RF Exposure Compliance Boundary Determination for Elongated Antennas

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Abstract
A measurement-based method for determination of RF exposure levels for elongated antennas is presented. The input to the method is a complex tangential electric far field. Spherical modes are obtained via a point match of the measured field. A spherical modal expansion and transformation to cylindrical modes is used to retrieve the electric field in a volume enclosing the antenna. By the use of a cylindrical modal expansion the method is capable of recovering fields at points inside the smallest sphere enclosing the antenna in contrast to using only spherical modes.

1 INTRODUCTION
Current standards, e.g. [2], for human exposure from radio base station antennas calls for methods to determine the electromagnetic field in a volume surrounding the antenna. Well-established and efficient methods are planar, cylindrical and planar transformation techniques based on radiating divergence-free solutions to the vector Helmholtz equation, see Leach and Paris [4], Blanch et al. [1] and Hansen et al. [3]. However, more efficient methods are obtained by combining the methods. To avoid spatial truncation of the measurement surface, a spherical measurement is used and converted into a spherical modal expansion. Thereafter, the spherical modal expansion is evaluated on a second type of surface and the corresponding modal coefficients/transforms are calculated. Now it is possible to evaluate the field on surfaces of a shape more conformal with the the geometry of the antenna, and hence get closer to the antenna. A combined spherical-cylindrical method for elongated antennas is presented in this report.

2 METHOD
2.1 Modal functions. The spherical radiating divergence-free solutions to the vector Helmholtz equation (spherical modes) are Hansen et al. [3] (θ and φ are the polar and azimuthal angles, respectively, and time dependence \(\exp(j\omega t)\) is used)

\[
\begin{align*}
    u_{lm1} &= \nabla s_{lm} \times r \quad (\text{TE}_r) \\
    u_{lm2} &= \frac{1}{k} \nabla \times u_{lm1} \quad (\text{TM}_r)
\end{align*}
\]

where
\[
\begin{align*}
    s_{lm}(r, \theta, \phi) &= h^{(2)}_l(kr)P^m_l(\cos \theta)e^{j m \phi} \\
    l &= 1, \ldots, \infty \quad m = -l, \ldots, l
\end{align*}
\] (1)

The corresponding cylindrical modes are found after a Fourier transform along the cylinder axis (z-axis) with kernel \(\exp(jk_z z)\)

\[
\begin{align*}
    U_{m1}(\rho, k_z) &= \nabla S_m \times \hat{z} \quad (\text{TE}_z) \\
    U_{m2}(\rho, k_z) &= \frac{1}{k} \nabla \times U_{m1} \quad (\text{TM}_z)
\end{align*}
\]

where
\[
\begin{align*}
    S_m(\rho, k_z) &= H^{(2)}_m(\sqrt{k^2 - k_z^2} \rho)e^{j m \phi} \\
    m &= 0, \ldots, \infty
\end{align*}
\] (2)

Here, \(n = 1, 2\) denotes TE/TM modes, respectively.

2.2 Spherical mode expansion. The measured tangential electric field components at \(N\) points and the \(J\) \(\text{TE}_r\) and \(\text{TM}_r\) spherical mode amplitudes are put into single-column vectors \(E\) (length \(2N\)) and \(a\) (length \(2J\)), respectively. The modal expansion is

\[
E = u \cdot a
\] (3)
where the elements of the $2N \times 2J$ matrix $u$ are the modal functions of Eq. (1) evaluated at the measurement points. The spherical modal series is truncated at $l = L = kr_{AUT} + 10$ and $|m| = M = k\rho_{AUT} + 10$ and an equi-angular $L \times (2M + 1), \theta \times \phi$, grid is used to recover the spherical mode amplitudes with high accuracy. Here, $r_{AUT}$ and $\rho_{AUT}$ are the spherical and cylindrical radii of the Antenna Under Test (AUT). The mode amplitudes $a$ are obtained by applying the method of least squares to Eq. (3). A matrix formulation (3), in contrast to using numerical integration Hansen et al. [3], makes it possible to use any angular grid and even non-spherical surfaces for the retrieval of spherical modes. For the above equi-angular grid the over-sampling ratio (number of points/number of modes) $OSR = \frac{L(2M + 1)}{[L(2M + 1) - M(M - 1)]}$ is close to 1 for elongated antennas ($L \gg M$). Confer with $OSR = 4/\pi$ in Hansen et al. [3].

