Implicit and Explicit schemes for Wire Frame
Time Domain Moment Method

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Abstract
A Wireframe implementation of the method of moments in time domain is presented, both
following an implicit and an explicit scheme. To this aim particular attention must be devoted
to wire junctions so to have implicit enforcement of current continuity in both schemes. Some
numerical tests shows the validity of the approach.

1 Introduction
Method of Moments field computation in Time Domain (MoM-TD) is not a new concept, for what
concerns theory [1], but only recently common access to powerful computers allowed for the method
exploitation. Some papers on surface patch MoM-TD exists as, for example, [2-5]. Fewer works are
available, on the other hand, for thin wire approximation (TWA) MoM-TD, as in [2] and [6].

In [2], for example, pulse functions are used as space-basis and weights and linear functions as time
basis. The N wires junction is treated by defining N − 1 bases and weights centred at the junction.
Current continuity at a node is implicitly guaranteed, a numerical explicit scheme is not possible
because bases are centred in the same point, hence the interactions between the current bases are not
delayed. In [6], on the other hand, linear bases and impulsive weights are used in a Predictor-Corrector
numerical scheme. The bases related with a wire junction in this approach does not enforce current
continuity automatically, so continuity must be enforced by an additional equation at any junction,
which makes implementation more cumbersome.

In this contribution a very simple set of modified basis functions that conforms to the current
continuity laws yet presents delays among each other are presented. This features allows for the
implementation of both implicit and explicit numerical schemes, the latter being more advantageous,
without additional complexity.

2 Formulation
Let be S a perfectly conducting wire structure. Each wire is divided into subsections S_m which are
straight wire segments of uniform radius [2]. The subsection geometry is depicted in Fig.1. If the time
coordinate is divided into discrete time instants t_n separated by a constant time step Δt the Electric
Field Integral Equation (EFIE) can then be recast in the form:

\[ \left[ \frac{\partial^2 A(r, t)}{\partial t^2} \right]_{t_n} + \nabla \psi(r, t_n) = \left[ \frac{\partial E_i(r, t)}{\partial t} \right]_{t_n} \]

being:

\[ A(r, t_n) = \frac{\mu}{4\pi} \int_S \frac{j(r', \tau^n)}{|r - r'|} \, dr'; \quad \psi(r, t_n) = \frac{-1}{4\pi\varepsilon} \int_S \frac{\nabla' \cdot j(r', \tau^n)}{|r - r'|} \, dr'; \quad \tau^n = t_n - \frac{|r - r'|}{c\Delta t} \]

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TWA imply considering the currents $\mathbf{j}(\mathbf{r'}, \tau^n)$ concentrated on the wire axis and directed only along it, while potentials and fields are computed on the wire lateral surface and tangential to it. $\mathbf{r}$ will indicate a point on such lateral surface, and $\mathbf{r'}$ a point on the wire axis.

For MoM application, currents need then to be expanded both on time bases functions $T_j(\tau^n)$ and on space bases $\mathbf{f}_k(\mathbf{r})$, and an internal product needs to be defined:

$$\mathbf{j}(\mathbf{r'}, \tau^n) = \sum_{k=1}^{N_r} \sum_{j=-\infty}^{n} I_{kj} T_j(\tau^n) \mathbf{f}_k(\mathbf{r}); \quad < \mathbf{v}(\mathbf{r}), \mathbf{w}(\mathbf{r}) > = \int_S \mathbf{v}(\mathbf{r}) \cdot \mathbf{w}(\mathbf{r}) d\mathbf{r} \quad (3)$$

being $N_s$ the number of subsections in which the whole structure is divided, and $\mathbf{w}_m(\mathbf{r})$ an appropriate set of weight functions. Testing the EFIE against each of the weight functions leads to a first discretization in space. At this point an implicit or explicit numerical scheme must be chosen for time discretization. In the explicit case the second order time derivative in $I$ is approximated as central differences, hence a forward time step is introduced:

$$< \mathbf{A}^{n+1}(\mathbf{r}), \mathbf{w}_m > + \Delta t^2 < \nabla \psi^n(\mathbf{r}), \mathbf{w}_m > = \Delta t^2 < \frac{\partial \mathbf{E}^i}{\partial t}|_{\mathbf{r}_n}, \mathbf{w}_m > + < 2\mathbf{A}^n(\mathbf{r}) - \mathbf{A}^{n-1}(\mathbf{r}), \mathbf{w}_m > \quad (4)$$

