FAST AND ACCURATE DETECTION OF HOMOGENEOUS BURIED TARGETS

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Abstract: In this paper we describe an approach to localize and (possibly) identify the shape of multiple homogeneous buried targets from measures of the scattered field at the air-earth interface, without explicitly solving the corresponding inverse scattering problem. The initial idea of Colton and Kirsch [1] is exploited to develop a new method which takes advantage of a suitable regularization scheme and of a properly defined compensation map. The proposed method remarkably improves performances with respect to previous approaches. Finally, a simple and effective extension to the case of multi-frequency data is also proposed.

INTRODUCTION
Inverse scattering techniques are particularly attractive in many non invasive diagnostics applications such as for instance subsurface sensing. In the most general sense, the aim of these techniques would be that of determining both electrical and morphological features of the buried targets. However, this is a very difficult task, since it implies the solution of a non-linear inverse problem.

On the other hand, only a partial knowledge of the properties of the investigated area is sufficient in many cases to the survey’s aim. In this respect, it is therefore of outstanding relevance the possibility of achieving partial information without solving the inverse scattering problem, but solving simpler auxiliary ones. Amongst the several approaches exploiting this circumstance, the methods based on the initial idea by Colton and Kirsch [1] appear to be very attractive, since in principle they allow to determine the shape of homogeneous targets by solving an auxiliary linear problem. On the other side, this latter is ill-posed so that regularization techniques have to be adopted. According to this, in this paper we propose a revised version of [1] in which the auxiliary ill-posed problem is regularized exploiting the scattering matrix introduced in [2]. In addition to this, some simple modifications are proposed, which allow to improve the quality of the reconstruction when increasing depth.

Finally, a simple and effective extension to the case of multi-frequency data is also proposed.

A SUPPORT ESTIMATION PROCEDURE, AND ITS PHYSICAL INTERPRETATION
The reference situation is shown in Fig.1. Assuming a 2D geometry, each homogeneous layer is described by a relative dielectric permittivity $\varepsilon_Ri$ and a conductivity $\sigma_i$, with a dependency on frequency given by $\varepsilon_i(\omega)=\varepsilon_Ri-\omega \sigma_i/\omega \varepsilon_o$. The magnetic permeability is everywhere equal to that of the free-space, $\mu_o$.

Let us suppose primary source and measurement probes are located on the same line $\Gamma$ located at the first interface. Then, the scattering equation can be written as:

\[ E_{\text{inc}}(r) = \int_{\Gamma} J_p(r',g_{12}(r,r')d' r' = A_p[J_p] \quad r \in D \]  
\[ E^v(r) = E_{\text{inc}}^v(r) + k_0^2 \int_{D} E^v(r',g_{22}(r,r')d' r' = E_{\text{inc}}^v(r) + A_i[\chi E^v] \quad r \in D \]  
\[ E^v(r) = k_0^2 \int_{D} E^v(r',g_{21}(r,r')d' r' = A_e[\chi E^v] \quad r \in \Gamma \]

wherein $J_p^v$ is the $v$-th primary source ($v=1,\ldots,V$), $E^v$, $E^v$ and $E^v$ are the incident, total and scattered field, respectively; $k_0^2=\omega \varepsilon_0 \mu_0$ is the background (complex) wave-number, $\chi(r)=\varepsilon_0(r')/\varepsilon_0-1$ is the contrast between the (complex) permittivity of the second homogeneous layer and that of the object; the functions $g_{12}(r,r')$, $g_{22}(r,r')$ and $g_{21}(r,r')$ are the proper Sommerfeld-Green’s functions [2], while $A_p$, $A_i$ and $A_e$ synthetically denote the radiation operators in (1), (2) and (3), respectively.

Note $A_p$ and $A_e$ are compact operators, and let $(u_{in}, \sigma_n, v_n)$ be the Singular Value Decomposition (SVD) of $A_p$. As
$A_p$ and $A_t$ have the same kernel (with reversed roles for integration and observation variables) the SVD of $A_e$ is given by $(u_n^*, \sigma_n, v_n)$. As a consequence of the above, $\{u_n\}$ ($\{v_n\}^*$) functions are a basis for the primary sources (the scattered fields), while $\{v_n\}$ ($\{u_n\}^*$) functions are a basis for incident fields (the induced currents).

By exploiting this circumstance, let us suppose from now on that our primary sources are the $\{u_n\}$ functions and denote by $E_{in}$ the corresponding scattered fields. Note that is no need of physically synthesizing these sources, as both $\{u_n\}$ and $E_{in}$ can be achieved by synthetically combining elementary sources and their corresponding scattered fields. Then, for each $r_p \in D$, consider the equation in the unknowns $\{a_n\}$:

$$\sum_{n=1}^P a_n E_{in}(r) = g_{21}(r, r_p)$$

As the term at the right hand side is the field of an elementary source located in $E_{in}$, enforcing Eq.(4) is an attempt to combine the different scattered fields in such a way that they appear as to emerge from $E_{in}$.

