NEW TOOLS AND SERIES FOR SCATTERING PROBLEMS IN LOSSY MEDIA

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Abstract. A convenient rewriting of the Electric Field Integral Equation in lossy media is introduced and discussed. In particular, a new series, similar in spirit to the Born series, is introduced. Such a series allows to solve in a simple and accurate fashion forward scattering problems in lossy media and its first term coincides, in particular situations, with the well known Extended Born Approximation. Theoretical tools and results are given on the range of applicability and rate of convergence of the new series. Moreover, a suitable ‘over-relaxed’ version of the new series is also proposed. Finally, an example showing the effectiveness of proposed tools and series is presented.

INTRODUCTION

Fast solutions to electromagnetic scattering problems by dielectric objects in lossy environments are of great interest in near surface geophysical explorations, environmental science, as well as in detection of buried objects and demining applications.

In order to reduce the computational burden and the difficulty of this class of problems, the recently introduced Extended Born (EB) approximation [1] is widely used in both forward and inverse scattering problems. On the other side, there seems to be a lack of theoretical tools able to provide for an a priori understanding of the range of applicability of this approximation. In the following, with reference to 2D scattering problems in lossy media and in the framework of the Contrast Source (CS) model [2], new theoretical tools and results are introduced and discussed. In particular, a new approximation, similar in spirit to the EB approximation, is introduced, and bounds are given on the applicability of this new approximation, denoted as Contrast Source-Extended Born (CS-EB) approximation. Moreover, a convenient extension into a series of the CS-EB Approximation is given, and its usefulness, applicability and rate of convergence are also discussed. Finally, a suitable ‘over-relaxed’ version of such a series is also introduced and discussed.

MATHEMATICAL FORMULATION

Let us consider the Electrical Field Integral Equation in the 2D scalar case. If \( \Omega \) denotes the scattering region under test, we have

\[
E(r) = E_i(r) + \oint_{\Omega} g(r-r') \chi(r') E(r') \, dr' \quad r \in \Omega. \tag{1}
\]

where \( E \) and \( E_i \) are the total and incident fields inside the scattering region, respectively. Moreover \( \chi(r) = \varepsilon_{eq}(r) / \varepsilon_{eq} - 1 \) is the usual “contrast function” relating the equivalent permittivities of the background and the scatterer. The integral operator in (1) relates the “contrast source”, defined as the product between the contrast function and the total internal field, to the scattered field inside the scattering region. \( g(r-r') = -j k_i^2 / 4 \pi H^{(2)}_0(k_i |r-r'|) \) is the Green’s function for homogeneous space where \( k_i \) is the complex wavenumber in the background while \( H^{(2)}_0 \) is the Hankel function of zero order and second kind.

If we multiply (1) by \( \chi \) and take into account the “contrast source” definition, we achieve

\[
J(r) = \chi(r) E_i(r) + \oint_{\Omega} g(r-r') J(r') \, dr'. \tag{2}
\]

Then, by adding and subtracting \( J(r) \) in the integral operator in (2), this latter can be written as:

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\[ \chi(r) \int_{\Omega} g(r-r') J(r') dr' = \chi(r) J(r) \int_{\Omega} g(r-r') dr' + \chi(r) \int_{\Omega} g(r-r') [J(r') - J(r)] dr'. \]  

(3)

In the presence of losses, because of the singularity of the Green’s function in the origin and its exponential decay far from the origin, the first term becomes the dominant one, while the second term is more and more negligible for increasing losses. If

\[ A_{\text{MOD}}(r) = \int_{\Omega} g(r-r') \left[J(r') - J(r)\right] dr'; \quad p(r) = \frac{\chi(r)}{1 - \chi(r)f_\Omega(r)}; \quad f_\Omega(r) = \int_{\Omega} g(r-r') dr'; \]  

(4)

a new integral equation in terms of the “contrast source” can be achieved which is given by:

\[ J(r) = p(r) E_i(r) + p(r) A_{\text{MOD}}(J) \quad r \in \Omega. \]  

(5)

Equation (5) has the same structure as the original equations (1) or (2), wherein (see eq. 2) the integral radiation operator is replaced from \( A_{\text{MOD}} \) and \( \chi(r) \) is replaced by \( p(r) \). Whenever the second term at the right hand side of (3) is negligible, only the first term at the right hand side of (5) can be considered. It is interesting to note that such an approximation exactly coincides with the result one would achieve within the Extended Born Approximation [1] in case of homogeneous scatterers.

