APPLICATION OF ADAPTIVE BASIS FUNCTIONS/DIAGONAL MOMENT MATRIX TECHNIQUE TO ARRAYS OF IDENTICAL ELEMENTS

Islam A. Eshrah and Ahmed A. Kishk
Department of Electrical Engineering, the University of Mississippi, University, MS 38677, USA

Abstract: In this paper, we propose an efficient algorithm for the application of the adaptive basis functions/diagonal moment matrix technique to arrays of identical elements. Adaptive basis functions constructed using clusters extending over an array element are used to generate a highly diagonally dominant MoM matrix. The new matrix equation is solved iteratively in a way that only significant mutual impedances are considered. The physical interpretation of the constructed set of adaptive basis functions is discussed and sample results are given.

I. INTRODUCTION
Many researchers have recently endeavored to overcome the difficulties encountered in the direct implementation of the method of moments (MoM) to large geometries, namely the prohibitively large, dense matrix for which the generation, storage, and inversion may require large memory and long computational time. Many techniques were developed and new types of basis functions were introduced to generate sparse or highly diagonally dominant matrices [1 and references therein]. Waller and Rao introduced the so-called adaptive basis functions/diagonal moment matrix (ABF/DMM) technique [1] to generate a highly diagonally dominant moment matrix, which uses an iterative algorithm that renders the matrix equation solution rather trivial.

In this paper, the ABF/DMM technique [1] is used to generate a set of adaptive basis functions (ABF), which proved to suit problems of arrays of identical elements. In the proposed algorithm, the cluster of the original basis functions spreads over a whole array element. Also, the way of determining the ABF coefficients does not involve testing each ABF all over the array, rather on the same element only. As such, the equations used to obtain the ABF coefficients constitute a determined set of linear equations. The set of ABFs representing the unknowns on one element is the same for all the array elements as they are identical, and the self-matrices are identical and exactly diagonal as per the requirements. This is not the case for the direct implementation of the ABF/DMM technique proposed in [1], however, where the ABF coefficients will depend on the element location in the array, and the self-matrices are not diagonal, as the coefficients are determined in a least-square-error sense after testing all through the geometry under consideration; a procedure that would be rather time-consuming for large arrays.

It is found that the generated ABFs have an elegant physical interpretation as discussed in section II. In section III, the implementation of the proposed algorithm is explained in detail. Also, the way the mutual impedances are successively taken into consideration is described in the same section. Sample results are given in section IV, followed by the conclusion in section V.

II. ABFs FOR EQUAL CLUSTER AND BODY SIZE
To illustrate the physical meaning of the constructed adaptive basis functions, consider the simple problem of a thin dipole. Direct implementation of the MoM using a set of subdomain basis and testing functions \(f_n, n=1,2,\ldots,N\), with \(N\) being the number of unknowns, yields the following matrix equation

\[
[Z][\mathbf{I}]= [\mathbf{V}].
\]

For ABFs defined on clusters spanning the whole dipole, we have

\[
g_i(z) = \sum_{k=1}^{N} a_{ik} f_i(z) = [a_i]^T [f], \quad i=1,2,\ldots,N
\]

where \([a_i]\) is the \(i^{th}\) ABF coefficients vector and \([f]\) is the vector of the subdomain basis functions. For each ABF \(g_i\), the function \(f_i\) will be referred to as the cluster central basis function and has a unit weight, i.e.
\( \alpha_0 = 1 \). Requiring that each ABF \( g_i \) produce a null field when tested with the subdomain basis functions \( f_j, j=1,2,\ldots,N, j \neq i \), we get

\[
[Z_i] [\phi_i] = -[\psi_i], \quad i = 1, 2, \ldots, N
\]  

(3)

where \([Z_i]\) is the matrix resulting from the elimination of the \( i^{th} \) row and column of \([Z]\) in (1), \([\psi_i]\) is the \( i^{th} \) column of \([Z]\) after eliminating the \( i^{th} \) row, and \([\phi_i]\) is the vector resulting from the elimination of the \( i^{th} \) row of \([\alpha_i]\) (the coefficients without the unit coefficient \( \alpha_0 \)). Noting that for each ABF \( g_i \), equation (3) represents \( N-1 \) linear equations in \( N-1 \) coefficients, it is solved uniquely for these coefficients. Once all the ABF coefficients are determined, the ABF MoM matrix equation can be obtained as

\[
[Z^{ABF}][I^{ABF}] = [V].
\]  

(4)

The MoM matrix in (4) is exactly diagonal, as per the enforced requirements in (3), and each diagonal element is a sum of the corresponding row of \([Z]\) in (1) weighed with the coefficients of the corresponding ABF, viz.

\[
Z_{n,n}^{ABF} = \sum_{i=1}^{N} \alpha_{i,n} Z_{n,i}, \quad n = 1, 2, \ldots, N.
\]  

(5)

Since \([Z^{ABF}]\) is a diagonal matrix, the solution of (4) is trivial, and for the case of a center-fed dipole with unit excitation and odd number of unknowns, the solution reduces to

\[
I_n^{ABF} = -\frac{1}{Z_{n,n}^{ABF}} \delta_{n,N+1}.
\]  

(6)

In view of (6), the following interpretation follows immediately: An adaptive basis function constructed using cluster covering the whole body represents the current distribution (with unit input current) due to a delta source excitation at the cluster centre. The elements of the diagonal impedance matrix in this case are minus the input impedances seen by the source. For the case of the dipole, the ABFs constructed above represent the “antenna mode” current \([2]\) due to delta source excitation at the corresponding feeding point. The “scatterer mode” current \([2]\) follows as a weighted sum of these antenna mode currents.

