NEW DISPERSION CHARACTERISTICS AND SURFACE-WAVE SUPPRESSION IN DOUBLE-NEGATIVE METAMATERIAL GROUNDED SLABS

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Abstract: In this work, the modal properties of a double-negative (DNG) metamaterial layer on a ground plane are extensively investigated. As is well known, such structure may support both ordinary waves (transversely attenuating only in air) and evanescent waves (transversely attenuating also inside the slab). On the basis of a simple graphical analysis of the dispersion equation for TE and TM ordinary and evanescent modes, fundamental conditions are presented which ensure the suppression of a guided-wave regime for both polarizations. Numerical simulations based on an experimentally-tested dispersion model for the permittivity and permeability of the considered DNG medium are reported, which confirm the existence of frequency ranges of surface-wave suppression as predicted by the theoretical analysis.

INTRODUCTION

Waveguiding structures involving metamaterial media have received considerable attention in recent years, due to the surprising propagation features that they may present [Alù and Engheta, 2003; Nefedov et al., 2002; Wu et al., 2003]. In the present paper, we focus our analysis on the modal properties of waves supported by double-negative (DNG) grounded slabs: two different kinds of waves are known to exist in such structures, i.e., ordinary waves (transversely attenuating only in air) and evanescent waves (transversely attenuating also inside the slab). On the basis of a simple graphical analysis of the dispersion equation of TE and TM modes, conditions are presented which ensure suppression of surface waves of both polarization and kind [Baccarelli et al., in press]. The existence of ranges of surface-wave suppression may find application for the realization of antenna substrates with reduced edge-diffraction effects. Numerical results on the dispersion properties of a specific DNG grounded-slab structure are reported here, which confirm the existence of such ranges and illustrate a number of novel and interesting modal characteristics.

STRUCTURE DESCRIPTION AND ANALYSIS

The structure considered here is a grounded slab of height \( h \), made of an ideal linear, stationary, isotropic, homogeneous, lossless metamaterial medium, with permeability \( \mu = \mu_0 \mu \) and permittivity \( \varepsilon = \varepsilon_0 \varepsilon \), both of which are negative (see Fig. 1(a), where the relevant coordinate system is also shown). As is well known, based on a transverse-resonance approach (see Fig. 1(b)), the dispersion equation for both TE and TM modes propagating along \( z \) with propagation constant \( k_z \) is:

\[
Z_{\text{TE,TM}}^0 + j Z_{\text{TE,TM}}^0 \tan (k_z h) = 0
\]

where \( Z_{\text{TE,TM}}^0 \) and \( Z_{\text{TE,TM}}^0 \) are the transverse modal characteristic impedances of a vacuum and of the slab, respectively, while \( k_z = \sqrt{k_z^2 \mu \varepsilon - k_y^2} \) is the transverse wavenumber inside the slab. Proper modes are considered, i.e., transversely attenuating at infinity in the transverse \( y \) direction. By expressing the characteristic impedances as functions of \( k_z \), Eq. (1) can be expressed in terms of adimensional variables: a graphical discussion of the resulting equation may then be performed, by distinguishing the cases of ordinary waves (in which \( k_z \) is purely real) and of evanescent waves (in which \( k_z \) is purely imaginary). In particular, the cases \( \mu, \varepsilon < 1 \) and \( \mu, \varepsilon > 1 \) have to be separately considered; the existence of a modal solution is then related to the existence of one or more intersections between two specific functions depending on polarization and kind of wave [Baccarelli et al., in press].

As a result of the above-described analysis, sufficient conditions may be derived for the absence of proper surface waves of each polarization and kind. In Table I we report such conditions for the case of a DNG medium with \( \mu, \varepsilon < 1 \) (in which ordinary waves cannot exist). In Table II we report conditions for surface-wave suppression for a DNG medium with \( \mu, \varepsilon > 1 \). By examining Tables 1 and 2, the following alternative conditions for the absence of any surface wave can be deduced:

\[
\mu, \varepsilon > 1, \quad |\mu| < 1, \quad |\varepsilon| > 1, \quad h < \frac{c}{2\pi f \sqrt{\mu \varepsilon - 1}} \ \text{min} \left\{ \arctan \frac{1}{|\mu|}, \frac{1}{|\mu| - 1} \right\}
\]

\[
\mu, \varepsilon < 1, \quad |\mu| < 1, \quad |\varepsilon| < 1, \quad h > \arctan |\varepsilon| \left( \frac{c}{2\pi f \sqrt{1 - \mu \varepsilon}} \right)
\]
It is to be noted that, when conditions given in Eqs. (2a) and (2b) are satisfied, even if proper real (surface) waves cannot exist, complex (leaky) waves are allowed to exist. In particular, it can be proved that only complex modes of proper type exist in DNG slabs, as will be clear from the numerical results.

