Abstract: This paper presents a Method of Moments (MoM) solution to the large-scale electromagnetic simulations of microstrip reflectarrays. A systematic procedure is outlined for the construction of entire-domain liked characteristic basis functions (CBFs) for each of the array element. Using these CBFs, the MoM matrix size can be substantially reduced as only a few basis functions are required to model the surface current on each element, making full-matrix inversion tractable for this large-scale problem.

INTRODUCTION

Microstrip reflectarrays often employ several thousand radiating elements to compensate for the phase differences in the path lengths so that an incoming plane wave can be focused at the receiving horn [1]-[5], making rigorous analyses difficult when the microstrip patches are modeled with subdomain basis functions. One approach to reducing the problem size is to ignore the mutual coupling effects when dealing with a large number of elements, as for instance in Huang[1] and Chang et al.[2], as though each of the elements were isolated scatterers, leading to discrepancy of a few dBs in the first sidelobe.

Pilz et al.[4] adopted a spectral-domain approach for a microstrip dipole reflectarray with 1,117 elements. It is reported that 2 entire-domain basis functions are sufficient, leading to a full-matrix inversion solution. The method, however, is limited to a few element geometries such as dipoles, and square and circular patches for which entire-domain basis functions are available. For patch element with a tuning stub, one still needs to model the surface current with subdomain basis functions, yielding a large matrix equation that can only be solved iteratively. Zhang et al.[5] employed the CG-FFT method to address this problem in which a uniform discretization of the problem domain is needed. To accurately model the inter-element spacing as well as the dimensions of the individual patches and stubs, a very fine discretization is used. Consequently, for the 900-element example in [5], the total number of unknowns rises to over 400,000, and the number of iteration steps exceeds 6,000. This difficulty of uniform discretization can be obviated via the use of the Sparse-Matrix Canonical Grid (SMCG) method as in Li et al.[6]. However, there is still no guarantee of a speedy convergent solution, in particular, when the matrix is ill-conditioned.

The characteristic basis function (CBF) approach suggested by Mittra et al. [7] provides a means to construct entire-domain liked basis functions from a subdomain discretization. In this paper, we present a systematic approach based on the Foldy-Lax equations for the construction of a different sets of CBFs for rigorous analysis of microstrip reflectarrays. For the examples shown in this paper, the total number of unknowns ranges from 20,000 to 200,000, requiring the inversion of \( N \times N \) matrix with \( N \) from 3,000 to 8,000.

CONSTRUCTION OF THE CHARACTERISTIC BASIS FUNCTIONS

Following the formulation in Li et al.[8] using the mixed-potential integral equation (MPIE), triangular discretization, and fast computation of the spatial-domain Green’s functions, we arrive at the matrix equation shown in (1).

\[
\begin{bmatrix}
Z_{11} & Z_{12} & \cdots & Z_{1M} \\
Z_{21} & Z_{22} & \cdots & Z_{2M} \\
\vdots & \vdots & \ddots & \vdots \\
Z_{M1} & Z_{M2} & \cdots & Z_{MM}
\end{bmatrix}
\begin{bmatrix}
I_1 \\
I_2 \\
\vdots \\
I_M
\end{bmatrix}
=
\begin{bmatrix}
F'_1 \\
F'_2 \\
\vdots \\
F'_M
\end{bmatrix}
\]

(1)

In (1) we have assumed that there are \( M \) elements in the reflectarray and the \( N \times 1 \) vector \( \{I_i\} \) denotes the \( N_i \) unknown weighting coefficients of the triangular basis functions representing the surface current induced on
The equations state that the final exciting field of the \(i\)th scatterer is equal to the incident field plus scattered fields from all scatterers except the scattered field from itself. Using the Foldy-Lax equations, the primary CBF for each scatterer corresponds to the incident field only, and its construction ignores the scattered fields due to all other scatterers, as though the scatterer was isolated. Using the primary CBF alone, as obtained in (2) is identical to the methodology adopted in the analyses of reflectarrays in [1]-[2]. To improve the accuracy of the numerical simulations, we can add secondary CBFs as computed in (3) and (4). The final surface current is the weighted sum of these CBFs shown in (5).

\[
\begin{align*}
[Z_u] \left\{ \tau^p_i \right\} = \left\{ Y_i \right\} & \quad (i = 1, 2, \cdots M) \\
[Z_u] \left\{ \tau^s_i \right\} = - \sum_{j=(i,j) \in M} [Z_g] \left\{ \tau^p_j \right\} & \quad (i = 1, 2, \cdots M) \\
\end{align*}
\]

