WEIGHTING WINDOWS BASED ON ATOMIC FUNCTIONS IN THE TWO-STAGE DIGITAL SIGNAL PROCESSING SCHEME OF SAR

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Abstract. A new method in the two-stage digital signal processing scheme of SAR, based on theory of atomic functions, is proposed.

INTRODUCTION

Two-stage signal processing method in SAR is used for the matching of frequency at the input and the output of digital system [1]. In that case, a digital synthesizing process passes with a low sampling frequency, that reduce requirement for a digital processing system. In the process of forming a radar image weighting functions are used for decreasing the sidelobe level. In the report, it is offered to use a new class of weighting windows, based on the theory of atomic functions [2-4]. A numerical experiment results are presented. Efficiency of the new method is shown.

BASIC PRINCIPLES OF THE SYNTHETIC RADAR SIGNAL PROCESSING

Consider the basic principles of SAR.

Synthesis of aperture using continuous ranging signal. The reflected signal has the form

\[ u_i(t) = U_i G(t) \cos(\omega_0(t - \tau_i) + \phi_0 + \psi_i) \],

where \( U_i \) is the maximal amplitude of the signal; \( G(t) \) is a normalized function characterizing modulation of sensing and reflected signals during their transferring and accepting by an antenna pattern; \( \phi_i \) is the change of the signal phase caused by reflection. To suppress noise during signal processing, the complex-valued analytical signal \( \hat{s}_i(t) \) is used

\[ \hat{s}_i(t) = U_i \exp\left\{-j\left[2\pi V^2 t^2 / (\lambda r_0) - \psi_i\right]\right\}. \]

The real signal which should be processed is usually a sum of a reflected signal and noise, i.e.,

\[ \hat{s}_i(t) = \hat{s}_i(t) + \hat{n}(t). \]

The optimal device must form a signal, corresponding to a radar image up to a constant factor

\[ J_i(\eta) = \left| \frac{\int_{-T/2}^{T/2} \hat{s}_i(t + \eta) \hat{h}(t) dt}{\int_{-T/2}^{T/2} \hat{s}_i(t) \hat{h}(t) dt} \right|. \]

where the radar image signal \( J_i(\eta) = J_i(\chi V) \); \( \eta \) and \( \chi \) are the temporal and spatial shifts between the accepted signal \( \hat{s}_i(t) \) and support function \( \hat{h}(t) \), respectively. As the support function a weighted function is used, which is complex conjugated up to the initial phase with the reflected signal,

\[ \hat{h}(t) = H(t) \exp[j\Phi_i(t)], \]

where \( H(t) \) is a weighting function. The SAR response at pointwise target, which is called signal function, is

\[ I_i(\eta) = \left| \frac{\int_{-T/2}^{T/2} \hat{s}_i(t + \eta) \hat{h}(t) dt}{\int_{-T/2}^{T/2} \hat{s}_i(t) \hat{h}(t) dt} \right|. \]

Digital signal processing in SAR. For ensuring high radar resolution in distance of SAR instead of a continuous signal (1) impulse signals are used. In the SAR with the digital signal processing the reflected signal, after its transform to a complex-valued form, digitizes in time

\[ \hat{s}_i[p] = \hat{s}_i(pT_d) = U_i G(pT_d) \exp\left\{-j\mu(pT_d)^2 - \psi_i\right\}, \quad p = 0, \pm 1, \pm 2, \ldots, \]

where \( T_d \) is sensing period. In that case, the reflected digital signal \( \hat{s}_i[p] \) is a sum
\[ \hat{\xi}_j[p] = \delta_j[p] + \eta[p] \, . \]  

where \( \eta[p] \) is a digital white noise. The support function is also presented in the digital form

\[ \hat{h}_1[q] = \hat{h}(qT_d) = H(qT_d) \exp\{j\mu(qT_d)^2\} \, , \quad q = 0, \pm 1, \pm 2, \ldots, \pm N/2 \, , \]  

where \( N = \text{int}\{T/T_d\} \) is a number of sensing period in the synthesis interval. Digital signal processing and production digital radar image signal is described with the discrete analog of (5)

\[ J_i[q] = |\hat{f}[q]|^2 = \sum_{p=-N/2}^{N/2} |\hat{\xi}_j[p + q]|^2 \hat{h}_1[p] \, . \]  

Two-stage digital signal processing scheme. Consider the two-stage processing scheme. On the first stage

the frequency reducing of sampling with less possible loss of power and bandwidth narrowing of processing signal at the same time is taken place

\[ \hat{\xi}[p_1] = \sum_{k=0}^{N_1} \hat{\xi}[p_1N_1 + k] \, , \]  

where \( N_1 \) is the number of samples, accumulating in the partial sum.