2.3 Conversion to cylindrical modes. The cylindrical mode amplitudes are obtained by evaluating the spherical modal expansion on a cylinder, summing over $l$ and Fourier transforming along the cylinder axis $z$. The cylindrical modal expansion is used to calculate the field near the antenna and the spherical modes are used on spheres farther out.

2.4 Back-transformations. With perfectly accurate modal amplitudes the entire electric (and magnetic) field can be recovered at any point outside the smallest sphere enclosing the antenna. However, with noisy erroneous data this is not true. The numerical performance of the spherical back-propagation is governed by the spherical Hankel function $h_2^{(2)}(kr)$. For $kr < l$ the imaginary (reactive) part dominates and noisy modes with $l > kr$ may completely dominate the field at $r$ if included in the modal series. Note, that this corresponds to cutting out the reactive part of the energy. Accordingly, the cylindrical modal expansion should be truncated at $|m| = k\rho$ and $kz = k$ to avoid exponentially increasing fields close to the antenna. All truncation limits imply a minimum spatial resolution approximately equal to $\lambda/2$. Hence, at the boundary of the reactive near field zone the back propagation breaks down whereas farther out the accuracy is maintained, i.e., a spherical far field can be used as input data. Furthermore, probe correction is obsolete, which simplifies the procedure significantly Hansen et al. [3, p. 230]. Using a far field, any radius in the far field region $r \gg 2L^2/\pi^2$ can be used as test distance, since the angular variation is negligible and the mode amplitudes will just re-scale with a complex constant. The mode amplitudes can then be re-scaled to any desired level of total radiated power.

3 RESULTS

3.1 Numerical tests. The numerical performance of the back-propagation is tested using arrays of Hertzian dipoles with known near field. The result of a numerical experiment with test distance 6 m is depicted in Fig. 1. Relevant error sources compliant with an existing far field range at Ericsson Microwave Systems Mölndal/Sweden are used. The horizontal dashed line indicate 30%=-5.2 dB error [2] and the vertical dashed line indicate the smallest sphere enclosing the antenna $r_{AUT} \approx 55$ cm. The radius $\rho_{AUT}$ of the smallest cylinder enclosing the antenna is approximately 10 cm.

Fig. 1: Errors for back-propagated near field at $f = 900$ MHz along a boresight ray $(\theta, \phi) = (\pi, 0)$.  

616 URSI EMTS 2004
3.2 Measurements. For further comparison the method has been applied to a near field and a far field measurement of a 17.7 dBi directivity UMTS antenna. The difference between the length, widths and heights of the compliance boundaries determined by the method using two different measurements are less than 25 cm, cf. Fig. 2.

![Near field range and Far field range](image)

Fig. 2: Side view of compliance boundary regarding an electric field strength limit of $E = 61.3$ V/m for a UMTS antenna @2140 MHz and with an output power of $P = 30$ W. The boundaries are determined using data measured at (a) a 6 m near field range and (b) a far field range. Box-shaped envelopes to the compliance boundaries have the following lengths, widths and heights (a) $l = 3.91$, $w = 1.88$, $h = 1.64$ and (b) $l = 4.11$, $w = 1.87$, $h = 1.88$ (unit m).

4 CONCLUSIONS

Combined use of spherical-cylindrical modal expansions is an efficient and accurate method for retrieving the field strengths anywhere outside the reactive near field of an elongated antenna. The critical step is the regularization of the back-propagation which is done by truncating the modal expansions at a spatial resolution of $\lambda/2$. For a high-accuracy measurement range the dominating error is the mode truncation error which implies a numerical break-down of the method at the boundary of the reactive near field region. The method allows for determination of field strength close to the antenna, e.g. in the rear region, where the radiation levels are very low, see Fig. 2.

REFERENCES