To compute the weighting coefficients of the CBFs, we follow the same procedure of using entire-domain basis functions. The same set of equations applies to the situation when a cluster of elements replaces each individual element. The final equation to be computed by full-matrix inversion is therefore:

\[
\begin{bmatrix}
\sum_{i=1}^{M} \left\langle \vec{\tau}^p_i, \vec{Z}_u \right\rangle \left\{ \vec{\tau}^p_i \right\} \\
\sum_{i=1}^{M} \left\langle \vec{\tau}^s_i, \vec{Z}_u \right\rangle \left\{ \vec{\tau}^s_i \right\} \\
\end{bmatrix}
\begin{bmatrix}
\sum_{i=1}^{M} \left\langle \vec{\tau}^p_i, \vec{Z}_u \right\rangle \left\{ \vec{\tau}^p_i \right\} \\
\sum_{i=1}^{M} \left\langle \vec{\tau}^s_i, \vec{Z}_u \right\rangle \left\{ \vec{\tau}^s_i \right\} \\
\end{bmatrix}
\begin{bmatrix}
\left\langle \vec{\tau}^p_i, \vec{Y}_i \right\rangle \\
\left\langle \vec{\tau}^s_i, \vec{Y}_i \right\rangle \\
\end{bmatrix}
= 
\begin{bmatrix}
\left\langle \vec{\tau}^p_i, \vec{Y}_i \right\rangle \\
\left\langle \vec{\tau}^s_i, \vec{Y}_i \right\rangle \\
\end{bmatrix}
\]

To reduce computer memory storage and CPU time, the right-hand side of (3) and (4) can be computed efficiently using the SMCG method [7].

**NUMERICAL RESULTS**

Two examples are considered in the paper. First, we compute numerical results for the dipole array depicted in [4]. We then replace the dipole element with the spiral element discussed in [8]. The simulations are computed using a cluster of Intel Pentium III 1GHz processor. The dipole array has 23,676 unknowns. Figure 1 shows the comparison of the far-field patterns using three different CBF approaches against experimental data. Method 1 is computed by the CBF approach with the r.h.s. in (3) and (4) by simple matrix-vector multiplication. For Method 2, the r.h.s. is computed efficiently by SMCG. For Method 3, we treat each 3x3 subarray as an element and the final matrix size can be substantially reduced. Table I shows the comparison of the CPU time using these methods. Table II shows the reduction of the L-2 norm for (1) using the solutions computed from (6). For the spiral array, the number of unknowns rises to 209,625, and both the CPU time and the computer memory are burdened considerably. Figure 2 shows the convergence of the far-field pattern against the number of CBFs. For this complex geometry, more CBFs are required. Table III shows the comparison of the CPU time using different number of CBFs when 16 nodes of the cluster are used. With these results, we have demonstrated that rigorous analysis of microstrip reflectarrays can be achieved via the use of CBFs.

**REFERENCES**


Figure 1. Comparison of far-field patterns of the dipole array using 3 CBF-based schemes against experimental data.

Figure 2. Convergence of the far-field pattern of the spiral array against the number of CBFs.

Table I. Comparison of the CPU time for the dipole array with 3 CBF-based schemes.

<table>
<thead>
<tr>
<th>No. of CBFs</th>
<th>Normalized L-2 norm error (%)</th>
</tr>
</thead>
<tbody>
<tr>
<td>3 CBFs</td>
<td>10.5</td>
</tr>
<tr>
<td>4 CBFs</td>
<td>6.56</td>
</tr>
<tr>
<td>5 CBFs</td>
<td>1.53</td>
</tr>
<tr>
<td>6 CBFs</td>
<td>0.82</td>
</tr>
</tbody>
</table>

Table II. Normalized L-2 norm error for the dipole array with different number of CBFs.

<table>
<thead>
<tr>
<th>No. of CBFs</th>
<th>CPU time in minutes</th>
<th>Final matrix size</th>
</tr>
</thead>
<tbody>
<tr>
<td>3 CBFs</td>
<td>180.79</td>
<td>3351x3351</td>
</tr>
<tr>
<td>4 CBFs</td>
<td>186.87</td>
<td>4468x4468</td>
</tr>
<tr>
<td>5 CBFs</td>
<td>193.31</td>
<td>5585x5585</td>
</tr>
<tr>
<td>6 CBFs</td>
<td>205.63</td>
<td>6702x6702</td>
</tr>
<tr>
<td>7 CBFs</td>
<td>228.83</td>
<td>7819x7819</td>
</tr>
</tbody>
</table>

Table III. CPU time of the spiral array with different number of CBFs.