INHOMOGENEOUS WAVES IN SEMICONDUCTING SUPERLATTICE

A. A. Bulgakov, O. V. Shramkova
Institute of Radiophysics and Electronics of the NAS of Ukraine
12 Ac. Proskura St, Kharkov, 61085, Ukraine

Abstract: The present work studies the inhomogeneous (complex) waves in infinite structure derived by the periodic recurrence of two alternation semiconductor layers. We have derived analytically and numerically the dispersion relation for plasma and complex waves in semiconducting superlattice. The special property of complex waves in semiconducting structure is the complex transversal wave vector component. The imaginary part of wave vector is much greater than the real one. The appearance of large amount of inhomogeneous modes is one of the most important properties of complex waves. We have discussed the field distributions and energy flows of complex waves.

Artificial materials in the form of multilayer structures and, in particular, semiconductor superlattices have received wide application. The electrodynamical properties of complex plasma polaritons are investigated in the present work. Complex waves are waves having a complex wave vector, even in the absence of dissipative processes [1]. The imaginary part of wave vector of complex plasma polaritons in a medium with translation symmetry is much greater than the real one. It is due to the fact that the dielectric permittivity of plasmalike medium can be negative in the area of plasma frequencies. An infinite periodic structure formed by the alternation of semiconductor layers of thickness \(d_1\) and another semiconductor layers of thickness \(d_2\) is considered in the work. We orient the system of coordinates so that the Oz axis is perpendicular to the layer boundaries. The layers with the thickness \(d_1\) are placed into an electric field parallel to the Ox axis produced a drift of particles with the velocity \(\vec{v}_0\). The interaction of electromagnetic waves is described by the Maxwell equations, the continuity equation and the equation of carrier motion. In this work the polarization with the nonzero \(E_x\), \(E_z\), \(H_y\) components is studied. The boundary conditions for the plane waves \(\exp(-i\omega t + ik_z x + ik_{z1,2} z)\) can be written as [2]

\[
D_{z2} - D_{z1} = -\frac{\omega_1^2k_zv_0}{(\omega - k_zv_0)^2}\frac{E_{z1}}{\omega}\bigg|_{\text{on the boundaries of layers}},
\]

\[
E_{z1} = E_{z2}\bigg|_{\text{on the boundaries of layers}},
\]

where \(k_{z1,2} = \sqrt{\frac{\omega^2}{c^2}\varepsilon_{s1,2} - k_x^2}\); \(\varepsilon_{s1} = \varepsilon_{01} - \frac{\omega_1^2}{\omega(\omega - k_zv_0)}\); \(\varepsilon_{s2} = \varepsilon_{02} - \frac{\omega_2^2}{\omega^2}\); \(\varepsilon_{01,02}\) are the dielectric permittivities of the semiconductor lattices; \(\omega_{1,2} = \sqrt{\varepsilon_{01,02}\omega_{pl,2}}\); \(\omega_{pl,2}\) are the plasma frequencies. Using the Floquet theorem and the transfer matrix technique, we get the dispersion relation for the unbounded periodic structure [3]:

\[
\cos \tilde{k}d = \cos k_{z1}d_1 \cos k_{z2}d_2 - \frac{1}{2}\frac{k_{z1}^2\varepsilon_2 + k_{z2}^2\varepsilon_1}{k_{z1}\varepsilon_2 + k_{z2}\varepsilon_1}\sin k_{z1}d_1 \sin k_{z2}d_2.
\]

Here \(\tilde{k}\) is so-called Bloch wave number, which is the “average” of the transverse wave numbers \(k_{z1}\) and \(k_{z2}\) of the layers. Equation (2) describes plasma waves in semiconductor superlattice. Let us show that this equation has a solution in the complex plane \(k_x = k_x^i + ik_x^r\) [4]. When \(k_x^r \gg \frac{\omega^2}{c^2}\varepsilon_{s1,2}\); i.e. \(k_{z1,2} \approx ik_x^i - k_x^r\), the dispersion relation takes the form

