PROPAGATION CHARACTERISTICS OF FINITE NON-RECIPROCAL
MAGNETIC PHOTONIC CRYSTALS

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Abstract: We present a numerical approach for calculating the $k$-$\omega$ diagram for finite nonreciprocal magnetic photonic crystals. This approach is necessary both to verify and to utilize the frozen mode regime in these magnetic crystals for real life application.

INTRODUCTION
Photonic and electromagnetic bandgap structures have been extensively studied in the past decade after the initial paper by Yablonovich [1]. The unconventional propagation characteristics of bandgap materials is of interest in developing integrated high Q RF filters and miniature antennas (e.g. see the special issue of IEEE Trans. Antennas Propagat., Oct. 2003). Recently, a new type of magnetic photonic crystal was introduced by Figotin et.al. [2] that incorporates magnetic materials to achieve even more interesting propagation characteristics. The characteristics of this one-dimensional non-reciprocal magnetic photonic crystal were investigated in [2] and [3]. Bloch mode analysis on infinite and semi-infinite composite stacks was carried out in [2] and [3] to generate the non-reciprocal band dispersion relations. The key characteristic of the new magnetic crystal is the support of the so called frozen mode. Basically, the wave entering the crystal slows down (has zero group velocity at the frozen mode frequency) and enlarges in amplitude. This phenomenon is connected to the stationary inflection point in the $k$-$\omega$ diagram. The frozen mode characteristic make this metamaterial of great interest for microwave applications. Further investigation is necessary in order to understand and realize the same exotic propagation behavior in finite size layered media.

BACKGROUND
The infinite periodic structure given in [2, 3] is composed of a unit cell that has two misaligned anisotropic dielectric layers and one ferromagnetic layer, as shown in Fig. 1 (a). The specific material parameters ($\mathbf{\varepsilon}$ and $\mathbf{\mu}$ tensors and physical dimensions) can be found in [2]. Using the transfer matrix of the assembled unit cell, one can generate the $k$-$\omega$ diagram for the infinite periodic structure [2] (see Fig. 1 (b)). We note the non-symmetric (non-reciprocal) response of the $k$-$\omega$ diagram and the inflection point satisfying $\omega'(k) = \omega''(k) = 0$ and $\omega'''(k) \neq 0$.

![Figure 1](image_url)

In this paper, we focus on the propagation characteristics of a finite magnetic photonic crystal. An $N$-layer crystal will have $N$ unit cells of the infinite structure assembled in free-space. We will excite this finite structure with a plane wave of frequency $\omega$, propagating along the positive $z$-direction. Using the transfer matrices in each individual layer, we can generate the field expressions everywhere inside the $N$-layer slab. From this field snapshot, we can extract the propagation constants of the Bloch-like modes via the fast Fourier transform (FFT). The details of the approach are given in the next section.

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The four modes supported by a 1-D anisotropic structure in the $x$-$y$ plane such as depicted in Fig. 1 (a) are given by $v_1 e^{i q_1 z}$, $v_2 e^{i q_2 z}$, $v_3 e^{i q_3 z}$, and $v_4 e^{i q_4 z}$, where $q_i$ refer to the in-layer wave numbers (an $e^{-i \omega t}$ time convention has been assumed and suppressed). Staring with (1), we proceed to enforce field continuity on both boundaries of each layer to obtain the transmitted wave as

$$
T \left\{ \begin{array}{c} E^{(i)} \\ H^{(i)} \end{array} \right\} = \left\{ \begin{array}{c} E^{(r)} \\ H^{(r)} \end{array} \right\} = \left[ \begin{array}{c} E_t \\ H_t \end{array} \right] _i.
$$

After expressing the incident, reflected, and transmitted waves in (2) in terms of the modes in (1), and assuming that the incident field propagating along the positive $z$-direction is known, the unknown coefficients of the reflected and transmitted fields are found to be

$$
\begin{bmatrix}
    a_1^{(t)} & a_2^{(t)} & a_1^{(r)} & a_2^{(r)}
\end{bmatrix}^T = \begin{bmatrix}
    v_1 & v_3 & -Tv_2 & -Tv_4
\end{bmatrix}^{-1} \begin{bmatrix}
    a_1^{(i)}Tv_1 + a_2^{(i)}Tv_3
\end{bmatrix}.
$$