**NUMERICAL RESULTS**

In order to illustrate the possibility to achieve proper surface-wave suppression in DNG grounded slabs, numerical results are presented in this section for the dispersion properties of TE and TM modes of different kinds supported by a specific structure with permeability and permittivity chosen as in Mojahedi et al. [2003]:

\[
\mu = 1 - \frac{\omega^2}{\omega^2 - \omega_0^2}, \quad \varepsilon = 1 - \frac{\omega^2}{\omega^2 - \omega_0^2} \tag{3}
\]

with \(\omega_0 = 21\) GHz, \(\omega_m = 25.4\) GHz, \(\omega_p = 28\) GHz. By observing Fig. 2, it can be seen that the medium is DNG from 21 GHz to 24.5 GHz. In this example, when \(\mu, \varepsilon > 1\), the conditions expressed in Eq. (2a) are never satisfied; when \(\mu, \varepsilon < 1\), the conditions expressed in Eq. (2b) are satisfied in the shaded range of Fig. 2 from 22.82 GHz to 24.5 GHz when the slab height \(h\) is greater than the lower limit represented by the dashed line in Fig. 2. By choosing, for instance, \(h = 5\) mm we have surface-wave suppression in the whole shaded range.

In Figs. 3 and 4 dispersion diagrams are reported for TE and TM modes, respectively, supported by the considered structure in a range from 21 GHz to 25 GHz. In particular, three distinct TE modes are reported in Fig. 3, two of which (those labeled TE\(_2\) and TE\(_3\)) have complex proper branches which extend inside the predicted range of surface-wave suppression with a very high attenuation constant; it is verified that no real proper modes exist inside such range. It can further be observed that these complex branches become improper when the medium is a single-negative metamaterial above 24.5 GHz. In Fig. 4 three TM modes are shown; all the three have complex branches and again no real proper solutions are seen to exist inside the predicted surface-wave-suppression range. The effect of surface-wave suppression on the field excited by finite sources in the presence of a DNG grounded slab of finite extension has been investigated (not shown here for brevity), confirming that the absence of surface waves allows us to eliminate any diffraction ripple in the radiation pattern.

**CONCLUSIONS**

In the present study, conditions are given to achieve surface-wave suppression in DNG grounded slabs, on the basis of a graphical discussion of the relevant dispersion equations. Numerical results on the dispersion properties of TE and TM modes supported by a specific structure confirm the theoretical predictions and demonstrate the existence of complex waves of proper type, which however show a very high attenuation constant. These findings and the investigation on fields radiated by finite sources in the presence of a DNG grounded slab of finite extension seem to be promising in view of possible applications of the considered DNG slabs as planar antenna substrates.

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**REFERENCES**


**Fig. 1** – (a) Metamaterial slab on a perfectly-conducting ground plane. (b) Transverse equivalent network for TE and TM modes of the grounded slab.

**Tab. I** – Summary of conditions for suppression of proper surface waves of different kinds on DNG grounded slabs with $\mu_r \varepsilon_r < 1$ ($b = k_s h \sqrt{\mu_r \varepsilon_r} - 1$). The condition for evanescent TE waves is necessary and sufficient, the condition for evanescent TM waves is only sufficient. Ordinary waves cannot exist in this case.

**Fig. 2** - Relative permeability $\mu_r$ (solid line with circles) and relative permittivity $\varepsilon_r$ (solid line with diamonds) as a function of frequency $f$ for the medium model described by Eq. (3). The product $\mu_r \varepsilon_r$ is shown with a gray solid line. The frequency range in which the conditions in Eq. (2b) are satisfied is represented as a shaded area. The minimum slab height $h$ to have surface-wave suppression in the DNG, calculated according to Eqs. (2b), is reported with a dashed line. Adimensional units (a.u.) are reported on the left vertical axis.

**Tab. II** – Summary of conditions for suppression of proper surface waves of different kinds on DNG grounded slabs with $\mu_r \varepsilon_r > 1$ ($a = k_s h \sqrt{\mu_r \varepsilon_r} - 1$). The condition for evanescent TM waves is necessary and sufficient, the other conditions are only sufficient.

**Fig. 3** – Dispersion curves of the TE$_1$, TE$_2$, TE$_3$ modes supported by a grounded metamaterial slab with slab medium as in Fig. 2 and slab height $h = 5$ mm. The shaded area represents the predicted range of surface-wave suppression for both TE and TM modes.

Legend of Figs. 3 and 4: Normalised phase constants $\beta/k_o$: Solid lines: Proper real ordinary waves; dotted lines: Improper real ordinary waves; light-gray solid line: Proper real evanescent wave; black dashed-dotted lines: Proper complex waves; Gray dashed-dotted lines: Improper complex waves. Normalised attenuation constants $\alpha/k_o$: Gray dashed lines: Proper complex waves; Black dashed lines: Improper complex waves. Thin solid line: $\beta_l = k_s \sqrt{\mu_r \varepsilon_r}$. 

**Fig. 4** - Dispersion curves of the TM$_1$, TM$_2$, TM$_3$ modes supported by a grounded metamaterial slab as in Fig. 3. The shaded area represents the predicted range of surface-wave suppression for both TE and TM modes.