EFFICIENT SIMULATION OF 2D ELECTROMAGNETIC CRYSTAL STRUCTURES WITH THE MULTILEVEL FAST MULTIPOLE ALGORITHM

Davy Pissoort (1), Dries Vandeghinste (1), Frank Olyslager (1) and Eric Michielssen (2)

(1) Department of Information Technology, Ghent University, St.-Pietersnieuwstraat 41, B-9000 Gent, Belgium
(2) Center for Computational Electromagnetics, University of Illinois at Urbana-Champaign, Urbana, IL 61801, USA

Abstract—The multilevel fast multipole algorithm is applied for the efficient simulation of large electromagnetic crystal structures. The total computational complexity of this approach is $O(PN\log(N))$, with $N$ the number of unknowns and $P$ the number of iterations required for the iterative solution to the desired accuracy. To reduce $P$, a simple block diagonal preconditioner is used. As an example a four-channel multiplexer-demultiplexer is simulated.

1 INTRODUCTION

In recent years, great interest arose in the simulation of electromagnetic crystals (ECs) because of their ability to control the lightwave propagation [1]. The two-dimensional ECs studied in this paper consist of a set of parallel dielectric or PEC cylinders placed on a periodic lattice in a homogeneous medium. Because of the periodicity of the lattice electromagnetic bandgaps exist in which electromagnetic fields cannot propagate in given directions. By removing or adding cylinders, it is possible to create localized EM modes, creating e.g. low loss waveguides, multiplexers, splitters, etc. As these structures can become very large, the need for an improvement of the simulation techniques is still present. In [2], we presented a rigorous semi-analytical technique for the simulation of 2D ECs. Contrary to the often used FDTD, this technique does not face phase errors for large structures. The main disadvantage of the method in [2] is that one has to solve a linear system whose dimension scales linearly with the number of cylinders, which means that when using a direct solver the overall computational complexity scales as the third power of the number of cylinders. To overcome this problem, we will apply in this paper the multilevel fast multipole algorithm (MLFMA) [3] to this semi-analytical technique. In this way, the computing time for one matrix-vector multiplication is $O(N\log(N))$. Also the memory requirements are reduced drastically. As in [2], we restrict ourselves to TM polarization, for which the electric field has only a component parallel to the cylinders ($z$-axis).

2 MLFMA FORMULATION

In [2] we expanded the unknown surface currents on every cylinder $j$ as a truncated Fourier series

$$J^j_z(\phi_j) = \sum_{m=-K}^{+K} \frac{I^j_m}{2\pi r_j} e^{i p \phi_j},$$

with $r_j$ the the radius of the $j$th cylinder and unknown coefficients $I^j_m$. All these terms generate cylindrical waves. The boundary conditions at the cylinders are enforced by using a Fourier decomposition in combination with an appropriate boundary impedance to obtain a linear set of equations

$$\sum_{j=1}^{N_{cyl}} \sum_{m=-K}^{+K} Z_{pm}^{ij} I^j_m = f^i_p, \quad i = 1, \ldots, N_{cyl} \text{ and } p = -K, \ldots, K,$$

with $N_{cyl}$ the number of cylinders and

$$Z_{pm}^{ij} = \frac{1}{4\pi^2} \int_{-\pi}^{+\pi} d\phi_j \int_{-\pi}^{+\pi} d\phi_i H_0^{(2)}(k|\rho_i - \rho_j|) e^{i m \phi_i} e^{-i p \phi_j}.$$
Normally, the matrix-vector multiplication would require \(N^2\) operations, with \(N = (2K + 1)N_{\text{cyl}}\). In the MLFMA one divides the cylinders into groups and a distinction between near- and far-field contributions is made. The near-field interactions are calculated in the classical way. For the far-field interactions the number of operations for a matrix-vector multiplication can be reduced using the following approximation for the Hankel function