For the implicit case a backward finite difference approximation is used:

$$< \mathbf{A}^n(\mathbf{r}), \mathbf{w}_m > + \Delta t^2 < \nabla \psi^n(\mathbf{r}), \mathbf{w}_m > = \Delta t^2 < \frac{\partial \mathbf{E}^i}{\partial t}|_{\mathbf{r}_n}, \mathbf{w}_m > + < 2\mathbf{A}^{n-1}(\mathbf{r}) - \mathbf{A}^{n-2}(\mathbf{r}), \mathbf{w}_m > \quad (5)$$

But it must be noted that the implicit approach easily leads to instabilities, hence the explicit one is preferred. Substituting (3) in (2) the following relations are obtained:

$$\mathbf{A}^n(\mathbf{r}) = \mu \sum_{k=1}^{N_r} \sum_{j=-\infty}^{+\infty} I_{kj} \int_{S_k} \frac{T_j(\tau^n) \mathbf{f}_k(\mathbf{r'})}{4\pi |\mathbf{r} - \mathbf{r}'|} d\mathbf{r}' \psi^n(\mathbf{r}) = -\frac{1}{\varepsilon} \sum_{k=1}^{N_r} \sum_{j=-\infty}^{+\infty} I_{kj} \int_{S_k} \frac{T_j(\tau^n) \nabla' \cdot \mathbf{f}_k(\mathbf{r'})}{4\pi |\mathbf{r} - \mathbf{r}'|} d\mathbf{r}' \quad (6)$$

Hence, by computing the integrals in (6) and casting them into (4) or (5) a linear iterative operator yielding the unknown coefficients $I_{kj}$ of the problem can be obtained.

Piecewise quadratic time bases are used for $T_j(\tau^n)$ [7]. These they yield good accuracy and low computational burden in evaluating delayed currents; stability being independent from the time bases in the present implementation.

It is then necessary to distinguish between wire segments and wire junctions for space bases:

$$\mathbf{f}_k(\mathbf{r}) = \begin{cases} \frac{1}{\Delta t_k} \rho^+ \hat{\rho}^- & \mathbf{r} \in S_k^- \\ \frac{1}{\Delta t_k} \rho^- \hat{\rho}^+ & \mathbf{r} \in S_k^+ \end{cases} \quad \mathbf{f}_k(\mathbf{r}) = \begin{cases} \frac{1}{2N} \Delta t_k \rho^+ \hat{\rho}^- & \mathbf{r} \in S_k^- \\ \frac{1}{2N} \Delta t_k \rho^- \hat{\rho}^+ & \mathbf{r} \in S_k^+ \end{cases} \quad (7)$$

Figure 1: Wire subsections and vectors used in its definition (left); regular basis (centre) and basis over a wire junction (right).
being, for the left (normal segment) equations: $\rho^\mp = |r - r_k^\pm|$ and $\hat{\rho}^\mp = \mp \frac{r - r_k^\pm}{\rho^\mp}$. And for the right (junction segment) equations: $\rho = |r - r_k|$, $\hat{\rho} = \frac{r - r_k}{\rho}$; and $\rho^{+t} = |r - r_k^{+t}|$, $\hat{\rho} = \frac{r - r_k^{+t}}{\rho^{+t}}$, $i = 1, \ldots, N_k$, where $N_k$ is the number of intersecting segments in the junction point minus one; (Fig. 1)

For what concern the weight functions, they are chosen equal to the base functions for regular segments, but a slightly modified version of (7) for junctions which differs from the corresponding basis for the amplitude on the secondary segments, so to have a continuous function. This choice has proven to be numerically more accurate than a plain Galerkin method, since it guarantees a better evaluation of testing.

3 Numerical Results and Conclusions

As a preliminary test a symmetrical three-pole whose arms are 1m long and subdivided into 20 subsections each is considered. Results of both implicit and explicit schemes are compared with the solution obtained, in frequency domain, by a commercial package.

The incident plane wave is a modulated Gaussian pulse with amplitude $E_0 = 377 \hat{x}$, lasting for $T = 2$ light meters (LM) normally impinging on the plane containing the three-pole (Fig. 2).

An excellent agreement between the present solutions and the time domain behaviour obtained by the frequency solution with a 2 MHz frequency step from 0 to 500 MHz is shown in Fig. 2, where the currents through a wire section near the junction point is represented. This shows the accuracy obtained with the present approach, both in an implicit and explicit scheme, for wire frame MoM-TD with a general number of junctions and subsections of variable length and thickness.

References