II. Estimated source coordinates |</p>
<table>
<thead>
<tr>
<th>x/λ</th>
<th>y/λ</th>
</tr>
</thead>
<tbody>
<tr>
<td>S1</td>
<td>-24.95</td>
</tr>
<tr>
<td>S2</td>
<td>-0.17</td>
</tr>
<tr>
<td>S3</td>
<td>9.64</td>
</tr>
<tr>
<td>S4</td>
<td>20.15</td>
</tr>
</tbody>
</table>

Fig.1. Geometry of the problem

Fig.2. Sliding Window Fourier Transform of the observed field (coloured matrix) and (theoretical, eq. (4)) PSSCs of the trial sources (white lines).

Fig.3. True (circles) and estimated (crosses) sources, observation curve (dotted)