Specifically, the fields are given as

$$
\begin{bmatrix}
    E^{(t)} \\ H^{(t)}
\end{bmatrix} = a_1^{(t)}v_1 + a_2^{(t)}v_3,
\begin{bmatrix}
    E^{(r)} \\ H^{(r)}
\end{bmatrix} = a_1^{(r)}v_2 + a_2^{(r)}v_4,
\begin{bmatrix}
    E^{(i)} \\ H^{(i)}
\end{bmatrix} = a_1^{(i)}v_1 + a_2^{(i)}v_3.
$$

The specific forms of the individual $T_i$ matrices for the $A$ and $F$ layers are given in [2].

Using these transfer matrices (as infinitesimal propagators for the incident field), we can generate the fields everywhere inside the finite slab as shown in Fig. 1 (c). In an infinite stack, this field snapshot consists of propagating Bloch waves with the propagation constants as given in the $k$-$\omega$ diagram of Fig. 1 (b). By the same token, we can extract the propagation constants of the modes inside this finite slab by looking at the field picture inside. This is done through an FFT operation with proper windowing to eliminate the spurious modes introduced by the windowing operation (due to the finite size of the slab). Here, we choose to apply a Chebychev window with the side lobe level set to -100dB. After the application of the windowed FFT, we retain only the largest amplitude harmonic components. This procedure is repeated for all incident wave frequencies to obtain the propagation constants inside the finite slab and thus generate a numerical $k$-$\omega$ diagram. From Fig. 2, it is seen that the numerical computations of the finite length slab (of total length $< 2\lambda_0$) exhibits the frozen mode characteristics since the inflection point is clearly identified.

Propagating and evanescent Bloch waves inside the material region are sufficient to analyze the behavior of infinite and semi-infinite crystals. Furthermore, the lack of a reflecting crystal-air interface prohibits coupling between forward and backward propagating Bloch waves. However, when we consider a finite crystal, the Bloch-like forward and backward propagating waves in the structure are inherently coupled due to the reflections from the crystal-air interfaces. Hence, although the finite crystal is excited by a forward (in positive $z$-direction) propagating plane wave, the propagation constants extracted form the field picture inside the slab using the FFT method described above will correspond to both forward and backward propagation waves. Hence, all four branches in the $k$-$\omega$ diagram are numerically constructed by just exciting the structure with a forward propagating monochromatic wave.

In general, the field inside the finite crystal is elliptically polarized. Depending on the polarization of the incident field, as well as the specific orientations of the anisotropy axes of the dielectric layers, the computed propagation constant will favor specific branches on the $k$-$\omega$ diagram. This is even more pronounced when only the strong propagating components are considered as in Fig. 2. Here, the propagation constants of field components with a normalized amplitude greater than 0.5 were plotted. For this specific example with 30 layers, it is seen in Fig. 2 (a) and (b) that an $x$-polarized incident field couples more to the $x$-component of the slower-propagating Bloch-like waves (i.e. the lower branches of the $k$-$\omega$ diagram). In contrast, the $y$-polarized incident wave favors the $y$-component of the faster-propagating waves (upper branches of the $k$-$\omega$ diagram). We also note that (see Fig. 2 (c) and (d)) the inflection point lies on one of the upper branches of the $k$-$\omega$ diagram. Hence, to better utilize the frozen mode regime, it is advisable to excite the finite crystal with a $y$-polarized incident wave. Using the presented simple analysis, we can generate an optimal incidence polarization to maximize the
fields coupled in the finite length crystal and minimize the reflections. This aspect and the behavior of wide band pulses around the frozen mode frequency is under investigation and will be discussed at the meeting.

The computation of the field picture inside the finite crystal requires many applications of the transfer matrix, depending on the sampling requirements. Away from the bandgap, the amplitudes of the eigenvalues of the transfer matrix are equal unity. However, in the vicinity of the bandgap the application of the transfer matrix many times can become numerically unstable. For this reason, the method described above cannot be used in the immediate vicinity of the band edges. However, of particular interest in this paper is the prediction of the stationary inflection point for finite crystal which lie away from the band edges. Hence, the above method can safely be used to predict the propagation behavior of finite and possibly very short length crystal.

REFERENCES