\[
H_0^{(2)}(k|\rho_i - \rho_j|) \approx \sum_{l=-L}^{L} \left[ e^{i\mathbf{k} \cdot \mathbf{\rho}_i} (\mathbf{\rho}_i - \mathbf{\rho}_j)^m e^{i\mathbf{k} \cdot \mathbf{\rho}_j} (\mathbf{\rho}_i - \mathbf{\rho}_j)^m \right], \tag{4}
\]

with \(\rho_i^m\) and \(\rho_j^m\) the middle points of the cylinders \(i\) and \(j\), \(\rho_i^c\) and \(\rho_j^c\) the centers of the groups of the cylinders \(i\) and \(j\), \(\mathbf{k}(\phi) = k(\cos \phi \mathbf{a}_x + \sin \phi \mathbf{u}_y)\) and \(\phi_l = \frac{2n\pi}{2L+1}\), see Fig. 1. The translation operator is given by

\[
T_l(k, \rho, \phi) = \frac{1}{2L+1} \sum_{l'=-L}^{L} H_0^{(2)}(kp)e^{il'\phi}. \tag{5}
\]

In this way we can write (3) as

\[
Z_{pm}^{ij} = J_m(kr_i)J_p(kr_j) \sum_{l=-L}^{L} \left[ e^{i[\mathbf{k} \cdot \mathbf{\rho}_j] + m(\phi + \theta)} ]_i \right] T_l e^{-jl[\mathbf{k} \cdot (\mathbf{\rho}_j - \mathbf{\rho}_j) + p(\phi + \theta) + j]}, \tag{6}
\]

which is similar to the standard fast multipole method expressions for 2D scattering [3].

### 3 PRECONDITIONING

The overall computing time is not only proportional with the time needed for one matrix-vector multiplication, but also with the number of iterations needed for convergence of the iterative solver to the desired accuracy. Because multiple scattering problems often lead to ill-conditioned matrices, a very large number of iterations can be required. To reduce this number of iterations, various preconditioning techniques can be used. However, one has to keep in mind that the complexity to calculate these preconditioners should not jeopardize the \(O(N \log(N))\) complexity. The use of a simple block diagonal preconditioner already reduces the number of iterations drastically. For this block diagonal preconditioner, the EC structure is divided into groups and the inverse of the small interaction matrix of every group is calculated by a direct method. The computing time for this block diagonal preconditioner is \(O(pM^3)\), with \(p \leq \frac{N}{M}\) the number of different groups and \(M \ll N\) the number of unknowns in one group. By consequence, if \(M\) is fixed, this computing time becomes \(O(N)\).

### 4 EXAMPLE

As an example we consider the four-channel multiplexer-demultiplexer [4] shown in Fig. 2. The radius and the relative dielectric constant of the dielectric cylinders are respectively 0.18\(a\) and 11.56, with \(a\) the lattice constant. The background medium is air. The interaction lengths are \(l_1 = 44a\), \(l_2 = 22a\) and \(l_3 = 24a\). The total number of cylinders equals 1303. For a convergent solution we need three unknowns per cylinder, resulting in a total of 3909 unknowns. On a 2GHz computer, the total time for filling in the interaction matrix and solving the linear system with a direct solver at \(f = 0.36\) is 442 seconds with a memory requirement of 580 MB. The groups that are used for the calculation of the block diagonal preconditioner are also indicated in Fig. 2. In total there are 8 different groups. The time to calculate the block diagonal preconditioner is 14.23 seconds. The total setup time (calculation of the translation matrices, outgoing plane waves, incoming plane waves and the near-field interactions) for the MLFMA is 13.48 seconds. For this example, we used the transpose free quasi-minimal residual (TFQMR) solver [5]. To converge to a residual error of \(1 \times 10^{-4}\), this solver needed 82 iterations, which took 58.5 seconds. This makes a total calculation time of 86.21 seconds. The total memory requirement reduced to 26 MB.
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Figure 1: Observation and source group.

Figure 2: Four-channel multiplexer-demultiplexer.

References