ASYMPTOTICALLY DRIVEN LOCAL BASIS FUNCTIONS WITH APPLICATION TO THE FAST MULTIPOLe METHOD

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Abstract: Asymptotic techniques are used to design basis functions for electrically large scatterers. These asymptotic basis functions are utilized on localized regions of a target, with the remainder of the target characterized by traditional piecewise basis functions. The scattering problem is analyzed via the fast multipole method, considering these hybrid basis functions and three-dimensional targets.

ASYMPTOTIC TECHNIQUES AND BASIS FUNCTIONS
Along the smooth and electrically large portions of a target the induced current has a relatively simple form, characterized by fast (traveling-wave) phase variation and an associated frequency- and geometry-dependent diffraction coefficient. There has been previous research in the context of the method of moments (MoM) in which such high-frequency basis functions are utilized on appropriate localized regions of a given target, with the remainder of the target characterized by traditional piecewise bases [1]. While such approaches have been demonstrated to work, they typically yield poorly conditioned matrices. When performing a direct (LU-decomposition) inversion of the associated matrix, the poor conditioning is not a significant problem. However, recently there has been much interest in iterative (conjugate-gradient) solvers, for which poor matrix conditioning may be a serious problem. The fast multipole method (FMM) [2], for example, is a fast numerical solver that employs an iterative matrix solver.

Rather than utilizing a traveling-wave asymptotic basis function directly, we use asymptotic techniques to design basis functions that yield well-conditioned matrices. Assume that the asymptotic current over a given region may be expressed as

$$J(r, \omega) = \sum_{m=1}^{M} j_m(r, \omega) \exp[-j \beta_m \cdot r]$$

where $j_m$ is the $m$th diffraction coefficient and $\beta_m$ represents the associated vector wavenumber. Note that the current amplitude $j_m$ is in general a function of position $r$, allowing consideration of curved surfaces. Rather than utilizing the currents in (1) directly as basis functions, we expand the slowly-varying (with $r$) amplitude $j_m$ in a traditional piecewise basis set, for example the RWG basis functions. Since the fast-varying phase $\exp[-j \beta_m \cdot r]$ has been explicitly extracted, the relatively slowly varying $j_m(r, \omega)$ may be expressed using a coarse basis representation (much coarser than the traditional ten-points per wavelength).

The overall procedure is as follows. Scattering from a given electrically-large target is first analyzed efficiently via an asymptotic ray-based technique. The main objective is to acquire the fast-varying current terms $\exp[-j \beta_m \cdot r]$ on smooth portions of the target. The asymptotic representation for $j_m(r, \omega)$ need not be accurate, since it will be characterized by a piecewise basis with coarse sampling (i.e. the numerical algorithm will accurately solve for $j_m(r, \omega)$). Over the complicated regions of a target, for which the asymptotic solution is less accurate, traditional fine-scale piecewise basis functions are employed. Over the smooth electrically large regions coarse-scale piecewise basis functions are used for representation of $j_m(r, \omega)$, and the fast phase variation is explicitly extracted (as in (1)).

It is important to note that throughout piecewise basis functions (e.g., RWG [1]) are being employed, and therefore the resulting matrices are relatively well conditioned. The computational gain is manifested in the fact that over the smooth electrically large regions relatively coarse spatial sampling is used, since the fast phase variation is extracted, and therefore the total number of unknowns is reduced significantly. The resulting matrix equation is solved using a multi-level fast multipole algorithm (MLFMA) [3,4] analysis.

EXAMPLE RESULTS
As an example we consider scattering from a perfectly conducting sphere, to address a curve structure. The
sphere in this example has a $5\lambda$ radius, and plane-wave excitation is considered, in free space. In this relatively small example a direct discretization of the spherical surface, at ten basis functions per wavelength, requires $N=19,101$ unknowns. Using the technique outlined above a total of $N=4,488$ unknowns are required.

In Figs. 1 and 2 we present the computed bistatic radar cross section (RCS) for VV and HH polarization, respectively, with a comparison between the direct and asymptotic-based analysis. Excellent agreement is observed between the two solutions.

![Figure 1](image1.png) **Figure 1.** Bistatic RCS for scattering from a perfectly conducting sphere, VV polarization.

![Figure 2](image2.png) **Figure 2.** Bistatic RCS for scattering from a perfectly conducting sphere, HH polarization.

**CONCLUDING REMARKS**

A technique has been presented wherein asymptotic techniques are integrated with numerical procedures, yielding a hybrid formulation that is solved via the MLFMA. The main contribution is development of local basis functions that explicitly accounted for the asymptotic fast phase variation, allowing a coarse piecewise representation for the associated spatially and frequency dependent diffraction coefficient. The procedure has been demonstrated here by considering scattering from a perfectly conducting sphere, and additional results on complex targets will be presented at the meeting.

**REFERENCES**


