Ray-Mode Methods for Coupling at Junctions in Urban Street Canyons

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Abstract: Propagation around corners in urban street canyons is modeled using hybrid ray-mode conversion. For low base station and mobile antennas, high-rise urban street canyons can be modeled as 2-D dielectric waveguide. By accounting for the mode diffraction at each of the four corners, we compute the coupling loss into crossing streets.

1. Introduction
Free propagation between radio transceivers has also been studied for radio communication in tunnels, which act as lossy waveguides. Both modal [1] and ray [2] approaches have been used to model propagation in tunnels. For long distances along a straight tunnel, the modal approach is appropriate even when the tunnel dimensions are much larger than the wavelength.

High-rise urban street canyons can be modeled as a simple 2-D dielectric parallel waveguide (i.e. tunnel) with the ground acting as an imaging plane, as suggested in Figure 1. Therefore, we can treat the urban street canyons for low base station and mobile antennas as hollow 2-D tunnels with dielectric walls. Ground reflection is accounted for by including an image source. We start by considering the mode expression in simplified case of a 2-D tunnel (analogues to parallel plate waveguide) and then show the 3-D modal expressions for diffraction at the corners and the modal field in tunnels and urban street canyons.

Kamel and Felsen [3] have shown for 2-D straight waveguide that the ray representation is asymptotically equivalent to the modal expansion of the fields radiated by a line source. In order to explain coupling at street junctions, the fields can be expressed in terms of rays generated by diffraction at the building corners. Subsequently, the building walls multiply reflect the diffracted rays. We show that the resulting ray sum can be converted into a mode sum, with the properties of the equivalent sources given in terms of the diffraction coefficients evaluated at appropriate mode angles. This approach is similar to the study of reflection at the open end of a perfectly conducting waveguide [4], but must account for the angle dependence of the reflection coefficient.

2. Ray-Mode Formulation for Coupling in 2-D Waveguides
Consider a source located in one arm of a cross junction of a 2-D tunnel, as shown in Figure 2. Since \( k_0 d >> 1 \), the modes excited by the source in the left arm will propagate to the gap and couple with little attenuation into right arm. They will also couple into the upper and lower arms as a result of the illumination in the aperture of the arms. By summing the diffracted ray contributions and using the Poisson sum formula, it is possible to translate the diffracted rays into a modal sum, with the amplitude of the modes given by the diffraction coefficient evaluated at the mode angle. Each mode incident on the junction will generate many modes into side arm. The source corresponding to the left hand edge C1 will have strength \( u_1 \) that is determined by the phase and amplitude of the incident modal field, and a pattern function given by diffraction coefficient \( D(\theta_n, \theta_{in}) \). Here \( \theta_{in} \) is the angle of the incident plane wave comprising the mode incident on the junction, and \( \theta_n \) is the angle of the diffracted ray. Similarly, the right hand edge C2 will have amplitude \( u_2 \) and pattern function \( D(\theta_n, \theta_{in}) \). The ray sum for the field due to diffraction at a corner is given by

\[
E_y(x,z) = \sum_{i=1}^{\infty} \sum_{n=-\infty}^{\infty} u_i \Gamma(\theta_n) D(\theta_n, \theta_{in}) e^{-jk_0 r_n} \sqrt{r_n} \tag{1}
\]

where
\[ r_n = \sqrt{z^2 + [x - (4n \pm 1)d]^2} \]
\[ \theta_n = \tan^{-1}\left(\frac{x - (4n \pm 1)d}{z}\right) \]  \hspace{1cm} (2)

Here the + sign applies for the right hand edge \( C_2 \) (\( i = 2 \)), the – sign for the left hand edge \( C_1 \) (\( i=1 \)). In equation (5), \( n=0 \) refers to the direct ray, while \( n = \pm 1, \pm 2, \pm 3, \ldots \) refer to rays coming from the images. The index \( p \geq 0 \) gives the number of reflections undergone by the ray corresponding to the particular image.

The ray sum in (1) can be converted into a mode sum using Poisson sum formula, where the modal amplitudes \( C_m \) are given by the Fourier transform of the function \( f(x) \) in the ray expansion given by

\[ f(x) = u_1\left[\Gamma(0)\right]^p(x) D(\theta, \theta_m) \frac{e^{-jkx}}{\sqrt{r}} \]  \hspace{1cm} (3)

but defined on the continuous infinite interval. The function \( p(x) \) in (3), and hence \( f(x) \) itself, is discontinuities because the number of reflection of ray increases discretely. However, we therefore approximate \( p(x) \) in (3) by the ramp function \( p(x) = |x + d|/2d \) in order to simplify the asymptotic evaluation of the integration for \( C_m \). This approximation distinguishes the method used here from previous results for metallic guides, where the reflection coefficient is unity.

3. Comparison With Measurements

The simulation results for 2-D waveguides are compared with the measurements by Sakai et al. [5]. The measurements were made in waveguides formed by Bakelite slabs backed by metal plates. The configurations considered were the intersection, as in Figure 2, as well as T-junctions and L-bends. The results of the comparisons are shown in Table 1.

For mode propagation in urban cross streets, we compared with the prediction with measurements by Erceg et al. in Manhattan cross streets [6]. The widths of the main street and the side street were 30 m and 20 m, respectively. A 10 m high transmitting and a 2 m high receiving omni-directional antenna located in the middle of the street was used. We have taken the dielectric constant of building walls to be \( \varepsilon_w = 6 \) with conductivity \( \sigma = 0.01 \), and use UTD diffraction coefficients for modal computations. The received power in the main and side streets is averaged over the cross sections of the streets. Figure 3 shows the numerical results for vertically polarized 900MHz signals. The solid line is the average received power in the main street, and the circles indicate the received power in the side street. Coupling loss for this case is about 40 dB, which is well matched with the measurements.

4. Conclusions

Radio propagation in urban street canyons can be described using modal analysis. We have evaluated the modal coupling for streets with junctions using hybrid ray-mode conversion. The coupling mechanism is explained through mode diffraction at the corners into the side tunnels/streets. We have found that the coupling loss is 50 dB for L bend junction, 42 dB for T junction and 37 dB for cross junction in 4m wide for 900MHz signals that have vertical polarization. For urban cross street case, the coupling loss is 40 dB for 900MHz. The coupling loss increases slightly with increasing street width.

5. References


Table 1. Comparison of simulations and measurements of coupling loss at junctions.

<table>
<thead>
<tr>
<th>Tunnel Type</th>
<th>Modal Analysis</th>
<th>Measurements</th>
</tr>
</thead>
<tbody>
<tr>
<td>Cross Tunnel [5]</td>
<td>20 dB</td>
<td>27 dB</td>
</tr>
<tr>
<td>T-junction Tunnel [5]</td>
<td>33 dB</td>
<td>32 dB</td>
</tr>
<tr>
<td>L-bends Tunnel [5]</td>
<td>30 dB</td>
<td>more than 28dB</td>
</tr>
</tbody>
</table>

Figure 1. Propagation in a high-rise urban street canyon.

Figure 2. Corner diffraction in 2-D guide.

Figure 3. Received Power in Urban Cross Street Canyons for 900MHz Vertical Polarization (Corner is located at 386 m from transmitter.)