Recent Advances in Nonlinear Inverse Scattering

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Abstract: This paper reviews recent advances in solving the nonlinear inverse problem for practical subsurface sensing applications. One of the pressing problems in solving the nonlinear inverse problem is the computer time. We will discuss various attempts to reduce this bottleneck, and show the use of the processing algorithm for inverting some real-world data.

INTRODUCTION
Inverse scattering is a field that finds wide applications in various areas in science and engineering. Inverse scattering empowers by allowing us to find the shape as well as the electrical properties of the object being investigated. Traditionally, tomography algorithms exist that are highly efficient. These algorithms are usually based on a linearization approximation of the relationship between the scattered field and the object being investigated [1-3]. The approximation is related to the Born approximation, or the single scattering approximation whereby one assumes that the incident wave from the transmitter just bounces once off the scatterer before it reaches the receiver. This approximation allows a Fourier transform relationship to be established between the scattered field and the object function used to describe the shape and the physical properties of the object. Hence, highly efficient algorithms involving $O(N \log N)$ computer operations can be derived that process the data rapidly. Such is the field of synthetic aperture radar (SAR), diffraction tomography, and ultrasound tomography.

NONLINEAR INVERSE SCATTERING
However, the single scattering approximation is often not accurate enough to describe many scattering processes in the real world. Data obtained in the real world often involve multiple scattering. When single scattering algorithms are used to process data with multiple scattering, ghost images often appear in the reconstructed images. When multiple scattering effect is included in modeling the relationship between the scattered field and the object function, a nonlinear relationship ensues. A way to process this data is to incorporate the nonlinear, multiple scattering effect into the algorithm. The multiple scattering effect can be accounted for by posing the problem as an optimization problem whereby a forward scattering model is used to generate the scattered field data to match the experimentally measured data [4-8]. This usually results in a gradient search algorithm involving three calls to a forward scattering algorithm. The cost of solving the forward scattering problem can be exorbitant, and hence, results in an inordinately large cost of solving the inverse problem. The strategy here has been to use efficient forward solvers to obtain rapid solutions to the forward scattering problem. The recent development of CG-FFT (conjugate gradient fast Fourier transform) solvers and MLFMA (multi-level fast multipole algorithm) seem particularly suited for this purpose [9]. CG-FFT seems particularly well-suited for volumetric scattering while MLFMA is more suitable for surface scattering phenomena. The computational complexity of such algorithms is approximately proportional to $N_{\text{iter}} N_{\text{trans}} N \log N$, where $N_{\text{iter}}$ is the product of the number of iterations needed to solve both the forward problem and the inverse problem, $N_{\text{trans}}$ is the number of transmitters used to illuminate the scatterer, and $N$ is the number of pixels in the image to be reconstructed [10]. Compared to linear, tomographic algorithms, nonlinear algorithms via a gradient search approach is about $N_{\text{iter}} N_{\text{trans}}$ times more expensive than linear algorithms. Since $N_{\text{iter}} N_{\text{trans}}$ can be large, the use of nonlinear inverse scattering algorithms has not been pervasive. Notice that the inversion algorithm is independent of the number of receivers, $N_{\text{rec}}$, used in the data. However, in order that we can construct an $N$ pixel image from $N$ independent data, we require that $N_{\text{rec}} N_{\text{trans}}$ to be proportional to $N$. By reciprocity, we expect that $N_{\text{rec}} \approx N_{\text{trans}} - \sqrt{N}$. Consequently, the computational complexity of the nonlinear inverse scattering algorithm is $N_{\text{iter}} N^{-\frac{1}{2}} \log N$. 

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Another nonlinear inverse scattering algorithm that has appeared in the literature is the linear sampling method [11]. This method requires a scattering matrix of the object to be known or measured. If the object function has N pixel of information, it is reasonable to assume that this scattering matrix has dimension of $M \times M$ where $M \approx \sqrt{N}$. For generating each pixel of the image, the linear sampling method operates on the $M \times M$ data on the scattering matrix requiring at least $O(M^2)$ or $O(N)$ operations. Consequently, to generate an $N$ pixel image, it needs $O(N^3)$ computer operations. Hence, it is more expensive than linear inverse scattering methods, and if $N > N_{min} = N_{min} \approx \sqrt{N}$, it can be slower than the gradient search method. The advantage of the linear sampling method has been its freedom from having to solve the forward scattering problem.

Since the computational time has been a bottleneck in the nonlinear inverse scattering, an alternative method to process voluminous real world data is to use one-dimensional or two-dimensional inverse algorithm to process real-world data which is usually collected in three dimensions, or to combine linear method with nonlinear method [12-14]. The use of reduced dimensions greatly expedites the forward solver solution time, and hence, can drastically reduce the CPU time needed to solve this problem. We can also combine 1D/3D inversion technique used in the context of distorted Born iterative method (DBIM) to reduce the computational labor of such algorithms. We will demonstrate and contrast the use of 1D/2D/3D nonlinear inverse problems in processing some realistic data.

CONCLUSIONS
A cost analysis show that nonlinear inverse scattering algorithms that account for multiple scattering effect is still much more costlier than linear inverse scattering algorithms that account only for single scattering. One way to cut down the cost of the nonlinear inverse scattering algorithms is to use 1D or 2D inverse scattering to solve the 3D inverse scattering problem, or to combine linear methods with nonlinear methods. The use of this strategy can greatly expedite the way that real world data is processed. Also, the use of fast solvers such as the CG-FFT method, the CG-Fast Hankel Transform method, as well as the multilevel fast multipole algorithm can be used to accelerate the solution to the nonlinear inverse scattering problem.

REFERENCES
