COUPLING BETWEEN THE WHISPERING GALLERY MODE IN A MICRORING LASER AND FUNDAMENTAL MODE IN AN ELLIPTIC OPTICAL FIBER

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Abstract
Experiments have demonstrated that a microring laser that is formed coaxial to a single-mode optical fiber couples light into the propagating mode of the fiber. The present paper focuses on the influence of now-common elliptic cross-section fibers on the coupling to a micro-ring laser. The angular Mathieu functions that satisfy the wave equation in one material medium are not orthogonal to those that satisfy the wave equation in a different medium. Consequently, coupling exists between an elliptic microring and the dominant mode in the fiber around which it is formed, so long as the refractive indices of the laser medium and the fiber are different. This stands in contrast to the situation when the micro-ring and the fiber are circular.

INTRODUCTION
Microring lasers can be formed from optically active polymers formed into a circular band, using an optical fiber as mandrel, as shown in Figure 1a [1]. Excitation of the fiber mode by light originating in the laser mode has been observed, as Figure 1b shows [2]. The structure is fabricated with a gold buffer layer between the fiber and the microring, which isolates the fiber from being a part of the laser resonance. However, Frolov, et. al. [1] suggest that light can pass from the laser into the fiber through evanescent tunneling into the fiber mode.

The coupling from the microring into the fiber through evanescent tunneling can be estimated by 1) assuming the laser mode to be that which would exist in a cylinder of infinite extent in the longitudinal direction, 2) truncating the structure as pictured in Figure 2, 3) assuming the field distribution across the truncated face to be that computed for the infinite-extent structure, and 4) computing the coupling integral between this approximate field and the modal field for the fiber.
CIRCULAR CASE
Based on experimental evidence, Frolov, et. al., have concluded that the resonance in the microring laser is a result of a whispering gallery mode (WMG) that is formed at the outer boundary of the optically active material forming the microring. We have been able to substantiate this fact theoretically. For the structure to resonate, the phase must change as a function of angle, passing through an integer number of $2\pi$ cycles. Thus, any WGM resonance is orthogonal to any propagating mode on a circular fiber. Any coupling between the laser resonance and the fiber must involve departure from circular symmetry. In an earlier paper, we have explored the possibility that refractive index change due to heating by the pump would introduce this coupling [3], but concluded that this effect could not produce appreciable coupling.

PROPAGATION IN AN ELLIPTIC FIBER
In elliptical coordinates, solutions for wave equation are combinations of Mathieu functions and Modified Mathieu functions. Solution of tangential E and H field in an elliptical fiber are as follows:

For region 1 ($0 \leq \xi \leq \xi_0$), which is inside the fiber:

\[
\begin{cases}
E_{z1} \text{ or } H_{z1} = \\
\begin{cases}
Ce_n(\xi, \gamma_1^2)ce_n(\eta, \gamma_1^2) & \text{ (even)} \\
Se_n(\xi, \gamma_1^2)se_n(\eta, \gamma_1^2) & \text{ (odd)}
\end{cases}
\end{cases}
\]

For region $O$ ($\xi_0 \leq \xi \leq \infty$), which is outside the fiber:

\[
\begin{cases}
E_{z0} \text{ or } H_{z0} = \\
\begin{cases}
Fek_n(\xi, -\gamma_0^2)ce_n(\eta, \gamma_0^2) & \text{ (even)} \\
Gek(\xi, -\gamma_0^2)se_n(\eta, -\gamma_0^2) & \text{ (odd)}
\end{cases}
\end{cases}
\]

where
The wave number $\beta$ is solved by enforcing continuity of tangential components of electric and magnetic field at $\xi = \xi_0$, the boundary between interior and exterior regions. The resulting infinite-dimensionals matrix is set to zero, thereby dictating $\beta$ [4]. It is noteworthy that different sets of Mathieu functions arise in regions $O$ and $i$.

**COUPLING COEFFICIENT BETWEEN MICRORING AND FIBER**

\[
\gamma_1^2 = \frac{1}{4}(k_1^2 - \beta^2)q^2; \quad \gamma_0^2 = \frac{1}{4}(\beta^2 - k_0^2)q^2
\]
\[
k_1^2 = \omega^2 \mu \epsilon_i \quad \text{and} \quad k_0^2 = \omega^2 \mu \epsilon_0
\]

where $E$ indicates the entire modal field of the fiber, spanning both interior and exterior regions. Figure 4 shows an example calculation as a function of ellipse eccentricity. It is seen that useful values of coupling occur in the vicinity of $e = 0.8-0.9$.

\[C_{wf} = \frac{1}{2\pi} \int_0^{2\pi} \int_0^\infty \tan \phi \tan \rho \rho d\rho d\phi \frac{E(\rho, \phi, z)}{E_f}
\]

**CONCLUSION**

Coupling occurs between a microring laser and the fiber that it encircles when material contrast exists between the two. This offers the possibility of design control of the coupling between the laser and the fiber.

**REFERENCES**


