Merging and Crossing Modes in Double Negative Metamaterials

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Abstract: We present a theoretical study of the consequences of the presence of a double negative metamaterial in a hybrid transmission line containing ordinary and negative refractive index (NRI) media. The cut-off frequency of propagating modes are found as solutions of a characteristic equation. Contrarily to what happens in ordinary propagation, mode merging and crossing are found versus negative small $\varepsilon$, $\mu$ and $n$ index, supporting low frequency propagation even at low $|n|$. The characterization has been focussed on high azimuthal index modes, where negative refractive index resonators resonating modes can be supported.

In recent years a new branch of physics has been started with the realization of negative refractive media by developing the properties of metamaterials. The most general definition of a metamaterial includes in such category all those composite materials containing elementary cells of dielectrics and conductors. For wavelengths much longer than the lattice spacing, the response is averaged on the cells and the effective $\varepsilon(\omega)$, $\mu(\omega)$ and $n(\omega)$ may realize values not obtainable in nature. Such materials can be designed to behave at a predictable frequency with peculiar characteristics, as negative effective permeability \cite{1} and permittivity \cite{2} materials, so the consequences for the wave propagation, calculated in 1968 by Veselago \cite{3}, differ from ordinary media. Some authors have investigated the quasi-zero \cite{4} and the negative index of refraction of metamaterials \cite{5, 6, 7} in the microwave range, as well as the connection with gyroelectromagnetism suggested by Veselago \cite{3, 8, 9}, in order to realize directive antennas \cite{4}, lenses \cite{10, 11}, filters \cite{12}. A study of crossover wavelength region through the cell spacing size has also been reported \cite{13}. Since the first paper \cite{1}, it has been pointed out that the inversion of $\varepsilon$ and $\mu$ causes the inversion of the sign of the refractive index; so a doubly negative metamaterial exhibits a negative $n$, implying left-handedness. NRI microwave propagation has been studied by many authors like Smith \textit{et al.} \cite{14} to determine the light scattering at an interface, Wu \textit{et al.} \cite{15}, to solve the guidance conditions of modes with both real and imaginary transverse wave numbers inside a doubly negative dielectric slab, Shadrivov \textit{et al.} \cite{16}, to differentiate quantitatively the propagation in a NRI slab between two ordinary media. In this work we study the propagation of light in a core/cladding guiding system when they separately hold negative parameters, referring to real case of metamaterials. The propagation in such system leads to peculiar effects, like transmission for $n \rightarrow 0$ and mode cut-off frequency merging and crossing, even in regime of high azimuthal index slow propagating modes, suitable to support resonant modes in whispering gallery resonators \cite{17}.

Let’s consider a cylindrical symmetry guiding system, where the core has $(\varepsilon_1, \mu_1)$ with the same sign, opposite to that of the cladding $(\varepsilon_2, \mu_2)$. The core has radius $R$, and the system is covered by a metallic wall of radius $R^*$, so the radiation is confined to propagate along the hybrid positive and negative guide. Both the cases where the core and the cladding separately have negative parameters are examined.

The metamaterial parameters can be expressed in function of the frequency $\omega$ as:

$$\varepsilon(\omega) = 1 - \frac{\omega_p}{\omega}, \quad \mu(\omega) = 1 - \frac{\omega_p^2}{\omega^2 - \omega_0^2}$$

where previously published values of $\omega_p/2\pi$ and $\omega_0/2\pi$ typical for known metamaterials are around a ten of GHz, $F$ around $\frac{1}{2}$. In this way both the parameters may be turned to be negative at the same time in the same GHz range \cite{16} and the NRI medium is realized \cite{1}. The numerical method (including the chance to insert gyrotropy) to extract the cut-off frequencies of modes is described elsewhere \cite{8}. For what concerns our discussion, it has to be noticed that this follows by the imposition of interface boundary conditions as a characteristic equation. The most general shape of the fields inside the waveguide expresses as:

$$\Psi = (\Psi_\varepsilon + \Psi_\mu n) \cdot e^{i\omega t} \cdot e^{i\beta z} \cdot e^{il\vartheta}$$

