Analysis of Skin Effect High Speed Interconnects Response by Wavelet Convolution

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Abstract: This paper describes a method for the analysis of high speed interconnects with skin effect by the use of Wavelet Expansion in the time domain. The need of a time domain analysis comes from the fact that when nonlinearities or long simulation times are present, the more standard frequency domain approach is no longer possible or convenient. The main characteristic of the proposed method is a new representation of the convolution operator in the wavelet domain, which allows the exploitation of all the good numerical properties of wavelets for this kind of problems.

INTRODUCTION
The constant increase of the speed of data processing in information systems is creating the need for simulation techniques that take into account behavior different from the quasi TEM mode of transmission lines. One of these non-ideal effects is the frequency dependence of the line parameters. Such problems can be easily solved in the frequency domain [1], since the frequency behavior of the phenomena is straightforward to describe. On the other hand this approach has mainly two disadvantages: first, in case of nonlinear terminations or distributed nonlinearities the superposition effect (used in the frequency domain) is no longer valid; furthermore (either with linear or nonlinear terminations) when several line transient times have to be considered the use of a great amount of frequency data is mandatory in order avoid the Gibb’s phenomenon. Previously proposed time domain methods can be seen in [2], [3], where FDTD, FETD and asymptotic impulse response calculations are used.

The method presented here is based on the use of Wavelet Expansion (WE) in the time domain, applied to the skin effect equations of a multiconductor transmission line. The wavelet approach to time domain simulation of MTL has been already presented by the authors in [4], where frequency dependence of the parameters was not taken into account. In this paper the convolution integral is efficiently treated by the use of wavelet expansion and operators. The main advantage of the method presented in [4], namely the low CPU times (due to the low number of wavelet functions necessary to obtain good results and due to the sparsity of the matrices) is confirmed also in this case.

MATHEMATICAL MODEL

Transmission lines with skin effect. For the sake of conciseness we will focus our formulation only on single conductor line, since the extension to multiconductor is straightforward.

The equations of a TL respectively in the laplace domain and in the time domain are reported in (1)

\[
\begin{align*}
\frac{\partial}{\partial z} v(z,s) &= -z(s)i(z,s) - sl i(z,s) \\
\frac{\partial}{\partial z} i(z,s) &= -gv(z,s) - sc v(z,s)
\end{align*}
\]

where \(z(t) = L^{-1} \left[ \frac{z_z(s)}{s} \right] \) is the modified transient impedance matrix [5].

As is explained in [5] \(z(t)\) can be expressed as a series of time dependent terms, using the Heaviside generalized expression to meromorphic functions. By introducing this expression in (1) we obtain the following set of equations for a TL with skin effect

\[
\begin{align*}
\frac{\partial}{\partial z} v(z,t) &= -l \frac{\partial}{\partial t} i(z,t) - r_{dk} i(z,t) - \sum_{i=1}^{\infty} \left[ r_k e^{-\mu_i t} \frac{\partial}{\partial t} i(z,t) \right] \\
\frac{\partial}{\partial z} i(z,t) &= -gv(z,t) - c \frac{\partial}{\partial t} v(z,t)
\end{align*}
\]

Wavelet expansion in time domain of the convolution. The concept of Wavelet Expansion is considered
known, together with the representation of the integral and differential operator for wavelets on the interval
developed by the authors [4]. In this work we use Daubechies wavelets on the interval, and the following
notation is introduced: \( \psi(t) \), \( \phi(t) \) is the chosen wavelet basis; \( f(t) = \psi(t) \) is the WE of
function \( f \), and \( f = [f_1, ..., f_n] \) is the vector of coefficients of the WE. The punctual product
\( y = gh \) between two functions in the wavelet domain (when the WE of \( h \) is known and the WE of \( g \) is unknown)
is calculated as \( y = h G \), where \( G \) is a diagonal matrix whose entries are the samples of function \( g \); the
primitive \( F \) of function \( f \) is calculated by the use of the integral operator as \( F = f T \) where \( T \) is the
representation of the integral operator in the wavelet domain.

Let us focus our attention to the term
\[
\sum_{k=1}^{N} r_k e^{-\alpha_k t} \psi(t) = \sum_{k=1}^{N} \int_{0}^{t} r_k e^{-\alpha_k (t-\tau)} \frac{\partial}{\partial \tau} \psi(\tau) d\tau = \sum_{k=1}^{N} r_k e^{-\alpha_k t} \int_{0}^{t} \frac{\partial}{\partial \tau} \psi(\tau) d\tau
\]

where \( f(\tau) = \frac{\partial}{\partial \tau} \psi(\tau) \) and let us indicate
\[
h(t) = \int_{0}^{t} r_k e^{-\alpha_k t} \psi(\tau) d\tau,
\]

By performing the WE of (4) we obtain
\[
h(z) = f(z) \cdot e_{2z} \cdot T
\]

where \( e_{2z} \) is the diagonal matrix whose entries are the sample of the exponential function, and \( T \) is the
integral operator in the wavelet domain. In the same way including the exponential term \( e^{-\alpha t} \) we can define
\[
g(z,t) = e^{-\alpha t} \int_{0}^{t} r_k e^{-\alpha \tau} f(\tau) d\tau,
\]