\[
\cos \tilde{k}d = \alpha + i\beta,
\]
\[ \alpha = \operatorname{ch} k'_d d_1 \operatorname{ch} k'_d d_2 \cos k_x'^* d_1 \cos k_x'^* d_2 - \operatorname{sh} k'_d d_1 \operatorname{sh} k'_d d_2 \sin k_x'^* d_1 \sin k_x'^* d_2 + \] 
\[ + \frac{1}{2} \frac{\varepsilon_1^2 + \varepsilon_2^2}{\varepsilon_1 \varepsilon_2} \left[ \operatorname{sh} k'_d d_1 \operatorname{ch} k'_d d_2 \cos k_x'^* d_1 \cos k_x'^* d_2 - \operatorname{ch} k'_d d_1 \operatorname{sh} k'_d d_2 \sin k_x'^* d_1 \sin k_x'^* d_2 \right], \]

\[ \beta = \operatorname{sh} k'_d d_1 \operatorname{ch} k'_d d_2 \sin k_x'^* d_1 \cos k_x'^* d_2 - \operatorname{ch} k'_d d_1 \operatorname{sh} k'_d d_2 \cos k_x'^* d_1 \sin k_x'^* d_2 + \] 
\[ + \frac{1}{2} \frac{\varepsilon_1^2 + \varepsilon_2^2}{\varepsilon_1 \varepsilon_2} \left[ \operatorname{ch} k'_d d_1 \operatorname{sh} k'_d d_2 \sin k_x'^* d_1 \cos k_x'^* d_2 + \operatorname{ch} k'_d d_1 \operatorname{ch} k'_d d_2 \cos k_x'^* d_1 \sin k_x'^* d_2 \right]. \]

The imaginary part of equation (3) is zero if

\[ k_x'^* = \frac{\pi M}{d_1 + d_2} = \frac{\pi L}{d_1 - d_2}, \quad M, L = 0, \pm 1, \ldots \]  
(4)

Equation (3) represents the dispersion relation in a general form taking into account that \( k_x \) is complex. Figures 1a and 1b show a numerical solution of the equation.

![Fig.1. Dispersion curves for inhomogeneous waves](image)

In Fig.1a we can see the real parts of complex plasmons. In Fig.1b we can see the imaginary parts of complex plasmons. A set of \( k_x'^* \) exists for each value of \( k_x' \). The solution of eq.(3) exists only if \( M \) and \( L \) have the same evenness, i.e. the relation \(|d_1 - d_2|/(d_1 + d_2) = L/M \) is a rational value. In this case we can see that the width of each slab equals an integer number of quantities \( \lambda = \lambda_z / 2 \), where the real value \( \lambda_z = 2\pi / |k_x'| \) characterizes the field spatial periodicity in \( z \) direction. Figure 2 shows the dispersion curves \( \omega(k_x') \) for \( L = 0, 2, 4, 10 \).

The most important characteristic of complex waves is the dependence of the energy flow on coordinates (Fig.3). The formulas for the energy flux components are:

\[ P_{x1,2} = \frac{c}{4\pi} \operatorname{Re}(E_z H_{y1,2}^* - \frac{\omega}{4\pi} |E_{x1,2}|^2) \operatorname{Re} \left( \frac{k_x e_{1,2}}{|k_{x1,2}|^2} \right), \]  
(5)

\[ P_{x1,2} = \frac{c}{4\pi} \operatorname{Re}(E_z H_{y1,2}^* - \frac{\omega}{4\pi} |E_{x1,2}|^2) \operatorname{Re} \left( \frac{k_x e_{1,2}}{|k_{x1,2}|^2} \right). \]
The equations indicate that $\mathbf{P}$ is parallel to the $\mathbf{k}$ vector in a lossless medium. The energy lines $(P(x,z) = \text{const})$ transport the energy of a wave. The function $|P(x,z)|$ constitutes chains of peaks along the z axis. Different peaks can exchange their energy by means of low-energy circuits.

The conditions of instability in the dependence on the parameters of lattice are investigated in the work. It is shown that the complex waves can be excited by the drift of carriers in the outside electric field.

REFERENCES