where $\Psi$ stays for $E$ or $H$, $\omega/2\pi$ is the frequency of the e.m. field, $l$ the azimuthal index, $\vartheta$ the azimuthal coordinate, and $\gamma$ is the propagation constant, whose imaginary part allows the propagation.
The mode propagation is analyzed, by extracting the cut-off frequency of the modes. Contrarily to what happens in a line with dielectric having refractive index \( n \) and air-like media (for us \( \varepsilon_p = \mu_p = n_p = 1 \) - where the cut-off frequency of the mode of the line diverges by decreasing the modulus of \( n \to 1^{+} \) - the modes are still supported at ordinary frequency in the whole negative domain even if \( n \to 0^{+} \) and they merge or cross in the \( \nu(\varepsilon < 0)|\mu = \text{const} \) plane, leading to a certain number of cases, displayed in the Fig. 1. The physical meaning is that there are regions of \( n \) where the number of supported modes suddenly changes, always of an even number, that is generally two. This fact is clearly displayed in the Figure 2.

We report two configurations, one with NRI core (Case I) and one with NRI cladding (Case II). Figure 2 shows a guiding system with \( R = 18.45 \) mm, \( R^* = 25.45 \) mm with NRI core or cladding, with fields at high azimuthal mode. In these mode cut-off frequencies charts the value of \( \mu_n \) has been fixed, while the values of \( \varepsilon_n \) (or \( n \)) sweep.

Figure 1. The cut-off frequency of the modes normally behaves continuously by varying the dielectric constant. In NRI media some peculiar effect appears, revealing that this is not generally true. The figures represent the cut-off frequency of modes versus the negative dielectric constant when the permeability is negative. The (a) case concerns the merging of two modes, the case (b) the crossing, the case (c) the chicane configuration of 3 modes and finally the faced merging of two couples of modes butterfly-like (d).

Figure 2. The upper figures represent the cut-off frequency of modes versus the negative dielectric constant when the \( \mu_n = -1 \) and the azimuthal index is \( l = 21 \). The core has \( R = 18.45 \) mm, with a metal wall far 7 mm from it. Lower: it has been taken track of the position of some merging points (c) (3-4) and (d) (1-2) of the Case I, by changing the permeability \( \mu_n \). The positions scales with \( \mu_p \) (left) but the effect is reduced to second order variations if the refractive index of the corresponding points is considered to plot.
Referring to the Case I, one observes the appearance of the mode merging and crossing resumed in Fig. 1. At high \(|\mu_1| = |\mu_2|\) the modes scale smoothly without intersections, but when the value is low enough (\(-3 < \mu < 0\) in the figure) the cut-off-frequencies of different modes follow peculiar behaviors. The same test has been repeated by varying \(\mu_1\) in the Case I, by taking track of the merging and crossing points. Such merging and crossing points on the \(v(\phi)\) chart depend strongly on \(\mu_1\) but are stable in the \(v(n)\) chart, so this effect depends mainly on \(\varepsilon_1\mu_1\). In the Case II the inner and the outer fillings have been exchanged, while all the other parameters remains the same. In this case still the propagation is allowed for all the values of \(\varepsilon_1\mu_2\), but the mode cut-off-frequency collapse by the configuration (a) two by two when \(\varepsilon_1\mu_2 > \varepsilon_1\mu_1\), and start crossing (b) for \(\varepsilon_1\mu_2 < \varepsilon_1\mu_1\). It is important to notice that in both the cases the propagation behaves differently than in ordinary media; this has been found to hold for low azimuthal index too. For the Case I, for instance, lowering the azimuthal index causes the reduction of the merging and mixing zone to lower frequencies and to lower \(|n_a|\) range. These results remind to ones from [16], where for positive media the propagation between an interface is allowed only if \(\varepsilon_p\mu_p < \varepsilon_n\mu_n\), but for NIR the propagation is always possible, even for \(\varepsilon_p\mu_p > \varepsilon_n\mu_n\), so modes can be supported at any value of the refractive index in the NRI medium. These effects confirm the great potential of metamaterials and NRI media in general. Whispering gallery modes in resonators [17] are supported by high azimuthal index modes and exhibit high merit factor. The mode permanency described in the low \(|n_a|\) range could find application in innovative ultra-high merit factor resonators with negative quasi zero index of refraction realized by opportune metamaterials or NRI crystals from the microwave up to the infrared ranges.

REFERENCES

[16] B.V. Shadrivov, A.A. Sukhoyrov, Yuri S. Kivshar, Ph. Rev. E, 67, 057602, 2003

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