NONLINEAR LEFT-HANDED METAMATERIALS

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Abstract: We analyze nonlinear properties of microstructured materials with negative refraction, the so-called left-handed metamaterials. We consider a two-dimensional periodic structure created by arrays of wires and split-ring resonators embedded into a nonlinear dielectric, and calculate the effective nonlinear dielectric permittivity and magnetic permeability. We demonstrate that the hysteresis-type dependence of the magnetic permeability on the field intensity allows changing the material properties from left- to right-handed and back. These effects can be treated as the second-order phase transitions in the transmission properties induced by the variation of an external field.

Recent theoretical [1-3] and experimental [4-6] results demonstrated a possibility of creating novel types of microstructured materials that demonstrate the property of negative refraction. In particular, the composite materials created by arrays of wires and split-ring resonators were shown to possess a negative real part of the magnetic permeability and dielectric permittivity in the microwave regime. These materials are often referred to as left-handed materials or materials with negative refraction. Such materials were introduced and analyzed theoretically by Veselago a long time ago [7], but they have been demonstrated experimentally only recently. As was shown by Veselago [7], left-handed materials possess a number of peculiar properties, including negative refraction, inverse light pressure, reverse Doppler and Vavilov-Cherenkov effects, etc.

Thus far, all properties of left-handed materials were studied in the linear regime of wave propagation when both the magnetic permeability and the dielectric permittivity of the material are assumed to be independent on the intensity of the electromagnetic field. However, the future efforts in creating tunable structures where the field intensity changes the transmission properties of the composite structure would require the study of nonlinear properties of such metamaterials, which may be quite unusual. We analyze, for the first time to our knowledge, nonlinear properties of left-handed metamaterials for the example of a lattice of the split-ring resonators and wires with a nonlinear dielectric. We show that the effective magnetic permeability of such a structure depends on the intensity of the macroscopic magnetic field in a nontrivial way, allowing switching between the left- and right-handed material properties by varying the field intensity. We believe that our findings may stimulate the future experiments in this field, as well as the studies of nonlinear effects in photonic crystals, where the phenomenon of negative refraction is analyzed now very extensively [8, 9].

We consider a two-dimensional composite structure consisting of a square lattice of the periodic arrays of conducting wires and split-ring resonators (SRR) shown schematically in Fig. 1. We assume that the unit-cell size $d$ of the structure is much smaller then the wavelength of the propagating electromagnetic waves and, for simplicity, we consider single-ring geometry of a lattice of cylindrical SRRs. The results obtained for this case are qualitatively similar to those obtained in the more involved cases of double SRRs or omega-type structures. This type of microstructured materials has recently been suggested and built in order to create left-handed metamaterials with negative refraction in the microwave region [4].

The negative real part of the effective dielectric permittivity of such a composite structure appears due to the metallic wires whereas a negative sign of the magnetic permeability becomes possible due to the SRR lattice. As a result, these materials demonstrate the properties of negative refraction in the finite frequency band, $\omega_0 < \omega < \min(\omega_p, \omega_{||})$, where $\omega_0$ is the eigenfrequency of the SRRs, $\omega_{||}$ is the frequency of the longitudinal magnetic plasmon, $\omega_p$ is an effective plasma frequency, and $\omega$ is the angular frequency of the propagating electromagnetic wave, $(E; H) \sim (E; H) \exp(i\omega t)$. The split-ring resonator can be described as an effective LC oscillator (see Ref. [10]) with the capacitance of the SRR gap, as well as an effective inductance and resistance (see the upper inset in Fig. 1). The nonlinear response of such a composite structure can be characterized by two different contributions. The first one is an intensity-dependent part of the effective dielectric permittivity of the infilling dielectric. For simplicity, we assume that the metallic structure is
embedded into a nonlinear dielectric with a permittivity that depends on the intensity of the electric field in a general form, \( \varepsilon_D = \varepsilon_D(|E|) \). We assume the linear dependence that corresponds to the Kerr nonlinearity. The second contribution into the nonlinear properties of the composite material comes from the lattice of resonators, since the SRR capacitance (and, therefore, the SRR eigenfrequency) depends on the strength of the local electric field in a narrow slot. Additionally, we can expect a nonlinear eigenfrequency detuning due to a resonant growth of the charge density at the edges of the SRR gap. The intensity of the local electric field in the SRR gap depends on the electromotive force in the resonator loop, which is induced by the magnetic field. Therefore, the effective magnetic permeability \( \mu \) should depend on the macroscopic (average) magnetic field \( H \).