Due to the linear relationship between scattered and incident fields, the above is equivalent to focus the radiating component of the induced current in $E_{in}$. In fact, while it is possible to focus the induced current in $E_{in}$ as long as it belongs to the support of the scatterers, say $\Sigma$, this is not any more possible elsewhere. In such a case, even assuming of being able to focus the total field in $E_{in}$, the induced current would be zero and the norm of the solution becomes unbounded in these points to compensate for such a null.

This latter circumstance suggests a method to determine the actual support of the scatterer by observing the norm $||A||$ of the $\{a_n\}$ vector in each trial point $r_p$. In particular, the points with a ‘low’ value of $||A||$ will belong to $\Sigma$, while the points with a ‘large’ value of $||A||$ will be outside $\Sigma$. Note the method exactly coincides with the one proposed in [1] when $P$ tends to infinity. Moreover, the above reasonings also furnishes a physical interpretation of [1] and related methods.

Now, the problem arises on how to choose $P$ and how to solve (4) in a way as robust as possible with respect to the unavoidable errors which are present in $E_{in}(r)$ (and in $u_n$, as well). To this end, let us project (4) on the $u_m$ functions $m=1,...,P$. If $S_{n,m}=<E_{in}(r), u_m>$, then the problem to be solved for each $r_p \in D$ is:

$$\sum_{n=1}^P S_{n,m} a_n(r_p) = f_m(r_p) \equiv (g(r_p, r) u_m(r))$$

(5)

The key point in achieving a regularized (or, equivalently, well conditioned) problem is a suitable choice of $P$. As the generic term $S_{n,m}$ of the scattering matrix $\Sigma$ (see [2]) is proportional to the product $\sigma_n \sigma_m$, it appears that a convenient choice can be accomplished by simply observing the behavior of the singular values $\sigma_n$ of the integral radiation operator [2] so to establish which is the index $N$ after which the singular values are lower than a given threshold. A possible choice for such a threshold is given by the estimated SNR. Accordingly, we choose $P=N$.

As the original method [1], our finite dimensional formulation is also affected by noise. On the other side, as we use the most significant part of the scattering matrix, which is by definition the one less sensitive to noise on data, our method allows to achieve an increased robustness against ill-conditioning problems.

EXTENSIONS AND PERFORMANCES OF THE PROPOSED PROCEDURE

When applying the above procedure to subsurface sensing, accuracy of the support estimation decreases with the distance from sources and probes [2]. This effect is related to the incident fields, which are “less intense” and “less variable” [2] when increasing depth. While the latter circumstance is somehow ineluctable, it is possible to modify the proposed method in order to compensate the former to some extent.

The basic idea is quite simple, since it amounts to perform an off-line compensation aimed to enhance the contributions related to the deeper parts of the region under test.

This is formalized by solving, for each point in $D$, the constrained optimization problem:

$$\text{MAX } |E_{in}|^2 \text{ s.a } \sum_{j=1}^N |c_j|^2 \leq 1$$

(6)

wherein $E_{inc}$ is the superposition of all the incident fields in the point under test and $c_j \in C$ is an excitation coefficient which is associated to the $j$-th primary source. By solving the problem (6), we determine which is the maximum total energy which can be concentrated in the point under test, say $E(r_p)$. Then, a map which
describes \( \Sigma \) by taking into account the different characteristics of \( E_{\text{inc}} \) when varying \( r_p \) can be obtained by plotting \( \| A(r_p) \| / E(r_p) \).

A further way to improve the reconstruction capabilities of our approach and to obtain stable solutions amounts to exploit multi-frequency data. Generally speaking, use of frequency diversity allows to obtain a larger quantity of independent information, since one usually assumes that the unknown quantity is independent of frequency or has a known dependence from it. In the present case, the actual unknown of the linear system (5) changes with frequency, without a definite law. However, since the support is actually the same, some advantage is expected. A very simple, yet effective processing (see below) amounts to adding all the maps obtained at the different frequencies, each one normalized to its maximum.

As an example, let us consider the profile depicted in Fig.2. The investigated domain is 1x1m\(^2\) wide, starting from the air-ground interface. Both the 25 transmitters and receivers are positioned on a line 2m long placed very close to the air-ground interface; the maximum working frequency is 500 MHz. The two homogeneous inclusions are lossless with relative permittivity \( \varepsilon_{R1} = 2 \) and \( \varepsilon_{R2} = 3 \) respectively, and are embedded in a layer 3m thick, whose average parameters are \( \varepsilon_{Rb} = 4 \) and \( \sigma_b = 5 \times 10^{-3} \text{[S/m]} \). Synthetic data have been generated via an exact forward solver and have been corrupted with a 10% additive noise; moreover, the background within the investigated region is assumed to be oscillating around the average value.

In Fig.3 it is shown the plot of \( \| A \| \) achieved when considering the regularized method without compensation of depth and with single frequency data (350 MHz), while the multi-frequency result (350, 400, 450, 500 MHz) with compensation is shown in Fig.4. Because of the adopted regularization scheme, the reconstruction in Fig. 3 already allows to get information about the location of the scatterers, which is not trivial in such a cluttered environment. As expected (see fig. 4), the quality of the reconstruction is remarkably improved when exploiting multi-frequency data as well as the compensation scheme.

REFERENCES