On the second stage, based on the (10), but for less number of samples, radar image signal is forming

\[ J_i[q] = \sum_{p_1=0}^{N_2-1} |\hat{\xi}_j[p_1 + q]|^2 \hat{h}_2[p_1] \, , \]  

where \( N_2 = N/N_1 \) is the number of partial sums in the synthesis interval, \( \hat{h}_2[p_1] \) is the support function, forming with the meaning of partial summing

\[ \hat{h}_2[p_1] = \sum_{i=0}^{N_1-1} \hat{h}[p_1N_1 + i] \, , \quad p_1 = 0, 1, 2, \ldots, N_2 - 1 \, . \]  

NUMERICAL EXAMPLE

In this work, application of atomic functions (AF) [3,4] for constructing support functions used in SAR is discussed. Main parameters of new weighting windows are presented and compared with those of classical windows. To compare different windows the following system of parameters will be used: the maximum sidelobe level, dB; angular distance to first zero, deg; and gain factor (aperture efficiency). Table 1 gives the important characteristics of several classical distributions in comparison with those based on AFs. Here, \( l \) is the total length of an antenna aperture. The asymptotic decay of sidelobes for all atomic windows is equal to infinity. The family of atomic windows is very flexible and allows us to obey different requirements. Recently, a wide class of new windows based on combinations (products and convolutions) of AFs with classical functions was proposed by V.F. Kravchenko [2]. Some of them possess outstanding properties making them to be useful in different problems of digital signal processing, including those connected with SAR. For example, Fig. 1 illustrates real and imaginary parts of support functions (left) based on atomic windows, together with corresponding synthesized radiation patterns (right).

Fig. 2 shows radar image signal in two-stage scheme in SAR for \( N_1 = 4 \) samples, accumulating in the partial sum (a) and without partial summing (b).
Table 1. Classical, Kravchenko-Rvachev, Kravchenko weighting windows and their characteristics.

<table>
<thead>
<tr>
<th>N</th>
<th>Type of distribution</th>
<th>Parameter</th>
<th>Maximum sidelobe level, dB</th>
<th>Angular distance to first zero, deg</th>
<th>Gain factor</th>
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<tbody>
<tr>
<td>1</td>
<td>rectangular</td>
<td>–</td>
<td>-13</td>
<td>-34.5/l</td>
<td>1.000</td>
</tr>
<tr>
<td>2</td>
<td>$T_{2x/l}$</td>
<td>–</td>
<td>-32</td>
<td>-69.0/l</td>
<td>0.5055</td>
</tr>
<tr>
<td>3</td>
<td>$\cos(\pi x/l)$</td>
<td>$n=1$</td>
<td>-27</td>
<td>-52.5/l</td>
<td>0.6636</td>
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<tr>
<td></td>
<td></td>
<td>$n=2$</td>
<td>-37</td>
<td>-70.0/l</td>
<td>0.5001</td>
</tr>
<tr>
<td></td>
<td></td>
<td>$n=3$</td>
<td>-40</td>
<td>-87.5/l</td>
<td>0.4244</td>
</tr>
<tr>
<td></td>
<td></td>
<td>$n=4$</td>
<td>-55</td>
<td>-105.0/l</td>
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<tr>
<td>4</td>
<td>$\exp(-\alpha(2x/l)^2)$</td>
<td>$\alpha=2$</td>
<td>-38</td>
<td>-58.4/l</td>
<td>0.5982</td>
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<tr>
<td></td>
<td></td>
<td>$\alpha=2.5$</td>
<td>-40</td>
<td>-72.9/l</td>
<td>0.5463</td>
</tr>
<tr>
<td>5</td>
<td>$up(2x/l)$</td>
<td>–</td>
<td>-28</td>
<td>-69.0/l</td>
<td>0.5000</td>
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<td>6</td>
<td>$fup_1(3x/l)/fup_1(0)$</td>
<td>–</td>
<td>-47</td>
<td>-103.5/l</td>
<td>0.4454</td>
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<td>7</td>
<td>$h_0[2x/(a-1)]/a/2$</td>
<td>$\alpha=1.5$</td>
<td>-43</td>
<td>-105.0/l</td>
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<tr>
<td></td>
<td></td>
<td>$\alpha=3$</td>
<td>-20</td>
<td>-52.4/l</td>
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<tr>
<td>8</td>
<td>$\Xi_a(2x/l)/\Xi_a(0)$</td>
<td>–</td>
<td>-16</td>
<td>-43.6/l</td>
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<td>9</td>
<td>$up(2t/l)$</td>
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<td>10</td>
<td>$fup_1(3t/l)/fup_1(0)$</td>
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<tr>
<td>11</td>
<td>$\Xi_d(2t/l)/\Xi_d(0)$</td>
<td>–</td>
<td>-42</td>
<td>-101.3/l</td>
<td>0.3825</td>
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</table>

CONCLUSION

The properties of atomic functions allow us to synthesize weighting functions with good parameters for the use in SAR digital signal processing. Results of numerical experiments prove the efficiency and flexibility of the novel approach. As a perspective of investigations we can note applications of two-dimensional and multi-dimensional weighting windows based on AFs for solving more complicated problems of SAR signal processing.

REFERENCES