III. IMPLEMENTATION OF THE ABF/DMM FOR ARRAYS

Once the ABFs are constructed for one element, an array of identical \( N_e \) dipoles with equal inter-element spacing may be analyzed as follows. Let the ABF MoM matrix equation be written in the form

\[
\left( [Z_0] + [Z_1] + [Z_2] + \ldots + [Z_{N_e-1}] \right) [I] = [V]
\]  

(7)

where \([Z_0]\) is a block diagonal matrix, with each block being \([Z^{ABF}]\) introduced in equation (4), which is in turn a diagonal matrix, and \([Z_n], n=1,2,\ldots,N_e-1\), is the \( n^{th} \) level mutual impedance matrix, i.e. \([Z_1]\) holds the block matrices representing the interaction between the first neighbors (closest neighbors to each element) in the array, \([Z_2]\) holds the block matrices representing the interaction between the second neighbors, and so on. Equation (7) may be solved iteratively by starting with the zeroth order solution \([I_0]=[Z_0]^{-1}[V]\), which represents the current distribution on the array elements assuming that they are isolated. Next, the first order mutual impedance is considered and the following iterative equation is used

\[
[I_{0}] = [I_0] - [Z_0]^{-1} [Z_1] [I_0], \quad [I_{1}]_0 = [I_0].
\]  

(8)

The converging solution of (8) (say at \( k=n_1 \)) represents the current distribution on the array elements, taking into consideration the interaction between the first neighbors. The second order solution may be found by applying the iterative formula

\[
[I_{2}]_0 = [I_0] - [Z_0]^{-1} ([Z_1] + [Z_2]) [I_0] + [I_0].
\]  

(9)

The procedure continues till, at the first iteration of some level, the error is less than a desired value.
IV. RESULTS

The previous algorithm was implemented for a 10-element array of half wavelength center-fed parallel dipoles, with half wavelength inter-element spacing. Two criteria based on the current root-mean-square error determine the accuracy of the solution: the error $\varepsilon_c$, which determines the convergence of the iterative formula, and the error $\varepsilon_l$ calculated at the first iteration in each level, which decides whether or not the corresponding mutual impedance should be taken into consideration. Table 1 shows the comparison between the input impedance obtained by the direct MoM solution and the ABF/DMM proposed algorithm. Due to symmetry, only five entries are listed, starting with the element on the array extreme. The value of $\varepsilon_c$ used is $10^{-3}$, which required only one iteration per level (the error actually achieved after this iteration is $10^{-5}-10^{-6}$). For the first case the error $\varepsilon_l=10^{-6}$ required that all the mutual impedance levels be considered, whereas the error $\varepsilon_l=10^{-5}$ in the second case required taking up to the second level of mutual impedance only.

<table>
<thead>
<tr>
<th>Element</th>
<th>Direct MoM Solution</th>
<th>ABF/DMM ($\varepsilon_l=10^{-6}$)</th>
<th>ABF/DMM ($\varepsilon_l=10^{-5}$)</th>
</tr>
</thead>
<tbody>
<tr>
<td>#</td>
<td>$R_{in}$ (Ω)</td>
<td>$X_{in}$ (Ω)</td>
<td>$R_{in}$ (Ω)</td>
</tr>
<tr>
<td>1</td>
<td>68.28</td>
<td>14.01</td>
<td>68.47</td>
</tr>
<tr>
<td>2</td>
<td>54.22</td>
<td>3.98</td>
<td>54.25</td>
</tr>
<tr>
<td>3</td>
<td>57.52</td>
<td>4.20</td>
<td>57.70</td>
</tr>
<tr>
<td>4</td>
<td>56.11</td>
<td>4.17</td>
<td>56.00</td>
</tr>
<tr>
<td>5</td>
<td>56.67</td>
<td>4.17</td>
<td>56.77</td>
</tr>
</tbody>
</table>

The comparison in terms of the computational time is totally in favor of the proposed algorithm. For the given case (with $\varepsilon_l=10^{-6}$), the execution time is 7 times less than the direct MoM solution, in spite of the fact that full advantage was taken of the matrix special properties in the filling process. As the array size gets larger, an even higher ratio is expected. For the memory storage requirements, the direct MoM solution requires allocation of a matrix of size $(N_u \times N_u) \times (N_u \times N)$, whereas the proposed algorithm requires allocation of matrices of total size $N_u \times (N_u \times N)$, i.e. the direct MoM solution requires memory storage larger by a factor of the number of array elements. The speed of convergence of the proposed algorithm depends on the inter-element spacing. Insufficiently distant elements (inter-element spacing less than 0.45$\lambda$ for the given case) may result in divergent results due to the violation of the diagonal dominance condition, which is necessary for the convergence of the iterative solution (Jacobi method). This problem can be easily overcome by defining the cluster on a group of elements rather than one element. In this case, the ABF will be interpreted as the current distribution on the dipole taking into consideration the effect of the mutual impedance up to a level related to the number of elements taken within the cluster.

V. CONCLUSION

An efficient algorithm was proposed for the analysis of arrays of identical elements using the ABF/DMM technique, and was applied to an array of thin dipoles to illustrate the implementation of the algorithm. The generation of the ABFs is based on equal cluster and body size and results in a set of functions that are directly related to the current modes of the body. The algorithm exhibits fast convergence, as a few iterations are required to reach a solution within an acceptable error. The obtained solution is in very good agreement with that obtained using the direct MoM implementation. Extension of the proposed algorithm to analyze planar arrays is straightforward.

REFERENCES
