INCLUDING APERTURES AND CAVITIES IN THE BLT FORMALISM

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Abstract. In the BLT formalism, based on the electromagnetic topology of a complex system, black boxes (junctions) and multiconductor transmission lines (tubes, especially uniform ones) have long been included. This paper discusses some features of including apertures and cavities in the formalism. This involves the definition of appropriate voltages and currents for inclusion in the scattering matrices.

INTRODUCTION

In the quantitative aspects of electromagnetic topology one uses various forms of the BLT equation (organized according to the topology) Baum, et al.[2], Baum[3], Baum[4], Baum[5], and Baum[7]. Fundamental to this is the use of scattering matrices to separate the variables for each topological piece in a form which readily allows the pieces to be incorporated into a description of the entire electromagnetic system. In special cases chain matrices can be used to collapse certain parts of the system into a more compact representation with the results then converted back to appropriate scattering matrices Parmantier, et al.[6]. In its original BLT1 form Baum et al.[2] the special case of uniform multiconductor transmission lines (MTLs) were treated as special “tubes” connecting junctions (general multiport linear and reciprocal “black boxes”) giving (more detail concerning these terms in the references)

$$\left( \left( 1_{n,m} \right)_{u,v} \right) \circ \left( \left( \tilde{S}_{n,m}(s) \right)_{u,v} \right) \circ \left( \left( \tilde{P}_{n,m}(s) \right)_{u,v} \right) = \left( \left( \tilde{S}_{n,m}(s) \right)_{u,v} \right) \circ \left( \left( \tilde{P}_{n}(s) \right)_{u,v} \right)$$ (1)

As discussed in Baum[5] one can redefine any tube as a junction (a multiport “black box”) with sources at the ports. However, increasing the number of junctions in the network increases the size of the supermatrices. For nonuniform MTLs (NMTLs) there is a special form of NBLT equation which preserves the supermatrix size but introduces additional terms into the equation Baum[5]. For completeness, if all tubes are replaced by junctions the BLT2 equation takes the form as above with the delay matrices reduced to identities and the sources reduced to discrete sources at the various ports. There is also a special form, the BLT3, in which the BLT1 is manipulated into a special form using geometric matrix series appropriate for early-time calculations Baum[7]. Practical implementation to date has used the BLT1 form Parmantier et al.[8]. While the electromagnetic topological description is quite general, certain pieces of real systems have not yet been practically included in the calculations. In particular the present paper is intended to discuss some aspects of apertures and cavities as pieces to be included in the BLT formalism.

For cavities with apertures and cables consider the illustration in Figure 1. The cavity might have any three-dimensional shape. It might have any number of apertures, with one shown here with aperture surface $S_a$. There may be any number of cables (single conductors or MTLs) entering or passing through the cavity. Here one is shown penetrating the cavity wall at boundary surfaces $S_{c1}$ and $S_{c2}$. There is also a surface $S_{cc}$ separating the cable from the cavity. It surrounds the cable except for a part consisting of the cavity wall (assumed perfectly conducting) serving as the reference conductor in transmission-line theory. There are situations in which the cable may pass through the cavity away from the wall for which $S_{cc}$ would completely surround the cable. By the electromagnetic uniqueness theorem we need know only appropriate fields on the closed cavity boundary, including the penetrations, to specify the problem. Our goal is appropriately to specify the fields on the aforementioned surfaces in the form of scattering matrices which fit into the BLT formalism.

SCATTERING MATRICES

To construct a scattering matrix we need to define incoming and outgoing waves. As illustrated in Figure 2 we can think of the aperture as being a degenerate case of a cavity with the two sides of the aperture, $S_{a1}$ and $S_{a2}$, coming together as $S_a$. Since it is the tangential components of the fields, $\vec{E}_t$ and $\vec{H}_t$, that must be specified on a surface, we need to think of these in terms of incoming and outgoing waves on both sides of the aperture.

Consider dividing up the aperture into N patches which we take as rectangular for simplicity of discussion as indicated in Figure 3. Other shapes (e.g., triangular) with special basis functions on them may be more useful numerically. The present choice is merely for illustration of the concept. Associated with each patch there are two polarizations, two incoming wave directions, and two outgoing wave directions, giving $8N$
variables to consider. The scattering matrix then relates the 4N incoming waves to the 4N outgoing waves giving a 4N \times 4N matrix. By defining voltages proportional to the tangential electric field (two components) and currents proportional to the magnetic field using the patch dimensions, we can in turn define combined voltage waves in the form

\[ \begin{align*}
\{V_{n,p}^{(\text{out})}\} &= V_{n,p} + Z_{c_{n,p}} I_{n,p} \\
\{V_{\ell,\ell'}^{(\text{out})}\} &= (S_{\ell,\ell'}) \cdot \{V_{\ell,\ell'}^{(\text{in})}\}, \quad (S_{\ell,\ell'}) = \begin{pmatrix}
0 & 1 \\
1 & 0 \\
0 & 1 \\
1 & 0
\end{pmatrix}
\end{align*} \]

where the normalizing impedance \( Z_{c_{n,p}} \) is just \( Z_0 \) times an appropriate ratio of patch dimensions, giving a block-diagonal scattering matrix (4N \times 4N) with determinant +1. Some work has been done concerning an aperture connecting two regions Harrington et al.[9]. In this case Green functions are used to generate admittance matrices for the two regions of the aperture, and then scattering matrices.

Turning our attention to the cable near the wall in Figure 1, let us consider the division of the cable and cavity volumes by a boundary surface \( S_{cc} \). Figure 4 shows a cross section. This boundary can take various shapes, the hemicylindrical one (radius a) being a convenient one. Let us consider what are sometimes
called entire-domain basis functions. For this purpose we can use the form of the electromagnetic fields in cylindrical (Ψ, φ, z) coordinates as found in various texts and papers, e.g., Baum[1]. Without going through the details, the vector wave functions in cylindrical describe the incident and scattered fields outside $S_{cc}$. Letting the radius a be electrically small leads to a set of functions on $S_{cc}$ consisting of exponentials in z times cos(φ) and sin(φ). These are in turn related to source terms in the MTL equations. This in turn goes into integrals of the form (for uniform MTLs)

$$
\tilde{\mathbf{V}}_n(s)\bigg|_u = \int_{L_u} e^{-j\mathbf{\tilde{\gamma}}_{n}(z)} \bigg[ \tilde{\mathbf{V}}(s)\bigg|_{u-z_u} \bigg] d\mathbf{z}_u
$$

Diagonalizing the propagation matrix we have Baum et al.[2]

$$
\tilde{\mathbf{V}}_{n,m}(s)\bigg|_u = \sum_{\beta=1}^{N} \tilde{\mathbf{\gamma}}_{\beta}(s)\bigg[ \tilde{\mathbf{V}}_{\beta}(s)\bigg|_u \bigg] \tilde{\mathbf{\gamma}}_{n,m}(s)\bigg|_u
$$

Then (4.12) splits into $N$ scalar integrals of the form (all analytic)

$$
\tilde{\mathbf{V}}_{\beta}(s) = \int_{0}^{L_u} e^{-j\mathbf{\tilde{\gamma}}_{\beta}(z)} \bigg[ a_{\beta\beta}e^{j\mathbf{\nu}\mathbf{z}u/L_u} + b_{\beta\beta}e^{j\mathbf{\nu}\mathbf{z}u/L_u} \bigg] dz_u
$$

CONCLUDING REMARKS

This paper explores some possible techniques for including apertures and cavities including MTLs as special junctions with appropriate scattering matrices in the BLT formalism. There are various other improvements that can likely be made.

REFERENCES