Performing the WE of (6) we obtain
\[
h(z) = f(z) \cdot e_{2z} \cdot T \cdot e_{1z}
\]

where \( e_{1z} \) is the diagonal matrix related to the exponential function \( e^{-\alpha z} \). Equation (7) represents
the wavelet expansion of each term of the sum present in equation (2). Taking into account that
\( f(z,\tau) = \frac{\partial}{\partial \tau} \psi(\tau) \) can be written in the wavelet domain as \( f(z) = i(z) D \) (where \( D \) is the matrix
representing the differential operator in the wavelet domain), we can now write the complete WE of (3) as
\[
u(z) = \sum_{k=1}^{N} \int_{0}^{t} r_k e^{-\alpha_k t} f(z,\tau) d\tau
\]

Equation (8) shows that the sum of the convolutions can be easily calculated in the wavelet domain by a sum
in which each term is a product between known and constant matrices (\( D \) and \( T \)) and diagonal matrices
easily calculated (\( e_{2z} \) and \( e_{1z} \)). The only unknown is obviously the WE of the current, which is
represented by its vector of coefficients and is out of the sum operation.

**Transmission Lines equation in the wavelet domain.** Once the WE of the convolution has been obtained it
is possible to treat the time dependence of (1) as reported in [4]. The voltages and currents along the line are
expanded in the wavelet domain while the time derivative terms are represented in the wavelet domain by the
\( D \) operator. The inclusion of (8) in such equation leads to the following representation of the TL equations in
the wavelet domain:
\[
\begin{align*}
\frac{\partial}{\partial \tau} \psi(t) &= -i(z) D - r_d i(t,\tau) - i(z) K \\
\frac{\partial}{\partial \tau} i(z) &= -g(v(z) - c(v(z) D)
\end{align*}
\]

\( v \) and \( i \) are the values in the considered time interval \( \Delta \tau \). Eq. (9) rewritten in a matrix form gives:
\[
\begin{bmatrix}
\frac{\partial}{\partial \tau} \psi(t) \\
\frac{\partial}{\partial \tau} i(z)
\end{bmatrix} = \begin{bmatrix}
0 \\
-\mathbf{D} - r_d \mathbf{I}_d - K
\end{bmatrix} \begin{bmatrix}
v(z) \\
i(z)
\end{bmatrix} \mathbf{H}
\]

where \( \mathbf{I}_d \) is the identity matrix of proper dimension. The solution to (10) has an analytical form, but in
order to obtain a good accuracy in the numerical evaluation of the exponential matrix it is necessary to
calculate it for line segments that are electrically short, so we can write for each \( \Delta \), as shown in [4]:
\[
\begin{bmatrix}
v(z + \Delta \tau) \\
i(z + \Delta \tau)
\end{bmatrix} = \begin{bmatrix}
v(z) \\
i(z)
\end{bmatrix} e^{\mathbf{H} \Delta \tau}
\]

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Let us suppose for simplicity that the line (of length \( L \)) is terminated with linear resistors: the boundary conditions in the wavelet domain are

\[
\begin{align*}
\mathbf{v}(L) &= R_i \mathbf{i}(L) \\
\mathbf{v}(0) + R_L \mathbf{i}(0) &= \mathbf{E}
\end{align*}
\]  

(12)

where \( \mathbf{v} \) and \( \mathbf{i} \) are evaluated in \( z = 0 \) and \( \mathbf{E} \) is the WE of the source generator.

The line is discretized in segments of proper length (this also allows us to deal with nonuniform lines) and an algebraic system where the unknowns are \( \mathbf{v} \) and \( \mathbf{i} \) at the near and far end of each space segment is obtained. The matrix of the system is a banded matrix where the submatrices are the ones shown in (11) plus the two equations in (12). Inverse WE of \( \mathbf{v} \) and \( \mathbf{i} \) gives the time domain values in the interval \( \Delta t \). If a linear load is considered then the result is obtained by a simple solution of the algebraic system. In case a nonlinear load is considered, the solution can be obtained by solving the voltage current nonlinear constitutive equation of the nonlinear load by a Newton-Raphson scheme at a number of consecutive time steps \( \Delta t \) as reported in [4]. The extension of this formulation to the case of a multiconductor line is straightforward.

RESULTS AND CONCLUSION

The test reported here is relative to a single conductor line, where the line itself is the RG-21 cable of length 9680 cm with the skin effect model described in [6]. The parameters of the cable under analysis are: \( z = r + k \sqrt{i} \omega + i \omega l \), \( y = g + ioc \), with \( r = 3.5 \Omega /cm \), \( l = 2.65 \mu H/cm \), \( g = 0 \ S/cm \), \( c = 0.9434 \mu F/cm \), and the value of the input and output impedances is \( R_L = R_S = 53 \Omega \). The simulation has been performed with a number of 128 wavelet functions in time. Figure 1 reports the input and output voltages with \( (k = 2.5E-6) \) and without frequency dependence. The method here proposed is characterized by accuracy and low CPU time consumption, if compared to standard time domain method.

![Figure 1. Input and output voltage of the test case](image)

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