**A NEW SERIES AND ITS APPLICABILITY AND RATE OF CONVERGENCE**

Then, the problem arises of the applicability and accuracy of this CS-EB approximation, as well as of how to solve (5) in more general cases. To this end, let us note (5) can be formally inverted to give:

\[ J(r) = (I - p A_{\text{MOD}})^{-1} p(r) E_i(r) \quad r \in \Omega. \]  

(6)

Then, if \( \|p A_{\text{MOD}}\| < 1 \), the inverse operator in (6) can be expanded into a series to give:

\[ J(r) = \left(I + p A_{\text{MOD}} + (p A_{\text{MOD}})^2 + \cdots + (p A_{\text{MOD}})^n + \cdots\right)p(r) E_i(r) \quad r \in \Omega. \]  

(7)

When \( \|p A_{\text{MOD}}\| \ll 1 \), only the first term can be considered, giving back the CS-EB approximation. Therefore, a key role is played from the norm of the operator \( p A_{\text{MOD}} \). As

\[ \|p A_{\text{MOD}}\| < \|p\| \|A_{\text{MOD}}\| \]  

(8)

a sufficient condition for the applicability of the new series, as well as information about its rate of convergence, can be gained by separately looking to \( \|p\| \) and \( \|A_{\text{MOD}}\| \). As far as the former is concerned, by taking advantage of its diagonal nature, one has

\[ \|p\| = \max[\text{abs}(\text{diag}(p))] \]  

(9)

Moreover, \( p(r) \) can be calculated in a closed form for circular cylindrical domains. As far as \( \|A_{\text{MOD}}\| \) is concerned, its behaviour with dimensions of the scattering region (which is supposed to be circular) and loss tangent of the background is given in figure 1. As expected, for a fixed dimension of the scattering region, this norm decreases for increasing losses in the background (see Fig.1.a) while, for fixed losses, the norm grows with the dimension of the region (see Fig.1.b). Also note the bound (8) is quite accurate, particularly in case of highly lossy environments and homogeneous scatterers.

The introduced tools (Fig. 1 and eq. 9) allow to obtain \emph{a priori} information on the series convergence. It is worth to note that for a wide range of scatterer sizes and contrast values the CS-EB series converges when traditional Born series diverges, and it generally converges more rapidly in those cases where both converge. Also note that (7) can be efficiently computed by iterated applications of the operator \( p A_{\text{MOD}} \), and exploiting FFT codes in the evaluation of this latter.

As a further contribution to the work, by following the same guidelines as in [3], we also have introduced and studied a further series solution which is based on a generalized over-relaxation method. In order to keep the communication contained in its length, no more details about this ‘modified CS-EB’ series are reported.

On the other side, it is worth to note that the modified series converges in a number of situations where CS-EB diverges. Moreover, it converges more rapidly than the CS-EB series in those cases where both converge.
A NUMERICAL EXAMPLE AND CONCLUSIONS
In order to show usefulness and effectiveness of the proposed tools and series, let us consider a cylindrical object \((\varepsilon = 40, \sigma = 0.08 \text{ S/m})\) of radius 12.5 m enclosed in a rectangular domain of size 25 m embedded in a lossy background medium \((\varepsilon = 10, \sigma = 0.02 \text{ S/m})\). 5 MHz is the adopted frequency. Now, by evaluating \(|p|\) through eq.9 and using Fig.1 to estimate \(|A_{\text{MOD}}|\), one can check that condition for the convergence is satisfied. In fact, by observing the relative error with respect to actual solution, one can see that while the traditional Born series does not converge at all, the CS-EB series does indeed converge. Moreover, it is interesting to note that the Modified CS-EB series converges faster than CS-EB.

Similar results have been found in a large number of cases, thus suggesting that extension of these tools and series to the 3-D vectorial case is worth to be pursued. The fast convergence of these series also suggest their possible exploitation in the solution of inverse scattering problems [4].

![Fig.1 Behaviour of the norm](image)

![Fig.2 Convergence rate](image)

REFERENCES