![Fig. 1. Schematic of the composite metamaterials structure. The lower inset shows a unit cell of the periodic structure while the upper inset shows the SRR equivalent oscillator.](image)

One can obtain the expression for the effective nonlinear dielectric permittivity

\[
\varepsilon_{\text{eff}}(|E|^2) = \varepsilon_D(|E|^2) - \frac{\omega_p^2}{\omega (\omega - i \gamma_\varepsilon)},
\]

where \( \omega_p = (c/d)[2\pi/\ln(d/r)]^{1/2} \) is the effective plasma frequency, and \( \gamma_\varepsilon = c^2/2\pi\ln(d/r) \). The second term on the right-hand side of Eq. (4) is in complete agreement with the earlier result obtained by Pendry and co-authors [1]. Dependence of the real part of magnetic permeability on the intensity of the magnetic field in four possible parameter regions [11] is shown in Fig. 2.

![Fig. 2. The real part of the magnetic permeability versus intensity of the magnetic field: (a, b) self-focusing nonlinearity; (c, d) self-defocusing nonlinearity; (a, c) electromagnetic field frequency is greater than the eigenfrequency of SRRs in linear limit; (b, d) electromagnetic field frequency is smaller than the eigenfrequency of SRRs.](image)

A complicated behavior of the magnetic permeability is observed in the cases shown in Fig. 2(b, c), where the jumps of the magnetic permeability with the growth of the magnetic field are predicted.

Due to the high values of the electric field in the slot of SRR as well as resonant interaction of the electromagnetic field with the SRR lattice, the characteristic magnetic nonlinearity in such structures is much stronger then the corresponding electric nonlinearity. Therefore, magnetic nonlinearity should dominate in
the composite materials that display the phenomenon of negative refraction. More importantly, the nonlinear medium can be created by inserting nonlinear elements into the slots of SRRs, allowing an easy tuning by an external field.

The possibility of strongly enhanced nonlinearities in left-handed metamaterials revealed here may lead to an essential revision of the concepts based on the linear theory, since the electromagnetic waves propagating in such materials always have finite amplitude. At the same time, the engineering of nonlinear composite materials will open a number of their novel applications such as frequency multipliers, beam spatial spectrum transformers, switchers, limiters, etc. We notice that the hysteresis behavior with jumps in the dependencies of the effective material parameters has been described above for stationary processes only. Such transitions will display a characteristic scale in time or space for initial or boundary problems, respectively. Such spatial or temporal scales are determined by the relaxation micro-processes in the SRR lattice.

In conclusion, we have presented, for the first time to our knowledge, a systematic analysis of nonlinear properties of microstructured materials which display negative refraction, the left-handed metamaterials. We have shown that the composite metamaterials composed of a lattice of wires and split-ring resonators possess an effective magnetic permeability that depends on the intensity of the macroscopic magnetic field in a nontrivial way. The magnetic nonlinearity is found to be much stronger than the nonlinearity in the dielectric properties due to the field enhancement in the split-ring resonators. The dependence of the effective magnetic permeability on the field intensity allows switching between its positive and negative values, i.e. a change of the material properties from left- to right-handed and back. Such processes can be treated as the second-order phase transitions induced by varying the external electromagnetic field.

REFERENCES